Effect of Random Disorder on the Critical Behavior of Josephson Junction Arrays

D. C. Harris, S. T. Herbert, D. Stroud, and J. C. Garland
Department of Physics, Ohio State University, Columbus, Ohio 43210
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Random percolative disorder has been introduced into 300×300 arrays of Nb-Au-Nb proximity-coupled junctions. Our measurements of dc transport properties show that large amounts of random disorder, although depressing T_c and broadening the resistive transition, do not alter the scale invariance of the phase transition. These results are described by a model which rescales the Josephson lattice by the percolation correlation length. The relevance of the observations to granular thin films is discussed.

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In this Letter, we explore the relationship between percolative disorder and the two-dimensional (2D) phase transition in proximity-coupled superconducting arrays. Such arrays are known to provide a near-ideal implementation of the X-Y model. Our findings—that large amounts of geometric disorder depress T_c and broaden the resistive transition, and yet do not otherwise modify the essential features of the phase transition—represent the first experimental confirmation of the persistence of the phase transition in a strongly disordered 2D X-Y model. Further, our results provide useful insight into the mechanism by which superconductivity is suppressed in high sheet resistance granular films. Our results illustrate the role that percolation can play in such films near the localization threshold, and show that the deterioration of the phase transition in this regime can be explained by rescaling the Kosterlitz-Thouless (KT) equations by a characteristic percolation correlation length. Other recent work has examined the role of disorder in the plaquette area of proximity-coupled arrays [1], and the superconducting-normal phase boundary $T_c(H)$ in bond-diluted wire networks [2], and did not consider the KT transition.

Our measurements were performed on a series of disordered proximity-coupled arrays consisting of Nb crosses arranged on a 300×300 square lattice decorating a Au film; percolative disorder was introduced by randomly removing crosses from the lattice. Several series of samples were fabricated, each consisting of arrays with 100%, 90%, 80%, 70%, and 60% of the Nb sites filled. For all samples, the cross arms were 1.2 μm wide, with a gap between adjacent sites of 0.4 μm. The lattice constant a_0 was 10 μm (see Fig. 1, inset). The same random number seed was used to generate samples within a given series, so that, for example, the 70% sample incorporated the same topography as the 60% sample, except that an additional 10% of the sites were filled. The critical percolation fraction p_c was empirically determined for each random number seed as that fraction of filled sites below which no connected path of junctions spanned the current electrodes of the sample. For the series of samples discussed here, $p_c = 0.5847$, in close agreement with the theoretical value of 0.593 for an infinite 2D square resistor network [3].

dc transport measurements were taken by applying a square-wave excitation current ($f = 23$ Hz) and synchronously detecting the voltage with a transformer coupled lock-in amplifier. Digital signal averaging yielded a noise floor below 0.3 nV; the temperature was controlled to ± 0.5 mK and the ambient magnetic field was < 3 mOe.

Figure 1 shows the resistive transition for five arrays with filled sites spanning 60%–100%. In the figure, the resistance of each sample is normalized to its value at $T = 8.85$ K, which was just below the Nb transition temperature. It is evident that the transition of the arrays to a zero-resistance state broadens appreciably with increasing site disorder, resulting in a value of T_c that decreases as the percolation threshold is approached from above.

The actual value of T_c is model dependent. In analyzing our data, we have defined T_c to be the temperature at which the exponent of the array current-voltage (I-V)

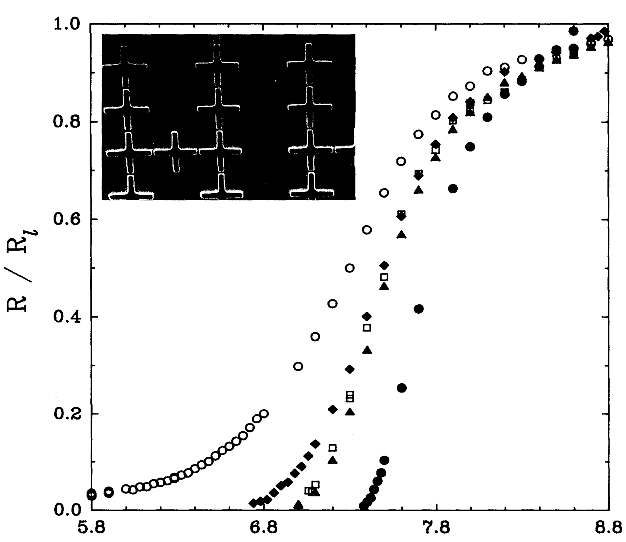

FIG. 1. Normalized resistive transition of the 100% (●), 90% (▲), 80% (□), 70% (◆), and 60% (○) arrays with a measuring current of 1 μA. R_l is the resistance at $T = 8.85$ K. Inset: A portion of a disordered array.

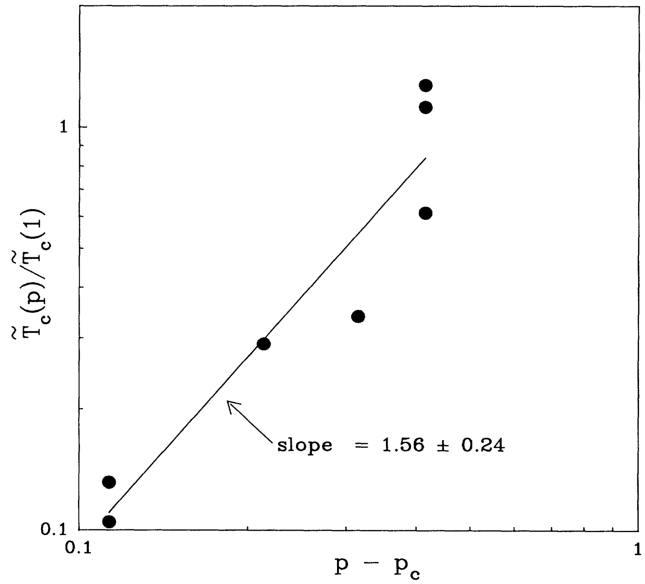

FIG. 2. Reduced phase transition temperature (see text) as a function of sample disorder, $p-p_c$. $\tilde{T}_c(1)$ is the average of three $p=1$ samples. The two $p=0.7$ samples were fabricated using different random number seeds.

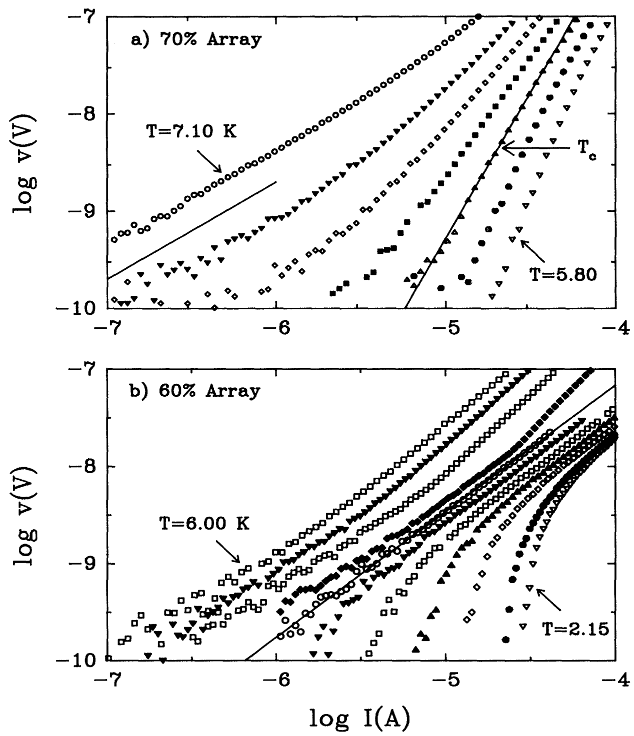

FIG. 3. Current-voltage characteristics. (a) 70% array: lines have slopes of 1 and 3, and unlabeled curves correspond to $T=6.80$, 6.60, 6.40, 6.20, and 6.00 K, respectively. (b) 60% array: the line has a slope of 1.29, and unlabeled curves correspond to $T=5.60$, 5.20, 4.40, 4.00, 3.60, 3.20, 2.91, 2.59, and 2.30 K, respectively.

characteristics $a(T)=3$. This choice anticipates our later finding that the phase transition of the arrays mimics a KT transition, even in the presence of appreciable site disorder. With this convention, T_c is suppressed from 7.26 K for the 100% array to 6.20 K for the 70% array. We could not extract T_c for the 60% array because that specimen did not display power-law behavior.

Figure 2 illustrates the depression of T_c with increasing disorder for the 70%–100% arrays. Note that T_c is plotted as a reduced temperature $\tilde{T}_c = k_B T_c/(\hbar/2e) i_c(T_c)$, after Lobb, Abraham, and Tinkham [4], in order to scale out the temperature dependence of the Josephson critical current $i_c(T)$ for an individual junction. Although there is appreciable scatter, the reduced transition temperature varies as a power law of $p-p_c$, with an exponent of 1.56 ± 0.24. Here, the critical current $i_c(T)$ was measured independently for three 100% samples. For $T \ll T_c$, the critical current was found to follow the de Gennes expression for a superconductor–normal-metal–superconductor junction in the dirty limit, $i_c(T) = i_c(0)(1-T/T_{c0})^2 \exp(-\alpha T^{1/2})$, where T_{c0} is the BCS transition temperature. This expression was then used for $i_c(T_c)$ in the 100% arrays as well as for the disordered arrays. We note that it is not possible to measure $i_c(T_c)$ directly because strong thermal fluctuations near T_c mask the individual junction behavior.

Several I-V traces of the 70% sample are shown in Fig. 3(a). The qualitative features of these curves are the same as those of the 80%, 90% and 100% samples: a linear slope in the low-current limit for curves above T_c, an abrupt increase to a cubic slope at T_c, and progressively higher power laws as the temperature is lowered below T_c. These features are generally associated with the KT transition and have been previously observed in fully ordered proximity-coupled arrays and in a host of 2D films [5,6]. Our results suggest that this transition also exists in arrays having a large degree of percolative disorder, and that the main effect of the disorder is to depress the vortex unbinding temperature and broaden the resistive transition.

This finding is reinforced by Fig. 4. Figure 4(a) shows the temperature dependence of the exponent $a(T)$ of the I-V characteristics for various degrees of disorder. It is evident that the 100% specimen shows a step increase from 1 to 3 that is in close accord with the universal jump discontinuity expected for a KT transition. A similar jump is observed for the disordered arrays, although the discontinuity is not so pronounced. In addition, the linear growth in $a(T)$ for $T < T_c$ is less rapid than for the 100% sample. This result is a natural consequence of the broadened transition. It is known that the linear dependence of $a(T)$ below T_c extrapolates to T_{c0} [6]. As T_c is depressed farther below T_{c0} with increasing disorder, the slope in $a(T)$ must decrease accordingly to reflect this behavior.

Figure 4(b) plots eighty I-V characteristics of the

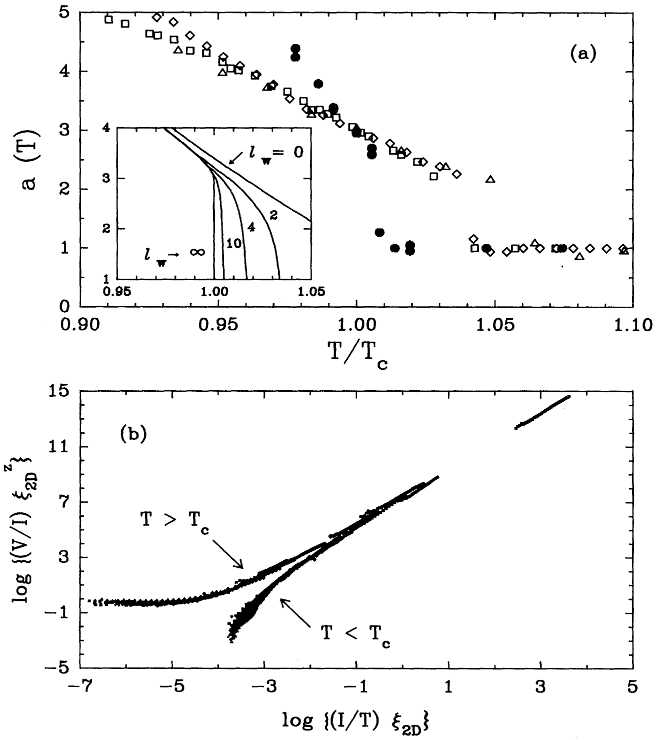

FIG. 4. (a) I-V exponent $a(T)$ vs temperature for the 100% (●), 90% (□), 80% (◇), and 70% (△) arrays. Inset: The calculated $a(T)$ vs T/T_c as a function of l_w (after Ref. [4]). (b) Condensation of I-V curves for the 70%–100% arrays, plotted in reduced variables z and ξ_{2D} (see text and Ref. [6]) to illustrate universal scaling (data for each array is shown for twenty different temperatures).

70%–100% samples in reduced coordinates, in order to illustrate universal 2D scaling. As discussed by Fisher [7], and Koch *et al.* [8], one signature of a phase transition is a universal condensation of the I-V characteristics onto two branches, each characteristic of one side of the transition region. In two dimensions, the reduced variables are $z=2$, the dynamical exponent associated with the coherence time [8] ξ^z, and the coherence length [6] $\xi_{2D} \sim \exp(C^{\pm} T_c/|T-T_c|^{-1/2})$. For our arrays, z and $C^{\pm}$ were treated as fitting parameters and were adjusted to give the best collapse of the data onto a universal curve. The optimal fit occurred with $z=2.0\pm0.1$ for all I-V curves for all samples, in agreement with theory. The value of $C^{\pm}$ (a nonuniversal constant expected to be of order 1) for each sample is given in Table I.

The I-V characteristics of the 60% sample are shown in Fig. 3(b) and do not show KT behavior. For this sample, power-law behavior was observed only at $T=4.00$ K (shown as a solid line in the figure), with an exponent of 1.29 ± 0.01. Below that temperature, the curves show a downward curvature associated with the development of an array critical current.

All of these results suggest that percolative disorder can greatly suppress T_c and broaden the resistive transition, without changing the scaling invariance of the 2D phase transition. This conclusion is valid over nearly the entire conducting range above p_c; very close to p_c, however, the phase transition disappears and is replaced by the formation of a critical current, analogous to that observed in single Josephson junctions.

TABLE I. Summary of array properties.

Array	100%	90%	80%	70%	60%
p	1.000	0.8997	0.7984	0.6979	0.5890
$\xi_p(a_0)$	1.00	1.47	2.47	5.75	575
l_w	5.70	5.31	4.80	4.50	0
T_c (K)	7.26	6.81	6.64	6.20	···
C^+	0.17	0.25	0.3	0.4	···
C^-	0.35	0.4	0.4	0.45	···

We believe the suppression of T_c in this system can be attributed to a disorder-induced reduction in the effective superfluid density $n_s(T,p)$ on the lattice. Ebner and Stroud [9] have shown analytically that $n_s(T,p)$ for a site-diluted Josephson lattice at $T=0$ is proportional to the effective lattice conductance in the normal state, and that both vary as a simple power law near the percolation threshold, i.e., $n_s(0,p)/n_s(0,1)=\sigma(p)/\sigma(1)\sim(p-p_c)^t$, where σ is the normal-state lattice conductance, and $t=1.30$ in 2D [10]. For a KT transition, $n_s(T_c)$ is related to T_c by the universal jump condition [6] $n_s(T_c)/T_c = 8k_Bm/\pi\hbar^2$. We propose a generalization of this relationship to site-disordered lattices having $p>p_c$:

$$n_s(T_c(p),p)/T_c(p)=8k_Bm/\pi\hbar^2 . \quad (1)$$

The plausibility of Eq. (1) is suggested by our observation, shown in Fig. 4(b), that highly disordered arrays continue to obey the scaling invariance of the KT transition. This observation also permits us to conjecture that the fractional reduction in superfluid density is independent of concentration, i.e.,

$$n_s(T_c(p),p)/n_s(0,p)=n_s(T_c(1),1)/n_s(0,1) . \quad (2)$$

This assumption, with Eq. (1), leads immediately to the relation

$$T_c(p)/T_c(1)\sim(p-p_c)^t , \quad (3)$$

whose predicted power-law behavior compares favorably with the data of Fig. 2 [11].

Equation (3) also suggests a relationship between $T_c(p)$ and the percolation correlation length $\xi_p(p)\propto(p-p_c)^{-\nu}$, where $\nu=\frac{4}{3}$ in 2D [12]. Rewriting Eq. (3) in terms of this parameter, we have $T_c(p)/T_c(1)\sim\xi_p^{-t/\nu}\approx\xi_p^{-1}$, which indicates that the reduction of T_c from its fully ordered value can be viewed as a direct measure of percolative disorder in the arrays.

Physically, $\xi_p(p)$ is the average size of the "holes" in the infinite cluster, and this observation suggests a mechanism for the broadening of the transition that accompanies the depression of T_c. de Gennes [13] and others [14] have shown that it is possible to rescale a percolating network by ξ_p to obtain critical exponents and other network properties. In the familiar "nodes-links-blobs" model [13], ξ_p is taken as the average distance between nodes in the rescaled lattice. Here we propose that percolative disorder also rescales the dimensionless length parameter $l_{\rm KT}=\ln(r/a_0)$ of the KT transition to become $l_{\rm KT}=\ln(r/\xi_p)$, which reduces to the conventional expression for $p \rightarrow 1$. This suggestion means that the phase transition is dominated by weakly bound vortex-antivortex excitations whose cores are centered on a disordered superlattice of holes in the infinite cluster. These excitations are weakly bound and thus dissociate at lower temperatures than the more strongly bound excitations centered on the unit cell of the underlying lattice.

An important consequence of this model is that size-effect broadening of the transition becomes increasingly important as array disorder increases. In fully ordered arrays, the broadening is governed by the size parameter $l_w=\ln(w/a_0)=5.7$, where $w=Na_0$ is the width of the array. In disordered samples, $l_w(p)=\ln[w/\xi_p(p)]$ and decreases to 4.5 for our 70% array. A comparison of the data of Fig. 4(a) with the calculations of Kadin, Epstein, and Goldman [6], shown in the inset, show that the observed broadening of the transition is in reasonable accord with this model. A sufficiently large amount of disorder will, of course, destroy the transition completely on a finite lattice. This crossover point will be reached when $\xi_p=w$, which for our arrays corresponds to $p=0.5905$; we believe this explanation accounts for the absence of KT behavior in our 60% sample. Table I lists the values of p, ξ_p, and l_w for the different samples. Although the models belong to different universality classes, we note that our analysis—which pertains to a disordered X-Y model—predicts qualitatively similar behavior to that expected for the disordered 2D Ising model considered by Harris [15].

We conclude with a comment on the applicability of our array results to the phase transition in 2D superconducting films. As first noted by Beasley, Mooij, and Orlando (BMO) [16] for granular Al/Al_2O_3, T_c of thin films is depressed as R_n nears the critical value $R_c=\hbar/e^2=4.1\ {\rm k}\Omega/\Box$. This depression, which is accompanied by a broadening of the universal jump in $a(T)$, is described by the expression $T_c/T_{c0}\approx(1+0.17R_n/R_c)^{-1}$, valid for BCS superconductors in the dirty limit. Our results suggest that one possible mechanism underlying this mean-field expression may be percolative disorder.

Specifically, we model an inhomogeneous 2D film as a random arrangement of Josephson-coupled small grains. Such a model is reasonable since inhomogeneities from various materials processing factors are likely to increase as R_n increases towards R_c [17]. For simplicity, we assume that the grains are arranged on a site-diluted square lattice with coupling energy $J(T)$, which near T_{c0} for a superconductor-insulator-superconductor array is given by [18] $J(T)=(23.1k_BT_{c0}/2\pi)(1-T/T_{c0})R_c/R$, where R is the intergrain resistance when the junction is in its normal state. Using Eq. (3), we may write $k_BT_c/J(T_c(p))=\alpha[(p-p_c)/(1-p_c]^t$, where α (≈ 0.95 on a square lattice [11]) is independent of p. Combining these relations, and using [9] $R_n/R=[(p-p_c)/(1-p_c)]^{-t}$, we obtain the BMO relation with a slightly different numerical coefficient $T_c/T_{c0}\approx(1+0.29R_n/R_c)^{-1}$. Thus the depression of T_c implied by the BMO relation follows naturally from a simple percolation model for an inhomogeneous thin film.

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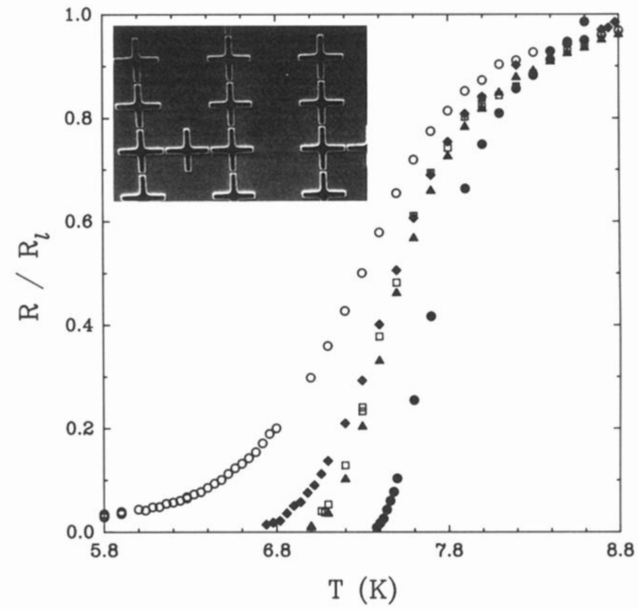

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