Issues related to the prediction of craniofacial growth

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The prediction of craniofacial growth involves four central issues: (1) What frame of reference should be adopted for measuring change? (2) What type of coordinate system should be used? (3) How should the change be described? (4) How can the change be explained biologically? In an effort to address these issues within a common framework, we are presenting a mathematical model for predicting the course of craniofacial growth in any given individual. The model is derived from a few basic assumptions about the long-range effects of gravitational pressure on the remodeling of bone and is expressed formally as a single geometric transformation. The validity of the model is examined empirically, using data for twenty individuals from the Denver Child Research Council's longitudinal growth study. The predictions of the model are found to be in close correspondence with the actual morphologic changes in each individual over periods ranging from 8 to 17 years. These findings suggest that a transformational approach to the study of human growth may provide clinicians with a valuable tool for long-range treatment planning.

Key words: Craniofacial growth, growth prediction, treatment planning, transformation, D'Arcy Thompson

A formal description of craniofacial growth can be of immense importance to orthodontists, oral and maxillofacial surgeons, and other clinicians who treat functional and esthetic anomalies of the human face. Development of a satisfactory treatment plan is often contingent upon predicting the outcome of growth in the absence of treatment and anticipating the product of the interaction between the desired treatment objective and craniofacial growth. The failure to anticipate the effects of growth in a long range treatment plan can result in a marked deterioration of the desired outcome over time. On the other hand, postponing treatment until the head has stopped growing can have disastrous consequences for the patient's self-image and ability to engage in social interactions during a period of the life cycle when peer relationships have a profound impact on the ability to form other relationships in later years.

Toward the goal of being able to make accurate growth predictions, many studies have already attempted to describe the remodeling of the craniofacial complex due to growth. At the outset, however, these investigations have confronted implicitly four fundamental questions, each of which poses a substantive problem for describing craniofacial growth:

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1. What frame of reference should be chosen for orienting and registering a profile (that is, the outline of a midsagittal section)?
2. What kind of coordinate system (grid) should be used to assign coordinates to points on a profile?
3. How should the remodeling of craniofacial morphology due to growth be described?
4. What theoretical explanation can be offered for the resultant description?

The present study examines these questions in an effort to elucidate criteria for selecting a frame of reference, a coordinate system, and a method for describing growth, in light of the various methods and conventions which might be employed.

The frame-of-reference problem

Cephalometrics has usually attempted to characterize growth in terms of the rate, amount, and direction of movement of craniofacial landmarks (points), as well as changes in angles and ratios which are based on the positions of those points. Numerous methods and conventions for their implementation have been used to describe craniofacial growth. For example, Walker[1] superimposed tracings of successive head films (registered on sella turcica and oriented on a line from the base of the occipital bone to the center of the palate) and plotted the positions of various landmarks throughout the growth period. The "track" produced by each landmark was averaged over many individuals to produce a normative "picture" of growth in a given population.

A second method that has been used to describe craniofacial growth and form is based on D'Arcy Thompson's[2] "method of coordinate transformations."[3-5] Typically, rectangular grids are constructed on a facial profile by inscribing the face in a rectangle and drawing horizontal and vertical lines through selected anatomic landmarks (see DeCoster's[4] Fig. 4). Changes in the initial profile due to growth can be represented by maintaining the original set of grid coordinates for each anatomic landmark in the older profile, thereby deforming the original rectangular grid in the same manner as the facial profile.

Regardless of the successes or shortcomings of any method for describing growth, each method has to employ certain arbitrary conventions for registering and orienting profiles. Unfortunately, different frames of reference often result in conflicting geometric descriptions and assessments of the same phenomena. These conflicts arise from the fact that a particular description depends critically upon the conventions used to orient and register profiles.[6-10]

Many cephalometricians have recognized the important implication of "orientation" and "registration" conventions for the interpretation of cephalograms. Krogman and Sassouni's[9] syllabus emphasizes that assessment of a given profile can vary significantly as a function of whether the Frankfort horizontal or the sella turcica–nasion line is taken as the reference line. Moorrees and Kean,[10] in evaluating the profile of an 11-year-old boy (their Figs. 8A, 8B, and 8C), for example, point out that, because of the downward inclination of the cranial base, the severe protrusion of the maxillary incisors is masked and a slight mandibular retrusion is exaggerated, if a horizontal reference line is taken to be sella turcica–nasion, rather than a "true vertical" that coincides with the direction of gravitation force. In this regard, it has been argued that the use of metal implants reveals relevant orofacial changes which often go unnoticed because implants establish a frame of reference whereby rotations of the mandible or maxilla are not masked by compensatory remodeling of those facial structures.[8] In summary, *a description of movements of points throughout growth is integrally related to the particular frame of reference from which movement is observed.*

A number of conventions are commonly used for orienting and registering profiles. Most procedures attempt to find an orientation that approximates "natural head position"—the position of the head when the visual axis is horizontal,[11] averaged over small variations due to postural adjustments, respiration, etc.[12] It must be emphasized, however, that natural head position is but a convention for orienting the head with respect to a reference line and, as such, it does not have a special status for describing craniofacial growth.[6, 8] Although the resultant reference line is not intended to pass through the same anatomic landmarks in all heads, anatomic approximations of natural head position are often employed as frames of reference. The most widely used "anatomic conventions" dictate that profiles are superimposed on either the Frankfort horizontal or the sella turcica–nasion line, with registration at porion or sella turcica, respectively. An alternative "convention" for determining natural head position has been devised by Moorrees and Kean.[10] They sought to determine natural head position by asking subjects to look at the image of their eyes in a mirror located at eye level. Their findings revealed that the head position obtained with this procedure was "remarkably constant at two observations within a week" and that this "functionally" derived natural head position results in less variation than anatomic conventions for establishing a frame of reference. A "relational convention" for approximating natural head position has been proposed by Graham Rabey,[13] whose analytic morphograph can produce frontal, lateral, and basal "radiograms" and "photograms" which are universally related in three dimensions. For the present purpose, it is sufficient to describe Rabey's conventions for establishing a frame of reference as based on fixed relations among certain anatomic landmarks. The frontal view, for example, is taken in such a manner that orbitale lies on a line connecting the right and left ear holes; radiograms or photograms can be compared longitudinally and in cross section when the pictures are placed so that the lines connecting the ear holes coincide, with registration occurring at the midpoint of each line.

Each of these methods for establishing a frame of reference was originally intended as a reliable procedure for orienting facial profiles, so that the same orientation could be established on different occasions by different investigators. It is important to keep in mind, however, that all of these methods are entirely arbitrary, and that their application to the study of growth begs an important question: By what criteria should the usefulness of any given frame of reference be evaluated? One important factor that must be considered is that different frames of reference can result in different descriptions of how the craniofacial complex changes as a function of growth. However, finding a reliable and economical description of growth is not merely a consequence of one's frame of reference; it is also a result of finding an appropriate coordinate system for "labeling" points.

The coordinate system problem

To appreciate the importance of establishing an appropriate coordinate system, it is useful to consider the popular technique of representing changes in craniofacial morphology as continuous deformations of a rectangular grid. In their survey of methods for examining facial growth and form, Moorrees and Lebret[5] note: "Of these methods, the mesh diagram is particularly suited for studying facial morphology because the findings

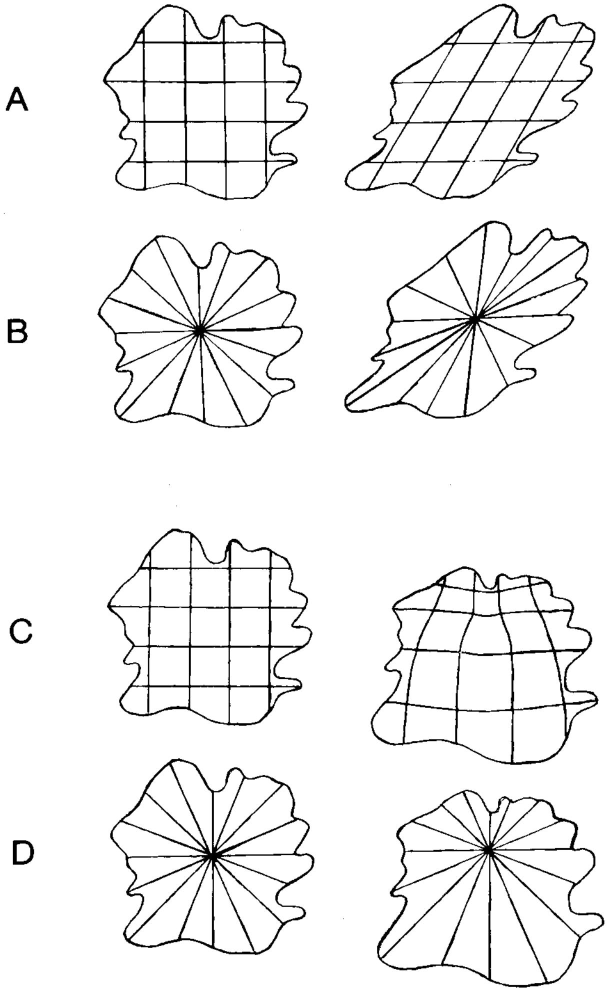

Fig. 1. The coordinate system used to represent a change can reveal characteristics of the change. A random shape on the left side of each pair of shapes is shown undergoing two transformations. The first transformation is depicted in **A** and **B,** the second transformation in **C** and **D.** Which coordinate system reveals more about each change? See text for details.

are shown graphically, facilitating interpretation. For research purposes it can be subjected to mathematical and statistical treatments.'' The use of ''mesh diagrams'' for describing craniofacial growth, however, begs an important question: What type of coordinate system (that is, mesh or grid) should be employed?[14]

In attempting to produce a graphic representation of craniofacial growth, cephalometricians have always worked with rectangular grids.[3-5] Despite the existence of D'Arcy Thompson's[2] elegant mesh diagrams using nonrectangular grids, no one until Bookstein[14] had questioned this critical assumption. Like the choice of most frames of reference, the selection of a rectangular grid for picturing craniofacial growth is an arbitrary convention, since properties of rectangular grids that grant them special status over other coordinate systems with respect to craniofacial growth have yet to be identified. In addition, selection of a grid or coordinate system, like the choice of a frame of reference, has an important consequence for the ease with which that change can be analyzed formally. These points are illustrated in Fig. 1.

Suppose we try to represent two changes using mesh diagrams (Fig. 1). Both changes are applied to a random shape depicted on the left side of each pair, and both changes are represented in rectangular coordinates (Fig. 1, *A* and *C*) and polar coordinates (Fig. 1, *B* and *D*). Which coordinate system directly reveals more about each change? The first change (Fig. 1, *A* and *B*) depicts a process which preserves all parallel lines of the rectangular grid and distances between all points with the same y coordinate, while changing the distance between all points with the same x coordinate (Fig. 1, *A*); none of these properties are revealed when the same change is represented in polar coordinates (Fig. 1, *B*). In contrast, characteristics of the second change are more easily apprehended in polar coordinates (Fig. 1, *D*). A polar coordinate system allows us to see that the radial axes maintain their positions, a property that is not revealed directly in the deformation of a rectangular grid (Fig. 1, *C*). Thus, the examples in Fig. 1 demonstrate the importance of one's choice of coordinate system for revealing characteristics of change. In selection of a coordinate system to represent a given change, the goal is to find a coordinate system that is *privileged*—privileged in the sense that it reveals geometric relations which are preserved over that specific change. Attainment of this objective is an important step in realizing the full value of mesh diagrams for representing a change such as craniofacial growth.[5]

Unfortunately, previous efforts to use mesh diagrams in the study of craniofacial growth have employed rectangular grids without considering whether rectangular grids are "privileged" with respect to craniofacial growth. That rectangular grids may not be the best choice for representing growth is suggested by a finding reported by a number of cephalometricians. Several investigators[15-30] have observed that anatomic landmarks move along straight lines that radiate from the vicinity of the cranial base; in polar coordinates, the angular coordinate of each point remains constant over growth. A polar coordinate system is well suited to represent this observation graphically, since the angle between radial vectors is maintained. A rectangular coordinate system, in contrast, cannot depict radial growth directly. It is not surprising that no growth study using rectangular coordinates has even examined the "radial growth hypothesis." In contrast, Ricketts[18] (his Fig. 12), who used a polar coordinate system to study mandibular growth, did, in fact, observe radial growth—what he referred to as ". . . polar phenomenon around the pterygopalatine area and the base of the body of the sphenoid at the root of the pterygoid plates" (p. 88).

In view of past difficulties in applying formal mathematical and statistical analyses to deformed rectangular meshes, we join with Bookstein[14] in asking whether the reason mesh diagrams have not been more useful to students of craniofacial growth might be a consequence of the pervasive choice of rectangular grids along with the somewhat arbitrary frames of reference used to orient the head in the coordinate system.

The problem of describing change

Once conventions are adopted for establishing a coordinate system and a frame of reference, it is necessary to devise a means for describing changes in craniofacial morphology. The traditional, cephalometric approach to this problem is based on measurements of simple metric relations among identifiable landmarks. These landmarks are analyzed as if they were connected by imaginary line segments. The length of each line segment and the angles formed by the intersection of line segments are measured at different stages of development for a large number of individuals. By averaging these measures across individuals of a particular race, sex, or nationality, it is possible to compile a table of cephalometric norms which are used as a standard for treatment planning and to make predictions about morphologic changes due to growth. There are at least two major problems with this "normative" approach: First, it does not adequately describe the changes that occur between landmarks.[6] Second, it is uneconomical; in order to represent a reasonable number of landmarks at different ages, the resulting table of cephalometric norms becomes large, cumbersome, and difficult to use.*

In an attempt to avoid this latter difficulty, Walker[1] has suggested an alternative method for describing craniofacial change: The positions of identifiable landmarks are plotted over time within a Cartesian coordinate system and the trajectory of each landmark is averaged over many individuals. With the aid of statistical regression techniques, each of the average trajectories is approximated with a polynomial equation. Walker's method provides a more efficient description of growth than tables of cephalometric norms, but it is equally uninformative about changes that occur between landmarks. Another problem with this analysis is that the motion of each landmark is considered to be an independent event, clearly a misrepresentation of what normally occurs during development. If all landmarks grew independently of one another, then the process of growth would ultimately result in a random deformation of the entire craniofacial complex.

A more reasonable hypothesis, first suggested by D'Arcy Thompson,[2] is that the head grows as a single unit so that the global outcome of growth can be described by a single mathematical transformation. Thompson's primary evidence for this claim was his ability to represent apparently complex changes in morphology as the geometric distortion of a grid placed over an evolving or growing organism. His unique insight was that presumably complex processes of growth and evolution could be adequately represented with a simple object, such as a square grid, just as easily as with a complex object, such as a human head. Unfortunately, Thompson rarely offered physical or biologic explanations for the phenomena he was modeling, and in most cases he did not even attempt to describe these phenomena formally with mathematical equations. The power of his approach lay primarily in the domain of visual perception. Thompson's drawings made it possible to "see" how different transformations operate, even though he did not formally analyze their effects with any degree of precision.

A possible strategy for quantifying Thompson's grid method has recently been proposed by Bookstein.[14, 20] In order to compare two homologous forms, Bookstein uses specially constructed biorthogonal grids, in which all lines intersect at right angles so that

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*A method that represents growth-related changes in a simpler manner would be preferable to these large tables of numbers. Furthermore, the fact that such tables can be put into computers for the purpose of making growth predictions does not make these norms any more economical. Before norms can be used to make growth predictions or even reveal characteristic changes in facial morphology associated with growth, relations that change reliably over growth in many individuals must be identified. To eliminate redundancies and to optimize the accuracy of predictions, these relations must be based on a theory of how landmarks change relative to one another.

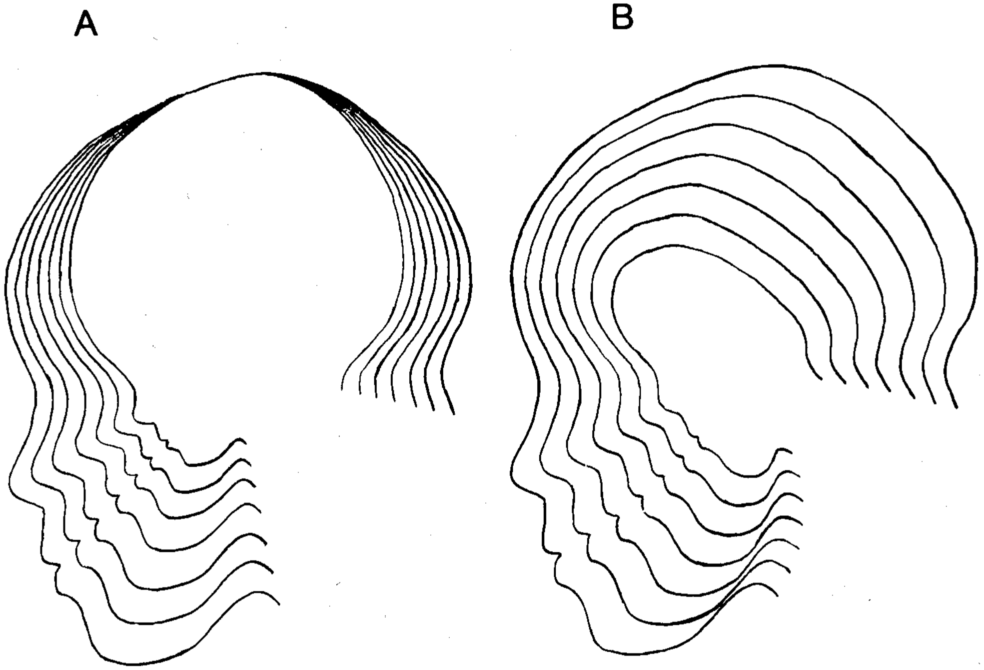

Fig. 2. Growth of a human head is simulated in the sequence of computer-generated profiles on the left that are obtained with the geometric transformation written in Equations 2 and 3; this transformation and others like it that preserve the same geometric invariants are almost always perceived as growth. In contrast, transformations that do not preserve the geometric invariants maintained by the transformation in Equations 2 and 3 are rarely, if ever, perceived as growth; the sequence of profiles on the right is produced by such a transformation.

the angles of intersection are unaffected by the particular transformation relating the two forms. This reduces the underlying change to graded dilatations along the principal axes of the grid. The changes are summarized quantitatively by recording the dilatation along each segment as a decimal ratio. Although Bookstein's analysis does provide a more systematic method of preparing grids than was employed by D'Arcy Thompson, we believe it still leaves much to be desired as a formal or quantitative description of change because the individual decimal ratios have no meaningful biologic interpretation and there is no attempt to describe how they are related to one another.

Other researchers have attempted to describe the global regularity of craniofacial growth in a more informal manner. Since publication of Broadbent's[15] work, it has become an accepted practice to examine the effects of growth by superimposing lateral cephalograms of a single individual taken at different ages. Most of the investigators who have employed this technique have been in general agreement that the over-all pattern of craniofacial growth is surprisingly stable across individuals. It has typically been observed that if one adopts a frame of reference that is roughly coincident with the cranial base, then most anatomic landmarks tend to move along straight lines emanating from a single point somewhere in the vicinity of sella turcica, and the amount of displacement tends to increase monotonically from the top of the head to the bottom.[15, 16, 18, 19]

It is important to keep in mind, however, that this type of description is valid for only a particular frame of reference. If, for example, these data from radiographic cephalometry had been interpreted using a line drawn between nasion and the anterior nasal spine as a frame of reference, then a very different-looking pattern of growth would have emerged. This argument has been taken by Moyers and Bookstein[6] to dismiss any conclusions drawn from the superimposition of cephalograms as mere "fabrications." We do not agree with their interpretation of this fact. Although selection of a frame of reference can dramatically affect the appearance of change, this observation does not mean that any conceivable pattern of growth can be created arbitrarily by aligning cephalograms in some appropriate manner. To illustrate this point, consider the changes represented in Fig. 1. There is no orientation from which the objects in Fig. 1, *A* and *B* can be aligned so that all points move radially from a single focus; nor is it possible to align the objects in Fig. 1, *C* and *D* so that all points move along parallel straight lines. *The invariant properties represented in these diagrams are not artifacts of a particular frame of reference; they are intrinsic to the underlying transformation to which the objects have been subjected.* Similarly, the over-all pattern of radial growth within the craniofacial complex, which has been revealed by superimposing cephalograms, is an intrinsic property of growth itself and should not be ignored.

It is interesting to note that a similar approach to describing changes in craniofacial morphology has proved to be especially useful in the study of visual perception. Todd, Mark, Shaw, and Pittenger[21] have recently demonstrated that radial transformations of human facial profiles, such as the one depicted in Fig. 2, *A* are generally perceived as growth by naïve observers, whereas other classes of transformation, such as the one depicted in Fig. 2, *B,* are almost never perceived as growth.[7, 22–24]

The problem of explaining change

What possible explanation could there be for heads growing in such a globally regular manner? One hypothesis that should probably be discounted is that the over-all pattern of craniofacial growth is controlled by a genetic plan. Although no one would doubt that growth is affected by hereditary influences, there is considerable evidence that predispositions imposed by genetics still allow a large degree of flexibility. Moore and Lavelle[25] point out: "Bone is indeed a most labile tissue capable of being rapidly and extensively modified by changes in its mechanical circumstances." In addition, other researchers have noted that the information content of human DNA is woefully inadequate to account for every minute detail in the microstructure of bone.[26]

An alternative hypothesis, more in line with the thinking of D'Arcy Thompson, is that the over-all pattern of craniofacial growth is primarily controlled by biomechanical influences. This hypothesis, known as Wolff's law, states that once the general form of a bone is established, "The bone elements place or displace themselves in the direction of functional pressure and increase or decrease their mass to reflect the amount of functional pressure."[27] The actual mechanism by which biomechanical influences are exerted is not entirely understood. Much of the research in this area has been concerned with the piezo-electric effect,[28] the cantilever-bending theory,[29] and the effects of muscle tension on the processes of deposition and resorption.[26] However, although each of these research efforts should be applauded, we must not lose sight of the immediate problem by focusing our attention at too local a level. *If biomechanical influences are to account for the global*

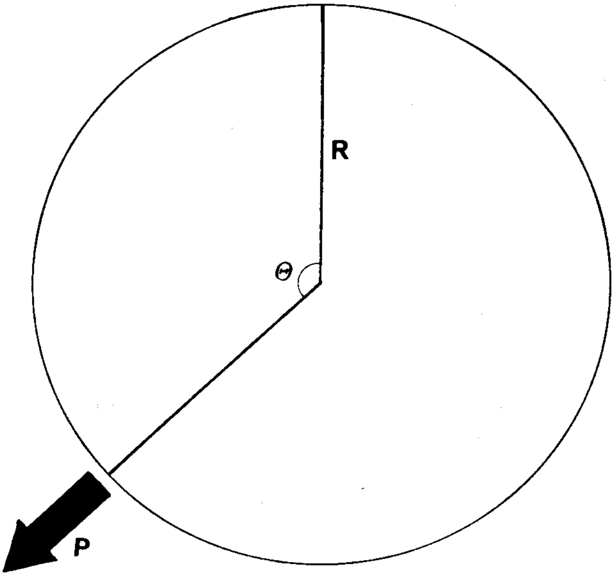

Fig. 3. The pressure distribution inside a fluid-filled spherical tank. The amount of pressure *(P)* at any point *(R,θ)* on the surface of the tank is determined by its vertical distance from the top of the sphere. The direction of pressure is always normal to the surface and can be expressed by the relation $P = a R (1 - \cos \theta)$, where R is the radius of the sphere and a is a constant product of the force of gravity and the density of the fluid. The pressure gradient relative to the surface can be represented by a single geometric transformation: $\theta' = \theta$, $R' = R + b P$, where b is a function that increases over time.

*regularity of craniofacial growth, then the over-all pattern of pressure and stress to which the growing material is subjected must also reflect that same regularity.**

One of the most obvious global influences on the biomechanics of growth is the force of gravity. This force is exerted on every point within the craniofacial complex, and it also provides a counter force for the action of muscles. In order to appreciate the over-all

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*Gould[30] has suggested at least two ways in which physical forces might influence growth. Such forces could act directly on the craniofacial complex, as is the case when an infant's head is swaddled to create a "culturally attractive" shape. According to Gould, it is in this sense that D'Arcy Thompson[2] sought to implicate physical forces as the *efficient* cause of form. But Gould also observes that physical forces can operate as ". . . blueprints of optimum shapes that determine the direction which natural selection (the true efficient cause) must take to produce adaptation." From Gould's perspective, and indeed our own, a physical force might be regarded as a *formal* cause of craniofacial growth. However, we carefully distinguish our position from Gould's on an important issue. Gould suggests that physical forces are determinants of optimal *shape*. In contrast, we contend, after P. S. Stevens,[31] that physical forces are determinants of optimal *paths of change*—paths of least resistence which minimize the amount of work performed by the growing structure. Thus, what is invariant over all instances of change resulting from a physical force is the resultant path of deformation and not the resultant shape. The "growth model" offered in Equations 2 and 3 is an approximation of the optimal path of change in response to gravitational stress; as such, this path is independent of the particular head (or object) undergoing change. In light of the importance of various "principles of least action" for understanding physical and biologic phenomena,[31] it would be most surprising if the same principles were not also involved in the ontogeny of the human head. As a consequence, genetic influences on craniofacial growth cannot be understood independently of the dynamic constraints that have acted on the organism throughout both ontogeny and phylogeny.

regularity of gravitational pressure, it is useful to consider the human skull as a spherical tank filled with fluid. From elementary hydrostatics, we know that the amount of pressure (P) at any point (R,θ) on the surface of the tank is uniquely determined by its vertical distance from the top of the sphere (Fig. 3). The direction of pressure is normal (perpendicular) to the surface at every point, and the amount of pressure can be expressed as a function of position by the following equation:

$$P = a R (1 - \cos \theta) \tag{1}$$

where a is a constant representing the product of the force of gravity and the density of the fluid. Thus, if all bone elements were placed or displaced in the direction of gravitational pressure, as suggested by Wolff's law, then they would all move outward along radial lines emanating from the center of the sphere. Moreover, if changes in mass reflect the amount of gravitational pressure, then the displacement along these radial lines of growth would increase monotonically over time as a function of pressure. If both of these assumptions are viable, then the over-all pattern of change can easily be expressed in polar coordinates with a single pair of equations:

$$\theta' = \theta \tag{2}$$

$$R' = R + b P \tag{3}$$

where b is an increasing function of time.* The over-all effect of this transformation on a human facial profile is depicted in Fig. 2, *A*. Not only does it create a perceptual impression of human growth, but it is also consistent with the data obtained through radiographic cephalometry.

This analysis, of course, is oversimplified. Heads are not perfectly spherical, there are other sources of stress operating on the craniofacial complex besides the force of gravity, and the relative orientation of the head with respect to gravity does not remain absolutely fixed. The resulting model should be thought of as a kind of ideal case, similar to analyzing the motion of a falling body without considering air resistence. Such a model can be quite useful if it helps us to appreciate the global influences on craniofacial growth or provides a means of approximating the course of growth in any given individual. This latter suggestion could have important applications for treatment planning and will be developed further in the next section.

Predicting the outcome of craniofacial growth

We now turn to examine how well this growth model is able to predict the morphologic outcome of craniofacial growth. For the purpose of testing this model, growth records of more than ninety people, duplicated from the Denver Research Council's Longitudinal Growth Study, were made available to us at the University of Connecticut Health Center.

Criteria for choosing an individual's records. For this initial study, pairs of lateral

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*The transformation given in Equations 2 and 3 has been discussed previously by Todd and associates.[21] In that work it was called "revised cardioidal strain" and written in the form, $\theta' = \theta$, $R' = R (1 + k (1 - \cos \theta))$, where $k = ab$. (See equations 1 and 3.)

head films from ten males and ten females were used. These individuals and their available head films satisfied the following criteria:

1. Records of an individual's annual examinations did not indicate that he or she had any unusual health or growth-related problem.
2. Two lateral head films were found that spanned a fairly large age range—the younger taken at roughly 5 years of age and the older at about 18 years of age. In addition, these films had to be well standardized, with no obvious perspective distortions. Any head film in which the mouth was open was also eliminated.
3. Good hard-tissue profiles were readily visible for the purpose of tracing the profile outline and identifying gnathion (Gn) and the anterior nasal spine (ANS).
4. There was no indication of orthodontic treatment over the span of the two x-ray films, either in the dental records or on intervening films.

Fitting procedure and results. The two lateral profiles for each individual were traced onto transparent acetate; gnathion and anterior nasal spine were identified on the x-ray film and tracing.

These profile tracings were oriented with respect to a sheet of polar graph paper by the following procedure: The younger profile was placed over the polar graph paper so that (1) ANS and Gn were placed on the 125-degree and 160-degree radial coordinates, respectively, and (2) the profile was roughly centered around the vertical axis so that the points on the facial mask and cranium that intersected the horizontal axis did so at equal distances from the center of the coordinate system. The younger profile was never moved after its position was fixed. The older profile was placed over the younger profile so that (1) the tops of both heads were superimposed on the vertical axis, (2) the older profile was centered on the horizontal axis around the vertical axis, and (3) ANS and Gn lay as near as possible to the radial coordinate assumed by each landmark on the younger profile.

After a profile was properly oriented, a graph pen tablet was used to record the polar coordinates of about 150 points along the outer boundaries of the cranium, facial mask, and mandible. These points were not anatomic landmarks but arbitrary points on the facial profile roughly 3 to 5 mm. apart. These data were used to generate a continuum of transformed skull outlines by applying the transformation given by Equations 2 and 3 with successively larger values of b. The resulting family of forms constituted a predicted path of craniofacial growth. According to our model, an individual might have grown at varying rates at different times but, barring some trauma, the direction of growth should have followed the predicted path. This hypothesis was tested by comparing the predicted skull outlines for any individual with a profile of that same individual at an older age.

Results. The twenty growth predictions that were made using this procedure are shown in Fig. 4. The reader is strongly encouraged to evaluate these predictions "by eye" prior to examining the statistical analysis discussed below.

In order to evaluate the accuracy of these predictions in an objective manner, it was necessary to devise a statistical procedure that was sensitive to differences in both size and shape. One possible strategy was to measure the distances between pairs of points with the same angular coordinate on different profiles. The average distance between all possible pairs on the younger and older profiles was taken as a measure of the total amount of variance that had to be explained. By comparing this measure with the average distance across all possible pairs on the older profile and on the transformed profile, it was possible

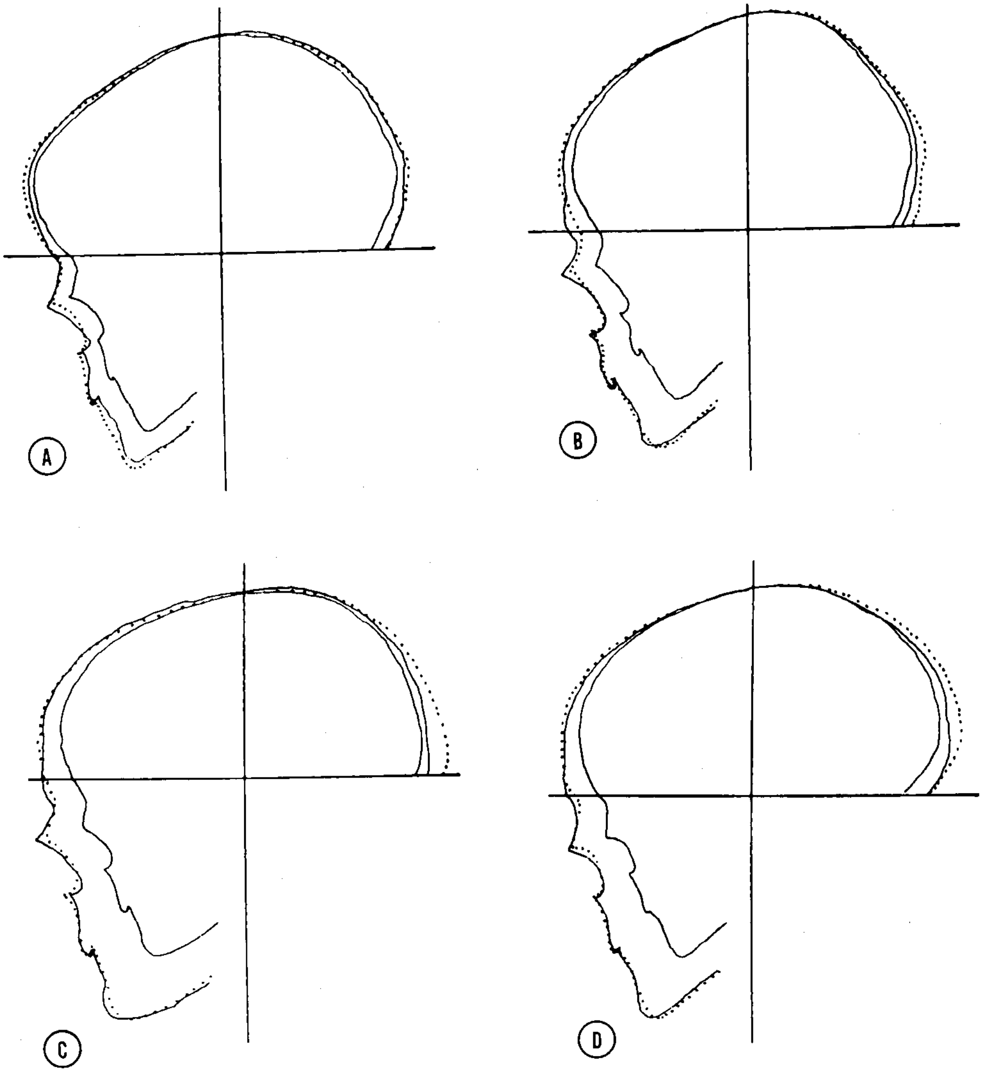

Fig. 4. Growth predictions of ten males and ten females. The two solid profile outlines show the younger (inner) and older (outer) profiles traced from lateral head films of a person's actual growth records. The dotted profile shows the growth prediction made by transforming the younger profile with the transformation in Equations 2 and 3. (See text for details of the "fitting" procedure and the statistical evaluation of the growth predictions.) **A,** Male, ages 5.9 and 13.9, 68.9 percent. **B,** Male, ages 6.3 and 19.0, 80.3 percent. **C,** Male, ages 4.3 and 18.6, 76.1 percent. **D,** Male, ages 5.1 and 17.0, 81.1 percent.

to determine the percentage of variance which our model accounts for. Because of our digitization procedure, however, there was no guarantee that any two points would ever share the same angular coordinate. We were able to circumvent this problem by dividing each profile into 5-degree sectors and computing a single average for the radial coordinates of every point within each sector. In comparing two profiles, we measured the

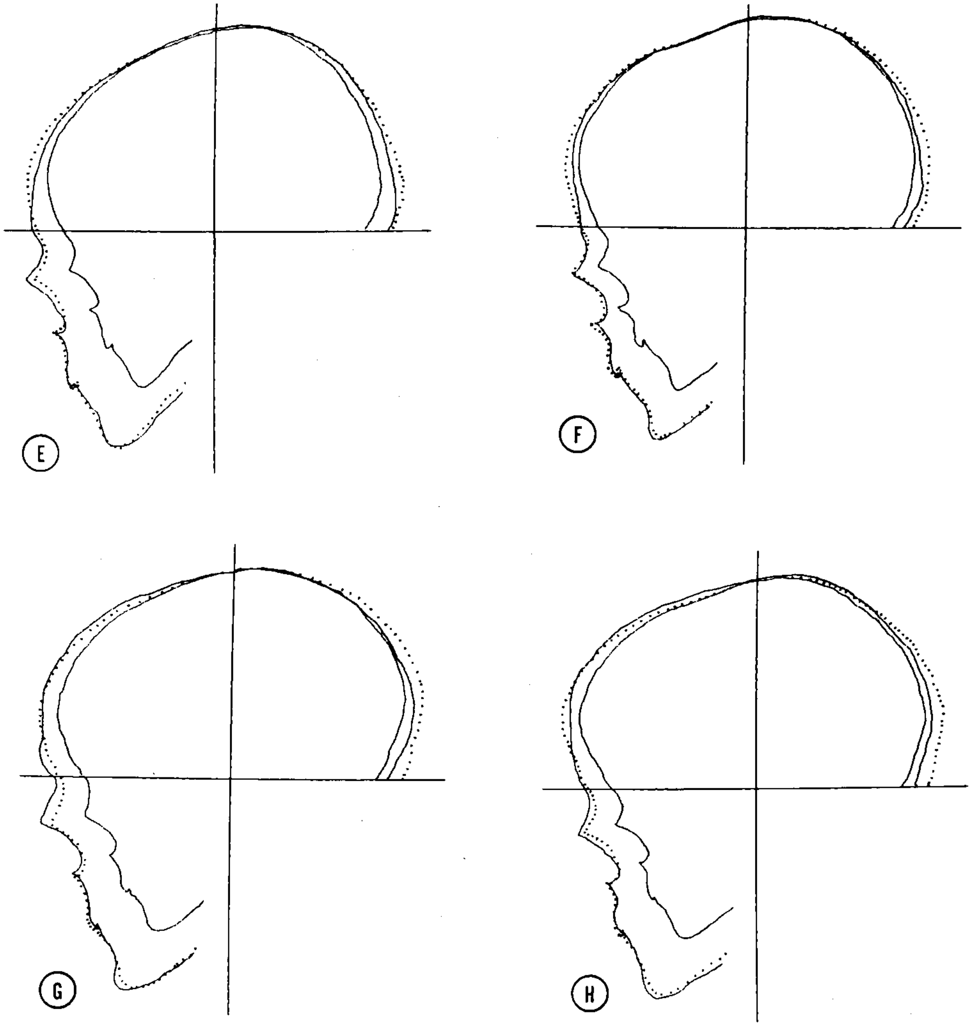

Fig. 4 (Cont'd). E, Male, ages 3.9 and 17.0, 82.7 percent. **F,** Male, ages 6.1 and 17.8, 71.2 percent. **G,** Male, ages 5.4 and 20.11, 80.6 percent. **H,** Male, ages 4.1 and 19.1, 77.3 percent.

difference between the averages obtained in homologous sectors as if they were homologous points. The average difference over all possible sectors provided a reasonable estimate of how much the profiles differed from one another.

This statistic showed that the transformation in Equations 2 and 3 accounted for an average of 75.6 percent of the variance in the females (S.D. = 5.99) and 77.7 percent of the variance (S.D. = 4.67) in the males. These percentages were even more impressive when the various sources of error introduced throughout the fitting procedure were taken into account.

Sources of error in the fitting procedure. Several points should be noted in evaluating the results of this fitting procedure. Only large age differences between the two profiles were used

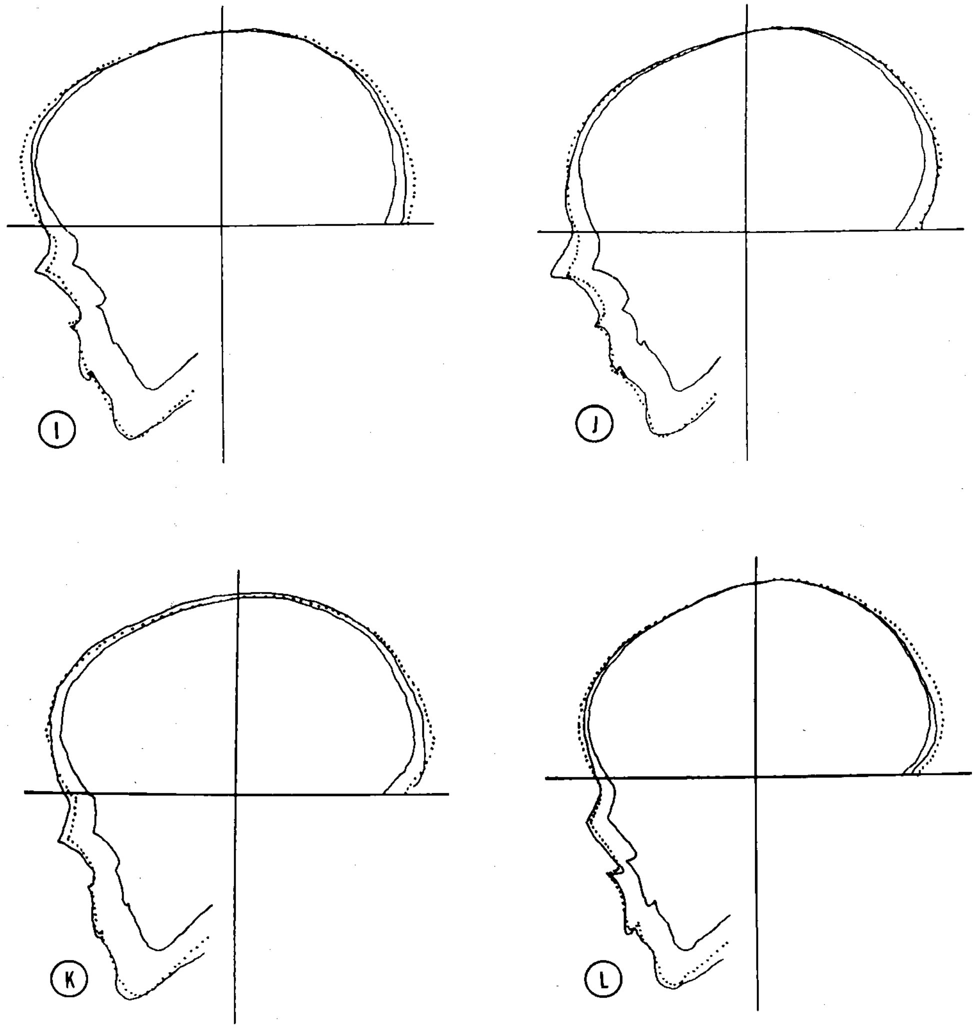

Fig. 4 (Cont'd). I, Male, ages 4.9 and 16.10, 76.3 percent. **J,** Male, ages 5.5 and 18.0, 82.2 percent. **K,** Female, ages 6.10 and 18.2, 71.9 percent. **L,** Female, ages 6.10 and 22.4, 63.7 percent.

for this initial study. This criterion was adopted in order to minimize the four sources of error that were introduced throughout the fitting procedure:

1. There were perspective distortions in the original x-ray films because of the poor standardization of head position with respect to the film.
2. A small amount of error was introduced when each profile was traced onto the acetate.
3. A small amount of error was introduced when the tracings were digitized into the computer.
4. There was also a small amount of error in orientation of the older profile with respect to the younger profile on the graph tablet. It should be noted, however, that for the purpose of making an actual growth prediction, the "older" profile obviously does not exist and thus there would be no orientation error. For the purpose of evaluating the growth transformation,

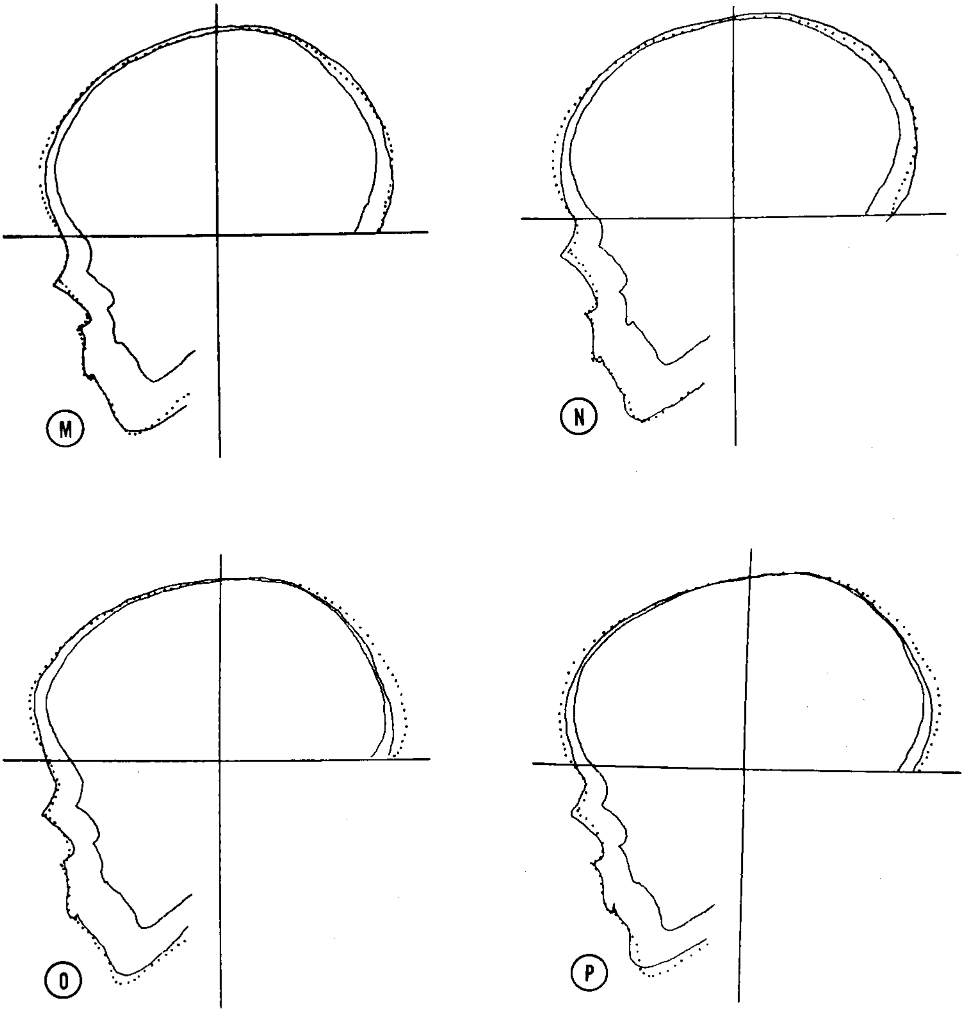

Fig. 4 (Cont'd). M, Female, ages 4.5 and 18.0, 83.3 percent. **N,** Female, ages 3.9 and 18.8, 80.8 percent. **O,** Female, ages 3.9 and 20.11, 75.6 percent. **P,** Female, ages 4.9 and 13.8, 68.5 percent.

a replicable procedure was needed to determine how well the predicted profile fit the actual profile. In any event, orientation of the younger profile in order to make a growth prediction followed a strict procedure that introduced virtually no error other than that associated with the identification of Gn and ANS.

If we assume that the four sources of error were roughly constant for any pair of profiles, then the variance introduced by the amount of error would constitute a smaller percentage of the total variance as the difference between the profiles increased. For the purpose of this initial test of our model as a predictive tool, it was thought better to maximize the percentage of variance for which the model could possibly account, thereby allowing us to determine whether the transformation defined by Equations 2 and 3 has a useful future in making growth predictions.

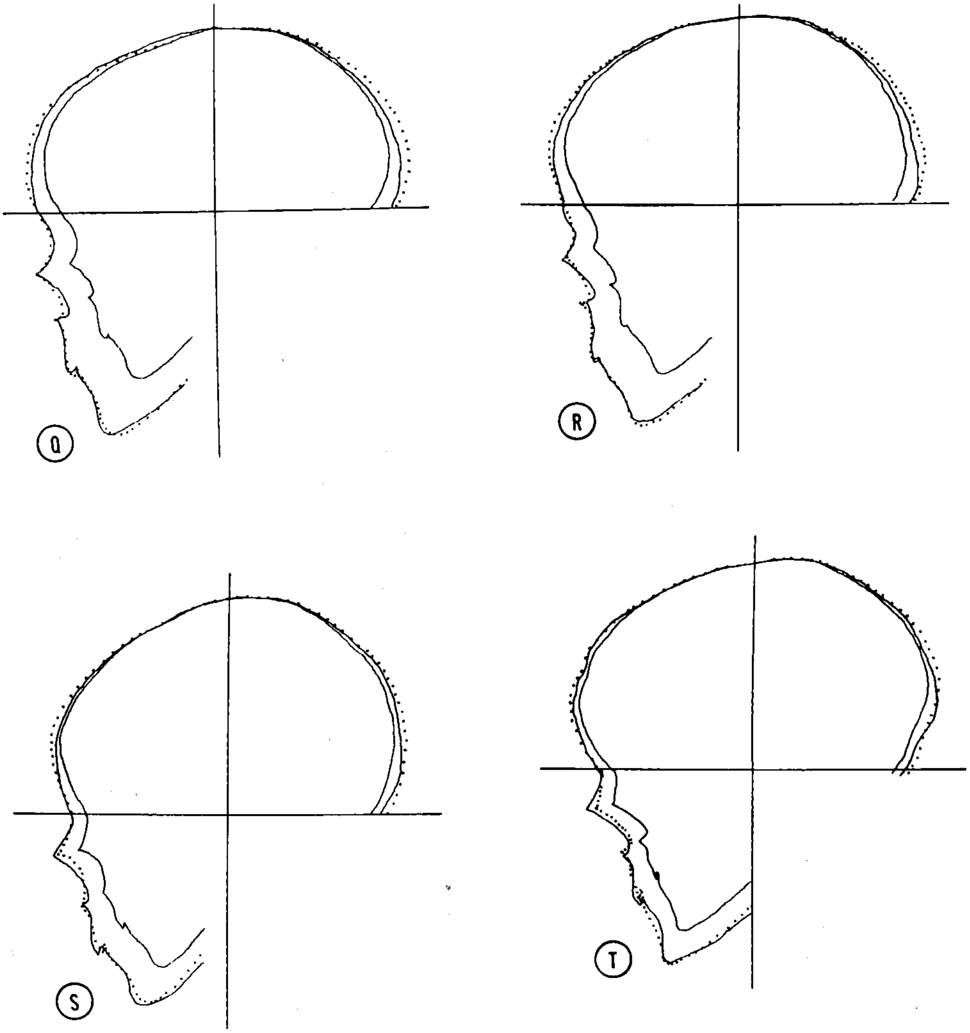

Fig. 4 (Cont'd). Q, Female, ages 3.10 and 19.1, 78.5 percent. **R,** Female, ages 5.2 and 16.0, 80.0 percent. **S,** Female, ages 5.4 and 16.4, 76.7 percent. **T,** Female, ages 7.0 and 19.0, 77.0 percent.

Eventually, both types of tracing error can be minimized through the use of optical scanners to digitize the profile directly from the x-ray film. And better standardization procedures for taking x-ray pictures, such as those employed by Rabey's Analytic Morphograph, could all but eliminate the standardization problem.

Conclusion

Let us reconsider the four problems that guided the development of our procedure for making growth predictions: (1) finding an appropriate orientation, (2) finding an appropriate coordinate system, (3) describing the change, and (4) explaining the change.

Most previous efforts to describe craniofacial growth have attempted to divorce the description of growth from the biologic explanation of growth. These methods typically start with the problem of finding an appropriate orientation[4, 5, 10] or coordinate system[14, 20] for representing growth. In our view, such problems are not empirical questions that can be studied independently of the biomechanics of growth. Rather, we argue that any description of craniofacial growth must reflect the global regularity of the biologic mechanisms that control growth. An analysis of one global regularity produced a mathematical transformation (Equations 2 and 3) for describing growth. In addition to suggesting a mathematical description, this model also addresses the remaining problems of finding an appropriate coordinate system in which to represent the change and orient the head within the chosen coordinate system. The optimal coordinate system for depicting any change is one which allows us to see the geometric relations that remain invariant under the transformation in question. Recalling Fig. 1, this principle explains why Fig. 1,*A* is a better representation of the first style of change than Fig. 1,*B* and why Fig. 1,*D* is a better representation of the second style of change than Fig. 1,*C*. Since our model says that growth is radial (Equation 2: $\theta' = \theta$), a polar coordinate system should be preferred to a rectangular coordinate system, as the former represents this geometric property more clearly.

In regard to the orientation within a polar coordinate system, our model suggests that the head should be oriented relative to some average position with respect to the direction of gravitational force rather than according to the position of specific anatomic landmarks. For future work, the position of the head with respect to gravity might be approximated by means of a procedure for taking radiographs similar to the one used by Moorrees and Kean[10] to determine "natural head position." However, existing evidence[12] that respiratory function can have a marked effect on head posture or position suggests that we should devise an orientation scheme that would take into account respiratory function (mouth breathing, nasal resistance) as well as other factors known to affect head position. In the absence of such x-rays or orientation scheme, for the present study we used Gn and ANS to maximize the reliability of the growth predictions.

The mathematical transformation, which was derived from consideration of the global stresses on an idealized human head, was shown to make reasonably accurate growth predictions over a span of about 10 to 15 years. This finding is very important because the transformation changes both shape and size of the profile and the transformed profile does not have to be normalized for size with respect to the actual profile resulting from growth. The predictions that were made with this growth model were not totally accurate because of mechanical sources of error and, perhaps, oral habits, such as thumb sucking, nail biting, mouth breathing, tooth clenching, or unusual facial expressions or facial postures. Nevertheless, they very closely predict the actual outcome of growth. The use of such a global transformation for this purpose may add an important dimension to treatment planning because facial profiles are represented as a continuous contour rather than as a collection of independent points. Clinicians who treat anomalies of the human face could make esthetic and other psychosocial judgments directly from an individual profile, rather than having to refer to "normative" patterns of craniofacial landmarks.

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