Putting the rate equations in a convenient form.

$$\frac{dN_2}{dt} = R_p - N_2/\tau - \begin{pmatrix}\text{stim em}\\ \text{rate}\end{pmatrix}$$

$$\equiv R_p - N_2/\tau - B\phi N_2$$

$$\frac{d\phi}{dt} = \begin{pmatrix}\text{stim em}\\ \text{rate}\end{pmatrix} - \begin{pmatrix}\text{loss}\\ \text{rate}\end{pmatrix}$$

$$\equiv B\phi N_2 V_a - \phi/\tau_c$$

Find B in terms of convenient parameters $\left(\Delta I \to \frac{dI}{dt} \to \frac{d\phi}{dt}\right)$

$$\Delta I \equiv I_{RT} - I = I R_1 R_2 (1-L_i)^2 e^{2\sigma N_2 l} - I$$

$$= I\left(e^{2\sigma N_2 l - 2\gamma} - 1\right)$$

Assume exponents $\ll 1$

$$\Delta I \approx 2(\sigma N_2 l - \gamma) I \checkmark$$

Work on Δt next:

$$\Delta t = 2\sum \frac{L_i}{(c/n_i)} = 2\sum \frac{L_i n_i}{c} = \frac{2L_{opt}}{c}$$

$$= 2\left(\frac{L - l + ln}{c}\right)$$

$$= 2\left(\frac{L + (n-1)l}{c}\right)$$

H2

$$\frac{dI}{dt} \approx \frac{\Delta I}{\Delta t} = \frac{2(\sigma N_2 l - \gamma) I}{2 L_{opt}/c} = \frac{(\sigma l N_2 - \gamma) c I}{L_{opt}}$$

$$\frac{d\phi}{dt} = V_a B \phi N_2 - \phi/\tau_c$$

$$V_a B N_2 = \frac{\sigma l}{L_{opt}} c N_2 \Rightarrow \boxed{B = \frac{\sigma l c}{V_a L_{opt}}} \checkmark$$

For a medium w/ uniform cross-section:

$$V_a = l A_a \Rightarrow \frac{l}{V_a} = \frac{1}{A_a}$$

Define "mode volume":

$$V = A_a L_{opt}$$

$$B = \frac{\sigma c}{V}$$

$$-\frac{1}{\tau_c} = -\frac{\gamma c}{L_{opt}} \Rightarrow \boxed{\tau_c = \frac{L_{opt}}{\gamma c}} \checkmark$$

Slope Efficiency

$$\eta_s = \frac{dP_{out}}{dP_p}$$

$$= A_b \, I_s \, \frac{\gamma_2}{2} \, \frac{1}{P_{th}}$$

Find a convenient form for P_{th}

$$R_p = \frac{\eta_p \left(P / h\nu_p \right)}{A\ell}$$

$$R_{pc} = \frac{\eta_p \left(P_{th} / h\nu_p \right)}{A\ell} \quad \text{but} \quad R_{pc} = \frac{\gamma}{\sigma \ell \tau}$$

$$P_{th} = \frac{A\ell \, h\nu_p}{\eta_p} \, \frac{\gamma}{\sigma \ell \tau} = \frac{\gamma}{\eta_p} \, \frac{h\nu_p}{\tau} \, \frac{A}{\sigma}$$

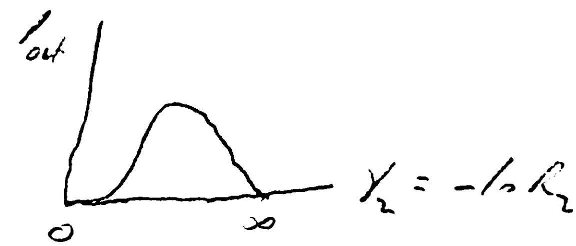
$$\eta_s = \eta_p \left(\frac{\gamma_2}{2\gamma} \right) \left(\frac{h\nu}{h\nu_p} \right) \left(\frac{A_b}{A} \right) \checkmark$$

Optimum Output Coupling

HW

Suppose we only couple out of mirror #2.

$$\left.\begin{array}{l} R_2 = 1 \\ R_2 = 0 \end{array}\right\} P_{out} = 0$$



Operate at $\dfrac{dP_{out}}{d\gamma_2} = 0$

$$P_{out} = A_g I_s \frac{\gamma_2}{2}\left(\frac{P_p}{P_{th}} - 1\right) \quad \text{but } P_{th} = \frac{\gamma h \nu_p A}{\eta_p T \sigma} \quad \left\{\gamma = \gamma_i + \frac{\gamma_1}{2} + \frac{\gamma_2}{2}\right\}$$

Define $P_{mth} \equiv P_{th}$ for the case $R_2 = 1$.

$$= \left(\gamma_i + \frac{\gamma_1}{2}\right)\frac{h\nu_p A}{\eta_p T \sigma}$$

$$P_{th} = P_{mth}\frac{\gamma}{\gamma_i + \frac{\gamma_1}{2}} \Rightarrow \frac{P_{th}}{P_{mth}} = 1 + \frac{\gamma_2/2}{\gamma_i + \gamma_1/2} \equiv S + 1$$

Define $x_m \equiv \dfrac{P_p}{P_{mth}}$

$$P_{out} = A_g I_s \frac{\gamma_2}{2}\left(x \frac{P_{mth}}{P_{th}} - 1\right)$$

$$= A_g I_s \frac{\gamma_2}{2}\left(\frac{\gamma_i + \gamma_1/2}{\gamma_i + \gamma_1/2}\right)\left(\frac{x_m}{S+1} - 1\right)$$

$$= A_g I_s \left(\gamma_i + \gamma_1/2\right) S\left(\frac{x_m}{S+1} - 1\right) \equiv C\, S\left(\frac{x_m}{S+1} - 1\right)$$

HS

$$\frac{dP_{out}}{d\gamma_2} = 0 \rightarrow \frac{dP_{out}}{dS} = 0$$

$$\frac{dP_{out}}{dS} = C\left(\frac{x_m}{S+1} - 1\right) + CS\left(\frac{-x_m}{(S+1)^2}\right) = 0$$

$$x_m(S+1) - (S+1)^2 - x_m S = 0$$

$$(S+1)^2 = x_m$$

$$S_{opt} = \sqrt{x_m} - 1$$

$$\gamma_{2,opt} = (2\gamma_e + \gamma_1)(\sqrt{x_m} - 1) \checkmark$$

$$P_{out,opt} = A_b I_s \left(\gamma_e + \gamma_1/2\right)\left(\sqrt{x_m} - 1\right)^2 \checkmark$$

For $\rho_p = \rho_{xB_h}$ $\Rightarrow$ $\gamma_{2,opt} = 0$

$P_{out,opt} = 0$

H6

Close to threshold:

$$x_n = \frac{P_p}{P_{pth}} = \frac{P_p}{P_{th}}(1+S)$$

Let $P_p = P_{th}(1+\Delta)$

$$x_n = (1+\Delta)(1+S) \approx 1+\Delta+S$$

At optimum coupling:

$$\gamma_{2,opt} = (2\gamma_L + \gamma_1)(\sqrt{x_n}-1)$$

$$= (2\gamma_L + \gamma_1)\,\frac{1}{2}(\Delta + S_{opt})$$

$$S_{opt} = \frac{1}{2}(\Delta + S_{opt})$$

$$S_{opt} = \Delta$$

$$\gamma_{2,opt} = (2\gamma_L + \gamma_1)\,\Delta \checkmark$$

$$P_{opt} = A_s I_s \left(\gamma_L + \frac{\gamma_1}{2}\right)\frac{1}{4}(\Delta + S_{opt})^2$$

$$= A_s I_s \left(\gamma_L + \frac{\gamma_1}{2}\right)\Delta^2 \checkmark$$