Physics 8820 **Homework 2** **Sept. 11**

(1) Starting from the relations derived in the notes for a beamsplitter:

$|R_{31}|^2 + |T_{41}|^2 = |R_{42}|^2 + |T_{32}|^2 = 1 \qquad R_{31}T_{32}^* + T_{41}R_{42}^* = 0$

show that:

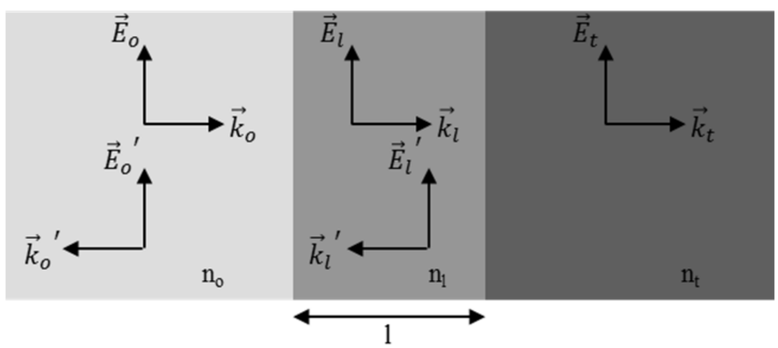
$\varphi_{31} + \varphi_{42} - \varphi_{32} - \varphi_{41} = +/- \pi$, $|R_{31}|/|T_{41}| = |R_{42}|/|T_{32}|$, and $|R_{31}| = |R_{42}|$, $|T_{32}| = |T_{41}|$.

(2) Dielectric thin film (review problem).

Suppose a thin dielectric is sandwiched between two semi-infinite dielectrics with light normally incident from the left, as shown. Using plane waves and dropping the time varying phase, we can chose a convention so that forward going waves are assigned phase factors of e^{+ikx} and backward going waves have e^{-ikx}. Show by matching boundary conditions that:

$$\begin{pmatrix} 1 \\ n_o \end{pmatrix} + \begin{pmatrix} 1 \\ -n_o \end{pmatrix} r = \begin{pmatrix} coskl & -\frac{1}{n_l} sinkl \\ -in_l sinkl & coskl \end{pmatrix} \begin{pmatrix} 1 \\ n_t \end{pmatrix} t$$

where $k = k_l$, $r = E_o'/E_o$ and $t = E_t/E_o$. (In case the font is unclear, k_l has a small L subscript which stands for "layer".) Note that this formulation cleanly separates incident, thin film, and transmitted quantities.



(3) A two color field.

(a) A beam of light has: $E(z,t) = E_1 \exp(ik_1z - i\omega_1t) + E_2 \exp(ik_2z - i\omega_2t)$.
Show that the light is 1st order coherent for all possible pairs of space-time points.

(b) Suppose, in the beam of light treated in (a), that each color suffers random amplitudes and phases such that the average intensities of the two colors are equal. Show that:

$$|g^{(1)}(\tau)| = |\cos[\tfrac{1}{2}(\omega_1 - \omega_2)\tau]|$$

This looks odd compared to our other examples of light with random fluctuations because the coherence does ***not*** go to zero for large τ (ie. large delays in our interferometer). Why?

(4) What is the frequency spectrum of collisionally broadened light?

(5) The lecture notes derive the variance of a single mode in a thermal distribution. In practice, many independent modes are present at the same time and this has a dramatic effect on the statistics. In this problem you'll show that for N modes of similar frequency: $(\Delta n)^2 = \langle n \rangle + \langle n \rangle^2/N$, where $\langle n \rangle$ now means the mean n for the entire distribution: $\langle n \rangle = \Sigma \langle n \rangle_i$, where $\langle n \rangle_i$ is for the i^{th} mode. This is a much smaller variance than for the single mode case.

To help you picture the problem, consider a blackbody source [a cavity with a small hole in it] that is spectrally filtered with an interference filter and directed to a detector some distance away. Interference filters have a narrow bandpass compared the center transmission wavelength.

(a) Let the mean occupation of each mode be $\langle n \rangle_i$ and the actual occupation of all the modes be represented as $\{n_i\} = \{n_1, n_2, \ldots\}$. What is the probability $p(\{n\})$ of being in the state $\{n_i\}$?

(b) Although the various n_i will be different and fluctuating, if only a narrow spectral range is present, the average occupation will be approximately the same across all modes (see the form we derived for $\langle n \rangle$ that has explicit frequency dependence): $\langle n \rangle_i = \langle n \rangle_{i+1} = \ldots$
Write $p(\{n\})$ in terms of the total $\langle n \rangle$.

(c) Find P(n) for this distribution. Note that for n total photons across all modes, there are multiple, equivalent ways to distribute them $\{n_i\}$, and each contributes to P(n). "Equivalent" here means that each $\{n_i\}$ has the same probability p. Remember also that photons are indistinguishable.

(d) Use the factorial moment generating function (or your preferred method) to find the variance.

(e) Suppose for a cavity at 5000 K, the interference filter has a bandwidth of 0.1 nm centered at 500 nm. Find N and the RMS.