Now, for the inhomogenous case.

We imagine the total population can be divided into sub-populations, each of a different line shape.

Eg: Atomic line. The atoms are in motion and their emitted radiation is Doppler shifted and the radiation they "observe" is shifted as well.

First draw

- V<0 -
- V=0 -
- V>0 -

Moving away from you

Eg: Solid state nuclei.

- Different states, defects, etc.

\[ \Delta N_\ell = N_\ell \chi (\nu - \nu_0) \Delta \nu \]

\[ \Delta \nu = \frac{\nu}{\nu_0} \]

With resonance frequency

between \( \nu_0 \) and \( \nu \)

Often a standard line shape around \( \nu_0 \)

Power absorbed just \( \approx \) by the \( \Delta N_\ell \)

\[ \frac{dP}{d\nu} = N_\ell h \nu \int \omega_h (\nu - \nu_0) \chi (\nu - \nu_0) \Delta \nu \]

\( \omega_h \) = \text{Line width}

\[ \frac{dP}{d\nu} = N_\ell h \nu \int \omega_h (\nu - \nu_0) \chi (\nu - \nu_0) \Delta \nu \]

\( \omega_h \) = \text{Line width}

\[ \frac{dP}{d\nu} = N_\ell h \nu \int \omega_h (\nu - \nu_0) \chi (\nu - \nu_0) \Delta \nu \]

\( \omega_h \) = \text{Line width}
For the homogeneous case we had:

\[
\frac{dP}{dv} = N_h \nu W_h
\]

So we write for the inhomogeneous case:

\[
\frac{dP}{dv} = N_k \nu W_{in}
\]

\[
W_{in} = \int w_b (v-v') z^* (v_0-v') dv'
\]

If we define:

\[
\sigma_{in} = \frac{W_{in}}{F}
\]

we get

\[
\sigma_{in} = \int \sigma_b (v-v') z^* (v_0-v') dv'
\]

This is an average cross-section that we can use to characterize the whole system! (For some problems)

Note:

\[
\sigma_1 = \sigma_b (v)
\]

\[
\sigma_{in} = \sigma_{in} (v)
\]

\[
\sigma_{is} = \frac{\sigma_{in}}{\nu}
\]

\[
\sigma_{is} = \frac{\sigma_{in}}{\nu}
\]
Recall, we had:
\[ \sigma_i = \frac{2 \mu_i^2}{3 \hbar \varepsilon_0} \frac{1}{4 \mu_i^2} \gamma_0 (\mu_i - \varepsilon_0) \]

We can write:
\[ \sigma_i \rightarrow \text{total line shift} \]
\[ \sigma_i = \frac{2 \mu_i^2}{3 \hbar \varepsilon_0} \gamma_0 (\mu_i - \varepsilon_0) \]

\[ \gamma_0 = \int_{-\infty}^{\infty} \gamma(x) \gamma(\mu_i - \varepsilon_0 - x) \, dx \]

**Limit case #1**: No inhomogeneous broadening
\[ \gamma = \delta(\varepsilon_0) \]
\[ \gamma_0 = \gamma(\mu_i - \varepsilon_0) \]

**Limit case #2**: Manley-Rowe line
\[ \gamma = \gamma(\mu_i - \varepsilon_0 - x) \]
\[ \gamma_0 = \gamma(x) \]

\[ \Delta \nu \approx \frac{\gamma_0}{2} \]

Due to shift improves all the broadening.

Following the text, from now on, we'll just use:
\[ \sigma_i = \frac{2 \mu_i^2}{3 \hbar \varepsilon_0} \frac{1}{4 \mu_i^2} \gamma_0 (\mu_i - \varepsilon_0) \]

\[ w = \sigma F \]

and not specify line or inhomogeneous explicitly. This's all in \( \gamma_0 \).
So, how does a general 2-level system affect the light?

We have $dF = -\sigma \frac{N_1}{N_2} F d\lambda$

All states in lower state, $N_1 = \# done,$

$dF = +\sigma N_1 F d\lambda$

All states in upper state

Then, $N_2 = \# in upper$

$\lambda F = -\sigma (N_1 - N_2) F d\lambda$

$\lambda > 0$ (assumption coefficient $> 0$

$\frac{dF}{d\lambda} = \lambda F$

$F(\lambda) = F_0 e^{\lambda \lambda}$

$A = \frac{1}{F_0} \lambda$

$\lambda = 2 \pi m (c \# light steps and)$

(recall that $F$ is $\frac{1}{\lambda}$ in $\omega m$)

$N_2 > N_1$ (population inversion)

$\lambda = \sigma (N_2 - N_1) > 0$

$\frac{dF}{d\lambda} = \lambda F$

$F(\lambda) = F_0 e^{\lambda \lambda}$

Of course, once $F$ becomes large enough, its absorption/bapt. color effect ($N_1 - N_2$)

It drives it to zero. No absorption or 50%.

$F = 0 \implies F(\lambda) = 0.$
Einstein Kinodynamic Treatment (non-degenerate)

Even with a greater treatment of light, we can use nature and a little physics to say some things about photon emission.

Suppose we have a blackbody at temp T.

By Jos, it is at equilibrium.
(We'll come until it is.)

We know (Eq 2.2.21)

\[
N(E) = \frac{8\pi E^2}{c^3} \frac{h^2}{\hbar^2 E^2 - 1}
\]

\[\text{Don't Erase!}\]

Now, we have two, stimuli, photon taking place in equilibrium.

Charaterize the poles as:

\[
W_{21}^{}\varphi = B_{21} \varphi_0 \quad \text{stim em, } B_{21} = \varphi(E_0)
\]

\[
W_{1}^{\varphi} = C_1 \varphi_0 \quad \text{abs}
\]

\[
W_{21}^{\mu} = A \quad \text{stim em}
\]

\[
W_{21}^{\mu,\varphi} = \text{nonline}
\]

We have:

\[
B_{21} \varphi_0 N_2 + A N_2 = B_{21} \varphi_0 N_1
\]

However, from the Boltzmann distribution:

\[
N_2 = \frac{N_2}{N_1} = e^{-\frac{B_{21} \varphi_0}{kT}}
\]

Non-degenerate case:

Existence of degenerate case not interesting for us.
\[ u_{z_1} P_{xz} + A \delta x = \delta_{12} P_{x_1} \delta x \]

\[ P = \frac{A}{\delta_{11} e^{\frac{\hbar}{\delta^2}} - \delta_{12}} = \frac{A / \delta_{11}}{\delta_{11} e^{\frac{\hbar}{\delta^2}} - 1} \]

Comparable to \( p(\omega) \) above:

\[ \frac{\partial^2}{\partial \nu^2} = 1 \quad \Rightarrow \quad \delta_{12} = \delta_{21} = 0 \quad \checkmark \]

\[ A = \frac{\pi h d^2 n^3}{c} \]

Text shows, \( \nu \propto 1/m^2 \)

so \( A \propto 1/m^3 \).

A shock transition will suffer large span. emission losses.
\[ P(\omega) = \frac{8\pi \nu^3}{c^3} \frac{1}{\nu - \nu'} \]

\[ P = \frac{A \nu}{\nu^2 \left( \frac{\nu}{c} \right)^2 - 1} \]

\[ \frac{\nu}{c} = 1 \quad \text{and} \quad \frac{c}{c} = 0 \]

\[ A = \frac{8\pi \nu^3 c^3}{c^3} \]

---

Text then goes on to show:

\[ A = \frac{16\pi^2 \nu^3 \omega^4}{31\varepsilon_0 c^2} \]

\[ B = \frac{2\pi^2 \omega^4}{31\varepsilon_0 c^2} \]

---

→ We have the interaction of EM wave time

\[ W_{hr}, W_{ol}, \text{by calculation, } E \to \nu, \gamma \]

A by time \[ \epsilon \to \nu, \gamma \]

not \[ \text{look at form of } S \]

\[ \text{Then } \text{recess some of our approximations: } 2\text{-level system} \]