Glass behavior of superconducting arrays: Novel finite-size effects

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We consider two-dimensional superconducting arrays in external magnetic fields with regard to the possibility of glass behavior. In particular, we perform Monte Carlo simulations on the incommensurate array, and obtain results supportive of the successive ordering picture suggested previously. Thus, the finite-temperature transition observed in simulations is attributed to novel "finite-size" effects. Relevance to the superconducting glass model for high-T_c superconductors is also discussed.

Recent experiments on two-dimensional (2D) superconducting arrays have revealed novel behavior in external magnetic fields.[1] In the high-capacitance limit, these arrays can be described by frustrated 2D XY models, in which the commensurate-incommensurate effects are reflected in the gauge-invariant frustration f ($=\Phi/\Phi_0$, the flux per plaquette in units of the flux quantum). The Hamiltonian for such frustrated XY models is given by

$$H=-J\sum_{i,j}\cos(\theta_i-\theta_j-A_{ij}) , \quad (1)$$

where J is the interaction energy, θ_i is the phase of the superconducting order parameter of the grain at site i, and A_{ij} is the bond angle given by the line integral of the vector potential $\mathbf{A}$. The plaquette sum of A_{ij} is equal to $2\pi f$. A number of studies on these models then reveal a wide variety of critical behavior as the frustration f is varied $(0\leq f\leq 1)$.[2-8] It is implied that the critical behavior, as well as the symmetries of the system, are highly discontinuous functions of f.

Among systems with various types of the frustration, there are two kinds of systems of particular interest, with regard to the possibility of glass behavior. One is the incommensurate system where f is an irrational; the other is the disordered system, i.e., arrays with quenched positional disorder. The possibility of a glass state in such systems has attracted much interest since it may be of relevance to the glass behavior of high-T_c superconductors observed very recently.[9] There is an attempt to explain those superconducting glass properties in terms of a model for granular superconductors or superconducting arrays with positional disorder.[10,11]

Such arrays with positional disorder have also attracted interest since any artificially fabricated samples should possess a certain amount of disorder in the positions of the superconducting grains. It has been suggested in recent Monte Carlo simulations that strongly disordered arrays indeed display glass behavior at finite temperatures.[12] Renormalization-group calculations,[13] however, have shown that the effective measure of disorder is given by the product of f and the amount of positional disorder, and that strong disorder in general destroys the phase transition while weak disorder preserves the transition, consistent with experiments.[14] Thus, renormalization-group calculations do not lead to any evidence for a glass state at finite temperatures. At zero temperature, on the other hand, the Edward-Anderson order parameter acquires long-range order while the spins still have algebraic order. This feature persists for strong disorder, which destroys any type of order at finite temperatures. Therefore, arrays with strong positional disorder might be considered to exhibit a zero-temperature glass transition, which possibly has some relevance to the apparent glass behavior observed in the simulations of finite-size systems.

For the incommensurate arrays, on the other hand, there are seemingly contradictory results so far: There are arguments leading to the absence of a finite-temperature transition,[3,5] while numerical simulations seem to favor the existence of a glass state at finite temperatures.[7] In fact, the interesting possibility of a glass state was also indicated in the analytical argument.[5] A recent analysis of the proposed glass state, with emphasis on the size of the system, suggested that the ordering in this state involves a series of "quasitransitions" into ordering at larger and larger length scales at successively lower temperatures.[8] It was also suggested that this glass transition temperature becomes arbitrarily low as the system size grows, implying a zero-temperature glass transition in the thermodynamic limit.[15] Therefore, according to this interpretation, the apparent finite-temperature glass transition observed in simulations is not inherent in the incommensurate array, but simply due to the novel "finite-size effects."

To investigate these effects, we have performed Monte Carlo simulations on the incommensurate array with irrational frustration, $f=1-\tau$, where $\tau\equiv(\sqrt{5}-1)/2$ is the golden number. In the simulations performed via the standard Metropolis algorithm, the specific heat per site C

was calculated using the fluctuation-dissipation relation and the helicity modulus γ, which is a measure of the stiffness of spins, was computed as in Ref. 13(b). Sample sizes of $N = L\times L$ with $L = 13$, 21, 34, 55, and 377 were used with periodic boundary conditions. As pointed out in Ref. 7, the use of periodic boundary conditions in the study of incommensurate systems constrains boundary plaquettes causing them to have the value of f different from that in the bulk. For those sample sizes used in these simulations, the difference is minimized and presumably does not affect the overall results. We cooled from a temperature $T/J = 1.0$ (Boltzmann constant exactly equal to 1), and performed runs of 10000 to 20000 Monte Carlo steps per site. As judged from the behavior of the energy, 10000 Monte Carlo steps seemed sufficient to attain the equilibrium. It should also be noted that the results for small samples ($L \leq 55$) are essentially identical to those obtained in much longer runs.[7]

The results for the helicity modulus (average of the x component and the y component) of small samples are shown in Fig. 1(a). Commensurate systems, in general, display discontinuous jumps in the helicity moduli due to the Kosterlitz-Thouless character of the transitions, although the sizes of the jumps may vary according to the value of f. [While the unfrustrated system ($f=0$) shows the universal jump, the fully frustrated system ($f=\frac{1}{2}$) is believed to exhibit a jump greater than the universal one.[6]] In sharp contrast, Fig. 1(a) shows neither appreciable size dependence nor definite evidence for the jump (except for the smallest sample). In particular, the helicity modulus seems to display an almost convex increase without an inflection point as the temperature is lowered, which might be regarded as a signal of a glass state.[12] However, it should be noted that such convex behavior of the helicity modulus observed in a finite-sized sample can also imply the absence of a finite-temperature transition.[16] Therefore, neither interpretation can be ruled out without detailed consideration of the finite-size effects. The results for the specific heat of the same small samples are shown in Fig. 1(b). A peak in the specific heat can be seen around $T/J \approx 0.2$, which is insensitive to the cooling rate. This behavior of the specific heat is in agreement with the previous data, which were interpreted as an indication of a finite-temperature glass transition.[7]

The corresponding results for the large sample ($L = 377$) are shown in Fig. 2. In this case the statistical fluctuations were rather small except in the "critical region" (around the peak in the specific heat), and the averages were taken over three independent runs. It is seen in Fig. 2(a) that the behavior of the helicity modulus has not

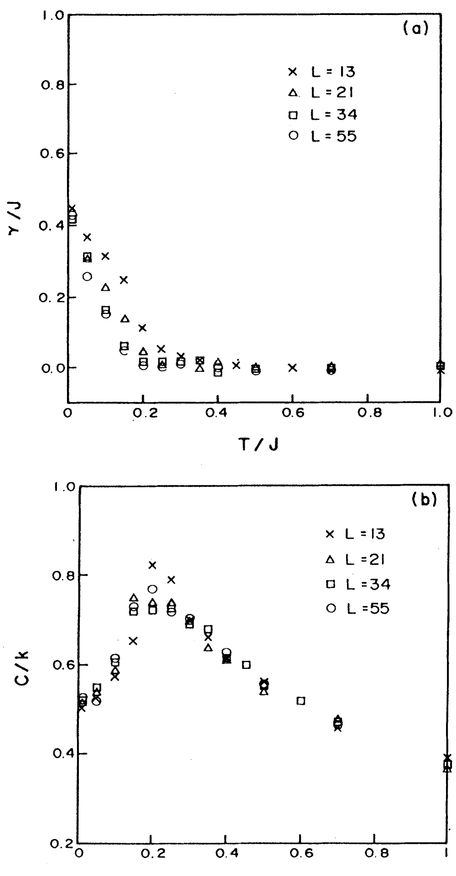

FIG. 1. Results of simulations for (a) helicity modulus and (b) specific heat of small samples ($L = 13$, 21, 34, and 55).

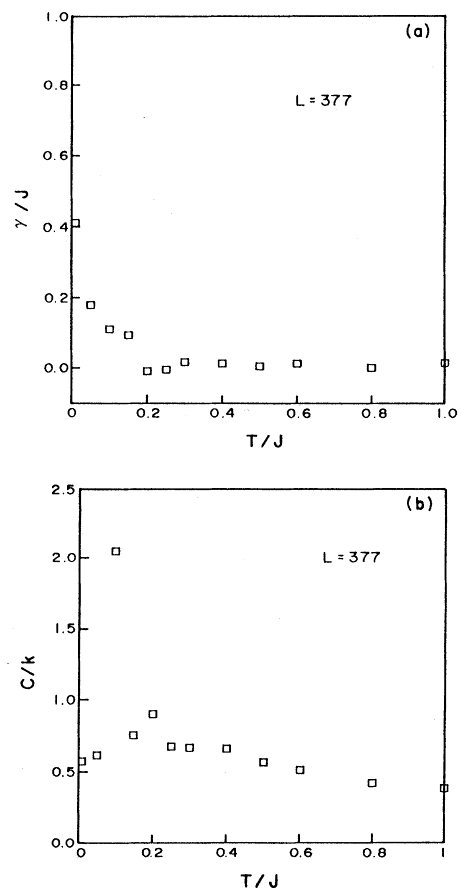

FIG. 2. Results of simulations for (a) helicity modulus and (b) specific heat of the large sample ($L = 377$).

changed qualitatively from the corresponding behavior in small samples. The specific heat, on the other hand, exhibits a striking difference, as shown in Fig. 2(b). It displays a sharp peak around $T/J \approx 0.1$ in addition to the relatively small peak around $T/J \approx 0.2$; the latter is the only peak observed in small samples. Except at the peak, the size of error bars estimated from the rms deviations is about the size of the symbols in the plot; it is rather large (≈ 0.5) for the peak. However, the existence of the sharp peak is evident at least in the cooling rate used. Thus, the transition temperature judged from the specific-heat peak seems to become lower with the system size, although there still exists the remnant of the higher-temperature transition observed in small samples. This apparently supports the successive ordering picture of the incommensurate system,[8] according to which the peaks around $T/J \approx 0.2$ and 0.1 are associated with the transitions in the commensurate systems with the frustration f given by appropriate rational approximants of the irrational value $1-\tau$. This picture is also consistent with the fact that the helicity modulus of the smallest sample exhibits the most apparent jump, perhaps at the transition temperature of the commensurate system with, for example, $f=\frac{2}{5}$.

Although these qualitative features were not sensitive to the available cooling rate, it is expected that the heights of the peaks decrease as we slow down the cooling rate sufficiently. In this case, there will be no true equilibrium transition, and the proposed glass transition should have a metastable effect. However, the relaxation time presumably becomes quite long, as for conventional glasses.[17] Then the two interpretations—glass behavior at finite temperatures and absence of a finite-temperature (equilibrium) transition—can be regarded as complementary to each other. Extensive Monte Carlo simulations with careful consideration of the finite-size effects are clearly needed to confirm this observation.

With regard to high-T_c superconductors, it should be noted that single crystals as well as polycrystalline samples have displayed similar glass behavior. Although single-crystal samples may have some defects, the "effective plaquette" area is expected to be very small due to the extremely short coherence length, implying that f is very small (around 10^{-4}) up to moderate strength of the external magnetic field. Hence, the effective disorder in single-crystal samples is presumably very weak; renormalization-group calculations, numerical simulations, and experiments are all in agreement on the irrelevance of such weak disorder, supporting the view that the origin of such glass behavior should not be attributed to disorder.[18] This observation naturally suggests as an alternative the incommensurate system, which should prevail in experiments since the rational numbers only form a set of measure zero in the interval (0,1).

In conclusion, we have considered a superconducting array with irrational frustration with regard to the possibility of glass behavior. Monte Carlo simulations have been performed on samples of various sizes, leading to results suggestive of the successive ordering picture. Thus, the glass transition temperature is expected to become lower as the system size grows, and the finite-temperature transition observed in simulations is presumably due to the novel finite-size effects. Real as well as numerical experiments on (finite-size) samples might then yield data suggestive of phase transitions in such incommensurate systems where statistical mechanics (of infinite systems) predicts no finite-temperature transition. This implies that the thermodynamic limit, which is taken as mathematical idealization, might be "unphysical" in the practical sense. It is also of interest to note that a touch of this should be widely applicable to experiments where the magnetic field or the frustration cannot be adjusted with infinite precision. In particular, high-order commensurate systems, i.e., arrays with rational $f=m/n$ (m and n relatively prime) but n very large, will display essentially the same behavior. Within the superconducting glass model, this seems to be a natural explanation of the glass behavior of high-T_c superconductors, which presumably have very small values of f.

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[1]R. A. Webb, R. F. Voss, G. Grinstein, and P. M. Horn, Phys. Rev. Lett. **51**, 690 (1983); Ch. Leeman, Ph. Lerch, G.-A. Racine, and P. Martinoli, *ibid.* **56**, 1291 (1986); R. K. Brown and J. C. Garland, Phys. Rev. B **33**, 7827 (1986).

[2]S. Teitel and C. Jayaprakash, Phys. Rev. B **27**, 598 (1983); W. Y. Shih and D. Stroud, *ibid.* **28**, 6575 (1983); **30**, 6774 (1984).

[3]S. Teitel and C. Jayaprakash, Phys. Rev. Lett. **51**, 1999 (1983).

[4]M. Y. Choi and S. Doniach, Phys. Rev. B **31**, 4516 (1985).

[5]M. Y. Choi and D. Stroud, Phys. Rev. B **32**, 7532 (1985).

[6]M. Yosefin and E. Domany, Phys. Rev. B **32**, 1778 (1985); M. Y. Choi and D. Stroud *ibid.* **32**, 5773 (1985).

[7]T. C. Halsey, Phys. Rev. Lett. **55**, 1017 (1985).

[8]M. Y. Choi and D. Stroud, Phys. Rev. B **35**, 7109 (1987).

[9]K. A. Müller, M. Takashige, and J. G. Bednorz, Phys. Rev. Lett. **58**, 1143 (1987); R. S. Razavi, F. P. Koffyberg, and B. Mitrovic, Phys. Rev. B **35**, 5323 (1987); A. C. Mota *et al.*, *ibid.* **36**, 4011 (1987); M. Tuominen, A. M. Goldman, and M. L. Mecartney, *ibid.* **37**, 548 (1988); C. Rossel, Y. Maeno, and I. Morgenstern, Phys. Rev. Lett. **62**, 681 (1989).

[10]G. Deutscher and K. A. Müller, Phys. Rev. Lett. **59**, 1745 (1987); I. Morgenstern, K. A. Müller, and J. G. Bednorz, Z. Phys. B **69**, 33 (1987); D. Stroud and C. Ebner, Physica C **153–155**, 63 (1988).

[11]There is another interpretation known as flux creep, and some controversy exists between these two types of interpretation. See, e.g., Y. Yeshurun and A. P. Malozemoff, Phys. Rev. Lett. **60**, 2202 (1988). The results of superconducting arrays presented in this paper are also applicable to high-T_c superconductors only if the superconducting glass interpretation is correct.

[12]A. Chakrabarti and C. Dasgupta, Phys. Rev. B **37**, 7557 (1988).

[13](a) E. Granato and J. M. Kosterlitz, Phys. Rev. B **33**, 6533 (1986); (b) M. Y. Choi, J. S. Chung, and D. Stroud, *ibid.* **35**, 1669 (1987).

[14]M. G. Forrester, H. J. Lee, M. Tinkham, and C. J. Lobb, Phys. Rev. B **37**, 5966 (1988).

[15]In this respect, disordered arrays and incommensurate arrays may be considered to display similar behavior.

[16]For example, we have obtained similar convex behavior in simulations of a one-dimensional XY model which clearly has no finite-temperature transition.

[17]See, e.g., *Heidelberg Colloquium on Glassy Dynamics,* edited by J. L. van Hemmen and I. Morgenstern, Lecture Notes in Physics, Vol. 275 (Springer-Verlag, Berlin 1987).

[18]Defects like twin boundaries can be regarded as correlated disorder. Such disorder with long-range correlations has been considered to exhibit metastable properties similar to those observed in high-T_c superconductors. See J. Choi and J. V. José, Phys. Rev. Lett. **62**, 320 (1989). On the other hand, such correlated disorder has also been argued to destroy the (equilibrium) transition, suggesting that the apparent finite-temperature transition displayed in the numerical simulations is possibly due to finite-size effects. See M. Y. Choi, in *Progress of Statistical Mechanics,* edited by C. K. Hu (World-Scientific, Singapore, 1988), p. 385; M. Y. Choi (unpublished).