PHYSICAL REVIEW B VOLUME 45, NUMBER 21 1 JUNE 1992-I

Fractional Shapiro steps in ladder Josephson arrays

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(Received 18 February 1992)

We study the response of Josephson ladder arrays ($1\times N$ plaquettes) to a combined ac and dc current. For $N\geq1$, the arrays are predicted to exhibit Shapiro steps at voltages $< V >=nN\hbar\omega/(2e)$ for n=1/2, 1, 3/2, ... , over a broad range of fields $f=\Phi/\Phi_0$ (Φ=flux per plaquette, $\Phi_0=hc/2e$). The calculated half-steps persist even in single-plaquette "arrays" at all fields $f\neq0$, and, when the critical currents parallel to the applied current are sufficiently unequal, even at f=0. Measurements of the step widths $\Delta I_{1/2}(f)$ and $\Delta I_1(f)$ for 1×600 ladder arrays of Nb-Au-Nb junctions are in good agreement with the calculations.

When an array of resistively shunted Josephson junctions is subjected to a current of the form $I(t) = I_{\rm dc} + I_{\rm ac}\sin(2\pi\nu t)$, the time-averaged voltage $< V >$ across the array exhibits a series of quantized plateaus known as Shapiro steps.[1,2] In an $N \times N$ square array subjected to a transverse dc magnetic field of magnitude $p/q \equiv f$ flux quanta per plaquette, the plateaus usually satisfy the rule[2]

$$< V >= nNh\nu/(2eq), \quad (1)$$

$n = 0, \pm1, \pm2, \ldots$. The basic features of the experiment, notably the rule (1), have been reproduced by calculations in which the array is treated as a collection of resistively shunted Josephson junctions (the "RSJ model") subjected to a combined dc and ac current and dc magnetic field.[3–8]

Besides these steps, various "anomalous" steps have also been reported by several groups. Most conspicuously, a half-integer step at $Nh\nu/(4e)$ has been found even at f=0, and other subharmonic steps have also been detected at zero field.[9,10] While several explanations have been proposed for these anomalies,[9–12] all remain unproven at present. The RSJ model also exhibits unexplained anomalies of its own. For example, extra ("anomalous") half-integer steps at $< V >= Nh\nu/(4e)$ have been found at f=1/3, 2/5, and 1/5,[3] but only with free transverse boundary conditions, not with periodic boundary conditions.[4,6] These steps have not been definitively confirmed in experiment.

In this Rapid Communication we consider Shapiro steps in ladder arrays of Josephson junctions consisting of $1\times N$ square plaquettes. Our goal is to shed some light on the origin of Shapiro steps in wider arrays, and on the anomalies mentioned earlier. We also consider the special case $N = 1$ ("single-plaquette" arrays). Besides carrying out calculations we present experimental results for ladders of resistively shunted junctions, and find good agreement between theory and experiment.

Our calculations are carried out as described previously.[3,4,13] We consider a $2\times(N + 1)$ lattice of grains coupled by resistively shunted junctions ($1\times N$ plaquettes, or $3N + 1$ junctions in all). A current $I = I_{\rm dc} + I_{\rm ac}\sin(2\pi\nu t)$ is injected into each grain at one end of the array, and extracted from each grain at the other end, with free transverse boundary conditions. The equations of motion[3,4,13] involve $2(N + 1)$ unknown phases ϕ_i, of which one may be arbitrarily fixed. A field of f flux quanta per plaquette is applied perpendicular to the array. This corresponds to a transverse magnetic field $B = f\Phi_0/a^2$, where $\Phi_0=hc/2e$ is the flux quantum and a is the lattice constant of a square plaquette.

The results of our calculations for the ladder arrays are shown in Fig. 1. They reveal integer and half-integer steps [$< V >= nNh\nu/(2e)$ with n integer and half-integer] over a broad range f. Figure 1(a) shows the time-averaged voltage $< V > /(NRI_c)$ as a function of dc current $I_{\rm dc}$ for several values of f and $N = 20$. $< V >$ is obtained by integrating in time steps of width $0.02\tau_0$ [$\tau_0=\hbar/(2eRI_c)$, where I_c is the critical current and R the shunt resistance of each junction], then time averaging over an interval $700\tau_0$. Figure 1(b) shows the widths $\Delta I_1(f)$ and $\Delta I_{1/2}(f)$ for the $n = 1$ and $n = 1/2$ steps, plotted as a function of f. Also shown is the critical current as a function of f, indicated as $\Delta I_0(f)$. The curves correspond to $I_{\rm ac}=I_c$ and ν/ν_0=0.6 where $\nu_0=2eRI_c/h$ is the characteristic frequency. All junctions are assumed identical.

Figure 1(b) shows that the $n = 1$ step persists over the entire range of f, but is widest at f=0. By contrast, the $n = 1/2$ step has its greatest width at f=1/2; $\Delta I_{1/2}$ is substantial even at rather small f, but always vanishes at f=0 or 1. There seems to be a fairly abrupt cutoff f below which $\Delta I_{1/2}$ vanishes. In all cases, the widths are obtained numerically, by visual inspection of plots of $< V > (I_{\rm dc})$.

In Figs. 2(a) and 2(b) we show the I-$< V >$ characteristics and step widths for a "ladder" consisting of a single plaquette ($N = 1$), again with $I_{\rm ac}=I_c$ and ν/ν_0=0.6. As in Fig. 1, there are only integer and half-integer steps. The step widths ΔI_1 and $\Delta I_{1/2}$ behave similarly to Fig. 1; $\Delta I_{1/2}$ is more sinusoidal than in Fig. 2, and has no clear cut-off at small f, but vanishes at f=0. There are no other clear steps other than integer and half-integer steps for the single plaquette at any f, although there are faint hints of an $n = 1/3$ step near f=1/3. The inset to Fig. 2(a) shows the time-dependent voltage across the two junctions parallel to the applied current. Just as in larger arrays,[6] each voltage has twice the period of the applied ac current. The two wave forms are identical but phase shifted by π radians, so that the *sum* has the period of the applied current.

When the I_c's in a single plaquette are sufficiently unequal, $n = 1/2$ steps appear even at f=0. Figure 3 shows the time-averaged voltages $< V > (I_{dc})$ across the two junctions parallel to the current, for a single plaquette in which three critical currents equal I_c and a fourth (parallel to the applied current) equals αI_c. To make the half-steps as clear as possible we show the extreme case α=0 (shunt resistance only). However, we have found both an $n = 1$ and an $n = 1/2$ step at f=0 at any $\alpha < 1.0$. The $n = 1/2$ step corresponds to a voltage drop of $(1/2)h\nu/(2e)$ across each of the two junctions

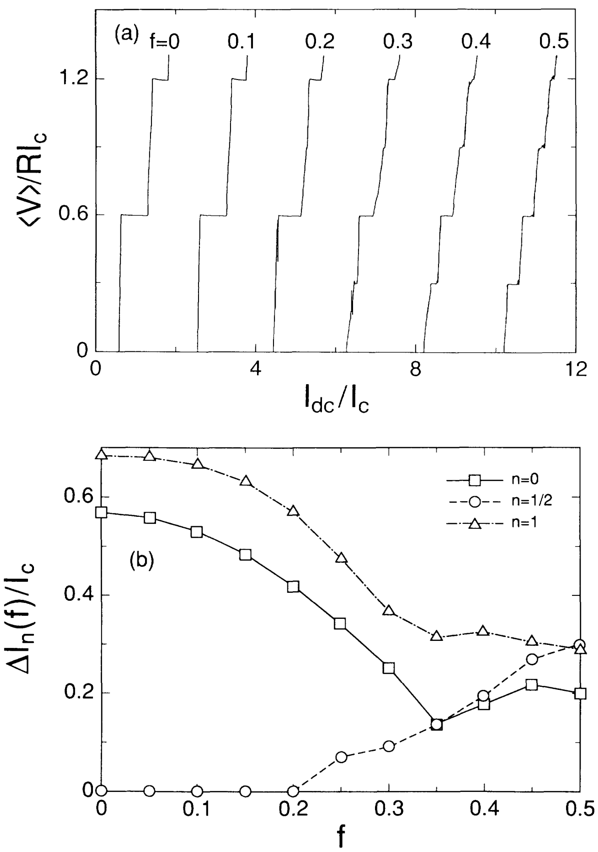

FIG. 1. (a) Time-averaged voltage $< V >$ vs dc current I_{dc} for a ladder array of length 20 plaquettes, for various values of the perpendicular magnetic flux. Flux is shown in units of $f=\Phi/\Phi_0$, where $\Phi_0=hc/(2e)$ is the flux quantum, and Φ is the flux through a single plaquette. The curves are offset horizontally. The amplitude of the ac current is $I_{ac}=I_c$; the frequency ν/ν_0=0.6, where $\nu_0=2eRI_c/h$, R is the shunt resistance, and I_c is the critical current of each junction. The $n = 1$ step corresponds to $< V > /(NRI_c)$=0.6. (b) Step widths $\Delta I_n(f)$ for step n, plotted as a function of f for the ladder array shown in (a). $\Delta I_0(f)$ represents the critical current. Calculated data points are shown as open symbols; the straight line segments connecting the points are merely to guide the eye.

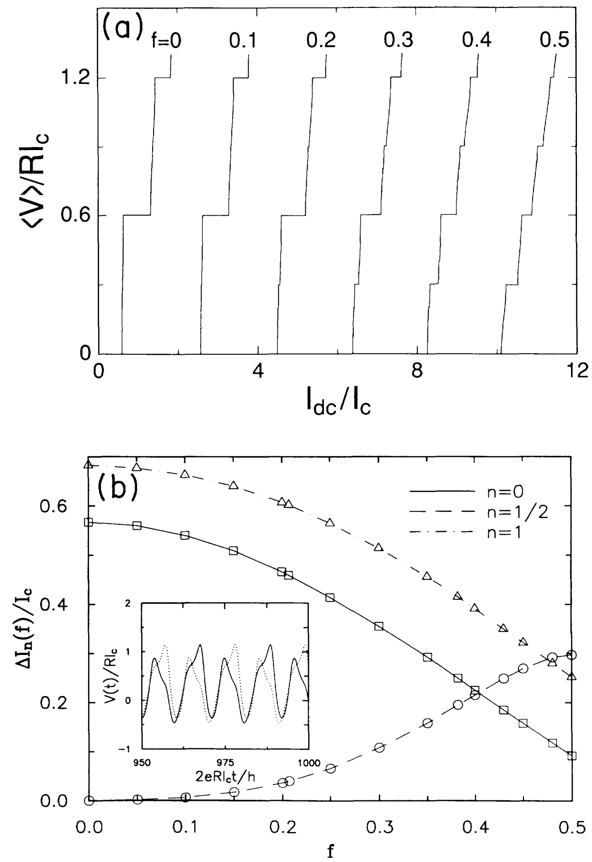

FIG. 2. (a) and (b) Same as Figs. 1(a) and 1(b) but for an "array" consisting of a single plaquette. Frequency and ac amplitude as in Fig. 1. Inset: time-dependent voltages $V_1(t)$ and $V_2(t)$ on the $n = 1/2$ step across the two junctions parallel to the applied current, at f=1/2.

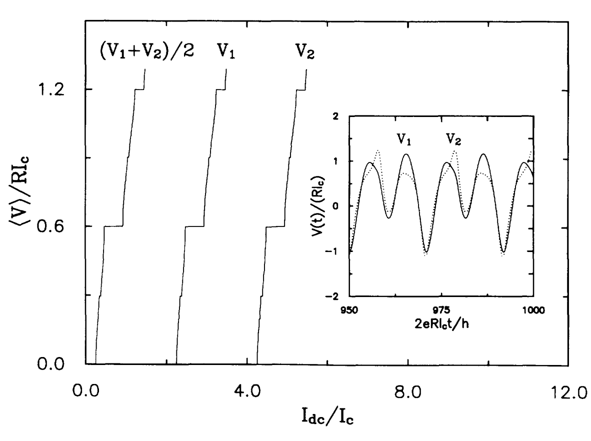

FIG. 3. Same as Fig. 1(a) for a single plaquette with unequal critical currents. All critical currents equal I_c with the exception of one critical current parallel to the applied current, which is taken to equal αI_c. Shown is the extreme case α=0 (shunt resistance only) which produces the widest half-integer steps. The voltages denoted V_1 and V_2 represent the time-averaged voltages across the two junctions parallel to the applied current. Inset: time-dependent voltages across these two junctions on $n = 1/2$ step. V_1 is the voltage across the junction with critical current αI_c.

parallel to the current. The inset to Fig. 3 shows the time-dependent voltage across these two junctions. As with the half-steps shown in Fig. 2, the period of these voltages is *twice* that of the applied ac current. We have confirmed that similar zero-field half-steps also occur in an N-plaquette ladder array, for various special patterns of unequal critical currents. In one such pattern, the critical current equals I_c for all the junctions perpendicular to the applied current, and for all the junctions parallel to the current on one edge of the ladder, but equals αI_c for all the other parallel junctions, with $\alpha < 1$. In this case the "anomalous" half-steps appear at a voltage of $(1/2)Nh\nu/(2e)$. We have found other patterns of unequal I_c's which lead to anomalous half-steps even in larger arrays.[14]

To test our calculations we have carried out experiments on 600-plaquette ladder arrays of lattice constant 5 μm. The plaquettes consist of proximity coupled Nb-Au-Nb junctions of thickness $\approx$2000 Å, approximately 1 μm width and 0.5$\pm$0.1 μm gap. The experiments were carried out at $T \approx 2$ K, far below both the Nb transition temperature of $\approx$8 K and the $\approx$5-6-K phase-locking transition typically found in larger arrays of Nb-Au-Nb junctions.[15] We estimate the average critical currents and shunt resistances of the junctions as $I_c \approx 20$ μA at T$\approx$2 K, 29 μA at T$\approx$1.75 K, and $R \approx 15$ mΩ at both temperatures, based on I-V measurements of the entire ladder.

The measured widths $\Delta I_{1/2}(f)$ and $\Delta I_1(f)$ are shown in Figs. 4(a) and 4(b) for T=1.75 and 2.0 K as a function of the applied transverse field $B \equiv f\Phi_0/a^2$. The ac amplitudes were $I_{ac} \approx 79$ μA at 1.75 K, 60 μA at 2 K. In both cases we use ν=402 MHz. The observed $n = 1/2$ and $n = 1$ Shapiro steps are quite rounded compared to those observed in larger square arrays.[9,10] Because of this rounding we have inferred the experimental widths by finding the points of inflection in each I-V characteristic, constructing parallel straight lines tangent to the curve on either side of the inflection point, and extracting the horizontal (current) distance between these lines.

There is generally good qualitative agreement between experiment and the calculations of Fig. 1, including, in particular (i) the persistence of the half-integer step over a substantial width in f around f=1/2 and (ii) the existence of a minimum in $\Delta I_1(f)$ near f=1/2. At the lower temperature, $\Delta I_{1/2}(f)$ shows sharp cutoff near f=0.2 and 0.8, in agreement with the calculations (which are symmetric about f=1/2). At the higher temperature, the measured $\Delta I_{1/2}(f)$ seems to be nonzero even at f=0 and f=1. $\Delta I_{1/2}(0)$ is, however, much smaller than that observed in wider arrays.[2,7,9,10] In the present case of a very narrow array, the anomalous half-step could hardly be due to a screening magnetic field produced by the current, as has been proposed by several groups in the case of wider arrays. Conceivably, it results from unequal critical currents in the Josephson junctions, as in our calculations presented above.

The present numerical results suggest several conclusions. First, in the ordered ladders, it is notable that the $n = 1/2$ step extends over a broad range of f (to at least f=0.15). This suggests a simple way to understand the previously mysterious appearance of $n = 1/2$ steps at f=1/3, 1/5, and 2/5 in the calculations of Lee *et al.*[3] These calculations are carried out in square arrays *with free transverse boundary conditions.* Hence, just as in our 1$\times$20 calculations, we might expect the $n = 1/2$ steps to occur over a fairly wide range of f, and not just at f=1/2. (We have calculated a half-step over a similarly wide range of f in 3$\times$10 arrays with free transverse boundary conditions.) The "anomalous" $n = 1/2$ steps previously reported in 10$\times$10 plaquette arrays, do not occur solely at f=1/2 in this interpretation, but instead simply correspond to a $\Delta I_{1/2}(f)$ which has a maximum at f=1/2 but which is still nonzero for f as small as 1/5.

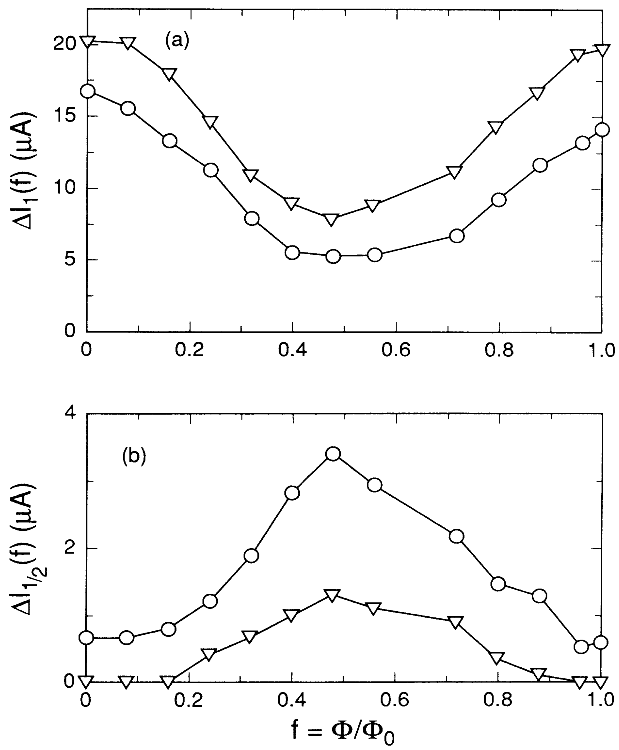

FIG. 4. (a) Measured step width ΔI_1 for a 1$\times$600 plaquette ladder array, plotted as a function of frustration $f=\Phi/\Phi_0$ where $\Phi_0=hc/2e$ is the flux quantum, at temperatures 1.75 K (triangles) and 2.0 K (circles), at a frequency of 402 MHz. (b) Same as (a), but for the half-integer step $\Delta I_{1/2}(f)$. The junction critical currents and shunt resistances I_c and R are estimated as 30 μA and 15 mΩ at T=1.75 K; 20 μA and 15 mΩ at T=2.0 K. $I_{ac} \approx 79$ μA at 1.75 K, 60 μA at 2 K. Step widths are believed accurate to within $\pm$0.5 μA.

It is somewhat surprising to find half-integer steps in an "array" consisting of a single plaquette. The occurrence of fractional steps in square arrays has often been interpreted in terms of a phenomenological picture of a vortex lattice which moves through an egg-carton potential formed by the lattice of grains.[2,3,16] While our single-plaquette time-dependent voltage patterns are similar to those found in large arrays, it is not obvious how to transcribe the vortex lattice picture to a single plaquette. Conceivably a more general picture, perhaps involving topological invariants,[16] may provide an alternative criterion for the occurrence of steps in various geometries.

To conclude, we have numerically studied Shapiro steps in Josephson ladders with free transverse boundary conditions and have obtained step widths as a function of magnetic field. We also present measurements of integer and half-integer Shapiro steps in ladder arrays in good agreement with the RSJ calculations. We find half-steps numerically even in single plaquettes at finite f, and, for unequal critical currents in the junctions of the plaquette, even at $f=0$. These half-steps persist at $f=0$ for larger lattices, provided that the critical currents are chosen to be unequal in a special way. It remains to be confirmed that this model accounts for the $f=0$ half-steps seen experimentally in larger arrays.

This work was supported by NSF through Grants No. DMR 90-20994 and DMR 88-21167, by the Department of Energy through MISCON Grant No. DE-FG02-90-ER45427, and by the National Nanofabrication Facility. Calculations were carried out, in part, on the Cray Y-MP8/864 of the Ohio Supercomputer Center. We should like to thank Dr. K. H. Lee for many valuable conversations.

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[14] S. E. Hebboul and J. C. Garland (unpublished) have also produced half-steps numerically at $f=0$, using a simple model of two "generalized" junctions in parallel, the generalized junctions consisting of a single resistively shunted junction in series with an inductance. If the critical currents of the two junctions are unequal, they obtain half-steps at $f=0$. This model bears some similarities to the single plaquette.

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