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Visual Information About Moving Objects

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A mathematical model of visual flow is presented, which could potentially account for an observer's ability to effectively interact with moving objects. The analysis demonstrates that there is visual information available to an observer about (a) a moving object's angle of approach, (b) changes in its velocity and acceleration, (c) whether its velocity and acceleration are positive or negative, (d) its time to collision with both the horizontal and vertical axes, and (e) whether it will cross the horizontal axis in front of or behind the point of observation. Several experiments are reported in which observers' sensitivity to these different aspects of visual information is examined using a variety of experimental tasks. The results suggest that human observers are highly sensitive to many abstract properties of visual stimulation, but that they are not sensitive to all of the information that is potentially available.

As a visually sensitive organism moves about in its environment, much of its activity is under the direct control of vision. One perplexing example of the visual control of action can be observed in a game of baseball. When a batter hits a fly ball toward the outfield, the ball travels at a speed well over 100 mi./hr. (160.9 km/hr.) along a parabolic trajectory. An outfielder detects light that has reflected from the moving baseball and is able to determine nearly instantaneously where the ball will land and whether there is sufficient time to move to the appropriate location before it touches the ground.

The outfielder problem contains several themes that are of general relevance to a theory of perception. One of these themes is the distinction between the properties of objects in an environment, called *physical variables,* and the properties of light that has reflected from those objects, called *optic variables*. Students of perception have traditionally assumed that the possible relations between physical and optic variables, which are referred to here as *visual information*, are exclusively composed of many-to-one mappings. According to this view, the optic variables are defined by reference to a static, two-dimensional retinal image and are considered to be an impoverished representation of a three-dimensional environment. Thus, to account for an observer's ability to accurately perform complex visually guided activities, such as catching a baseball, it assumes that perception is mediated by the observer's expectations, which are developed through experience and stored permanently in memory. An alternative conception of visual information that does not rely on the mediation of memory has been developed over the past three decades by James Gibson (1950, 1966, 1979). Gibson has shown that optic variables can be defined with reference to a richly structured and continuously changing optic array, so that many of the relations between physical and optic variables form one-to-one mappings. One of the goals of this paper is to demonstrate formally that there are a sufficient number of these one-to-one relations to uniquely specify whether an observer must move forward or backward to catch a free-falling projectile.

A second theme of the outfielder problem is the concept of rigid motion. A primary aspect of our perception of a moving baseball is that the size and shape of the ball appear to remain invariant at all times. These are the defining characteristics of rigid motion, but it is important to keep in mind that there are many other recognizable changes that one is likely to encounter to a natural environment (e.g. Bassili, 1978; Cutting, Prof-

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fitt, & Kozlowski, 1978; Jansson, 1977; Jansson & Johansson, 1973; Jansson & Runeson, 1977; Johansson, 1973, 1975; Pittenger & Shaw, 1975; Pittenger, Shaw, & Mark, 1979; Todd, Mark, Shaw, & Pittenger, 1980). Rigidity can be violated in two ways: (a) An object can undergo an elastic deformation or (b) it can be split into two or more rigid objects moving along different trajectories. Recent analyses of visual information have formally demonstrated that both of these violations are directly specified in terms of optic variables (Lee, 1974; Ullman, 1977). These analyses have been confirmed by empirical evidence that human observers can accurately distinguish between rigid and nonrigid motion in shadow projections of real objects (Gibson, Gibson, Smith, & Flock, 1959; von Fieandt & Gibson, 1959) or in computer generated simulations (Mace & Shaw, 1974; Ullman, 1979). Given that an object is undergoing rigid motion, it can also be demonstrated that its three-dimensional form is uniquely specified within an undetermined scale factor (Gibson, Olum, & Rosenblatt, 1955; Koenderink & van Doorn, 1977; Nakayama & Loomis, 1974; Ullman, 1977, 1979). That is to say, when an observer is stationary, there is visual information about the shape and orientation of moving objects, and when an observer is moving, there is visual information about proportional relations of size and distance among stationary objects. The perceptual salience of this information is demonstrated by the fact that human observers can accurately judge the orientation of a rigid object in motion (Flock, 1962, 1964; Gibson & Gibson, 1957) as well as its three-dimensional form (Lappin, Doner, & Kottas, 1980; Ullman, 1979; Wallach & O'Connell, 1953.)

A third theme of the outfielder problem involves the concept of an event (see Gibson, 1979). When one perceives the flight of a baseball, its past, present, and future are somehow merged into an extended unit. The outfielder is immediately aware of where the ball has been, where it is going, and how soon it will get there. Indeed, it is this temporally global aspect of perception that is most essential to a skilled outfielder and most problematical to a perceptual theorist. How does one perceive where and when a baseball will land in time to complete the appropriate sequence of actions that will result in a successful catch? Before attempting to answer this question, it is useful to consider how similar problems have been successfully analyzed in other contexts. For example, it has been formally demonstrated that there is visual information available to an observer about the direction of an object's rectilinear motion relative to the point of observation and about whether a collision is imminent (Gibson et al., 1955; Gordon, 1965; Lee, 1974, 1976; Nakayama & Loomis, 1974). This information can be exploited by a variety of animals including monkeys, kittens, chicks, frogs, fiddler crabs, and humans (Lee & Aronson, 1974; Lee & Lishman, 1975; Lishman & Lee, 1973; Schiff, 1965; Schiff, Caviness, & Gibson, 1962; Warren, 1976). It has also been demonstrated that there is visual information about time until collision during rectilinear motion (Lee, 1974, 1976) and the trajectory of curved motion when an observer is moving along a rigid ground surface (Lee & Lishman, 1977). Schiff and Detwiler (1979) have recently demonstrated that human observers are sensitive to information about time until collision during rectilinear motion, but the perceptual salience of curvilinear trajectories along a rigid ground surface has not been investigated. The outfielder problem is inherently more difficult than any of the situations described above because the flight of a baseball has a curved trajectory that is independent of the global context of visual flow provided by the ground plane. Thus, the goals of this paper are twofold: first, to demonstrate mathematically that the trajectory of a moving baseball can be directly specified by optic variables, and second, to empirically validate this analysis by examining observer sensitivity to the potentially informative properties of a continuously changing visual display.

Mathematical Analysis of Moving Objects

The analysis presented in this section is an extension of previous investigations (e.g. Gibson, 1950, 1958, 1966, 1979; Gibson et al., 1955; Gordon, 1965; Lee, 1974, 1976; Nakayama & Loomis, 1974; Ullman, 1977, 1979), and is primarily an attempt to achieve greater generality by considering moving objects that are not necessarily confined to a planar ground surface. A number of notational conventions borrowed from computer science are used for convenience. All variables are designated by single letters: Greek letters for angles (e.g., ϕ), uppercase letters for distances (e.g., X, Y, R), and lowercase letters for points (e.g., a, b). The letter appears by itself for physical variables and is accompanied by a prime for optic variables (e.g., R', a'). Arithmetic operations are designated by special symbols: * for multiplication, / for division, and ** for exponentiation. Finally, derivatives with respect to time are designated using the following conventions: $V = d/dt$ (e.g., VX, VR'); and $A = (d/dt)**2$ (e.g., AX, AR').

The analysis is concerned with the translatory motion of a single line segment ($\overline{ab}$) connecting two identifiable points in a three-dimensional environment (see Figure 1). The points must be part of a rigid configuration whose orientation relative to the observer is optically specified (see Ullman, 1977, 1979). The problem is to describe the motion of $\overline{ab}$ solely in terms of optic variables that are defined with reference to a particular projection surface. The choice of the projection surface is purely a matter of mathematical convenience, since the optic variables associated with one surface can always be uniquely transformed into the optic variables associated with any other surface (Lee, 1974). The one that is most convenient for the present analysis is a planar surface that is parallel to $\overline{ab}$ and one unit distance from the point of observation. This type of projection is analogous to viewing a natural event through a windowpane, or a computer simulation of a natural event on a video display monitor.

The resulting coordinate system is shown in Figure 1. The horizontal axis is a line passing through the point of observation that is perpendicular to both $\overline{ab}$ and the projection surface. Since $\overline{ab}$ is part of a rigid object, its length (R) cannot vary over time. All other physical variables such as the angle of approach (ϕ) and the horizontal and vertical distance from the point of observation (X and Y) are assumed to be varying continuously. As a consequence of this assumption, the optic variables (R' and Y') defined by polar projection must also be varying continuously. The distance (H) from the projection surface to the point of observation is assumed to be a fixed unity.

Ignoring rotation, the motion of a free falling object ($\overline{ab}$) relative to the projection surface is completely described by six equations:

$$X = R/R' \tag{1}$$

$$VX = -(R * VR')/(R'**2) \tag{2}$$

$$AX = ((2 * R * (VR'**2))/(R'**3)) - ((R * AR')/(R'**2)) \tag{3}$$

$$Y = X * Y' \tag{4}$$

$$VY = (X * VY') + (VX * Y') \tag{5}$$

$$AY = (X * AY') + (2 * VX * VY') + (AX * Y'). \tag{6}$$

It follows from this analysis that the individual physical variables describing the translatory motion of a free-falling object are not optically specified, because they cannot be uniquely defined in terms of optic variables. However, it also follows that the ratios of all possible pairs of these physical variables are optically specified, and that many of these ratios are potentially useful to perception. For example, Equation 7 demonstrates that there is optical specification of an object's angle of approach relative to the horizontal axis:

$$VY/VX = \tan \phi = Y' - ((R' * VY')/VR'). \tag{7}$$

Equations 8 through 11 relate to an object's velocity and acceleration:

$$VX/R = -VR'/(R'**2) \tag{8}$$

$$VY/R = VY'/R' - (Y' * VR')/(R'**2) \tag{9}$$

$$AX/R = (2 * (VR'**2)/(R'**3)) - (AR'/(R'**2)) \tag{10}$$

$$AY/R = (AY'/R') - ((2 * VY' * VR')/(R'**2)) + (Y' * (AX/R)). \tag{11}$$

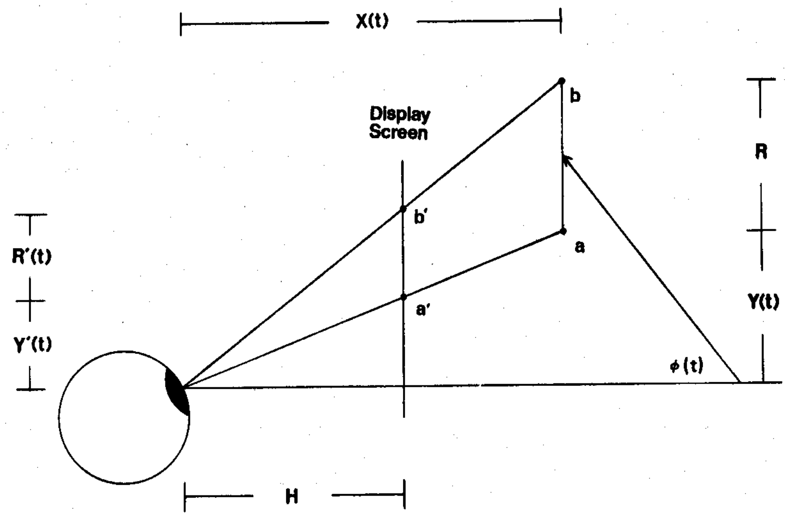

Figure 1. The optic relations between a simulated moving object and a visual display screen.

These equations are all scaled in terms of object size (R), so they do not define velocity and acceleration in any absolute sense. The information they describe is only useful because the size of a rigid object is always positive and must always remain constant. Thus, the equations demonstrate that there is visual information about changes in velocity and acceleration and indicate whether their values are positive or negative.

Time Until Contact

Other ratios are particularly useful for determining where and when a moving object will cross the horizontal axis relative to the point of observation:

$$VX/X = -VR'/R' \qquad (12)$$

$$AX/X = (2 * ((VR'/R')**2)) - (AR'/R') \qquad (13)$$

$$VY/Y = (VY'/Y') - (VR'/R') \qquad (14)$$

$$AY/Y = (AY'/Y') - ((2 * VR' * VY')/(R' * Y')) + (AX/X). \qquad (15)$$

Let TX be the amount of time before $X = 0$ and let TY be the amount of time before $Y = 0$. As long as AX and AY are not changing over time (see Equations 10 and 11), then the following relations are obtained from elementary kinematics:

$$0 = ((TX**2)(AX/X)/2) + (TX * (VX/X)) + 1 \qquad (16)$$

$$0 = ((TY**2)(AY/Y)/2) + (TY * (VY/Y)) + 1. \qquad (17)$$

Using appropriate substitutions and the quadratic formula, it is possible to demonstrate that TX and TY are optically specified for all possible values of distance, velocity, and acceleration. The ratio of these variables provides additional information about where an object will first contact the horizontal axis: if $TY/TX = 1$, then the initial contact will occur at the point of observation; if $TY/TX < 1$, then the initial contact will occur in front of the point of observation; and if $TY/TX > 1$, then the initial contact will occur somewhere behind the point of observation. Such information could be useful for a variety of activities such as judging a fly ball or deciding if it is safe to cross the street at a busy intersection (cf. Ebbesen, Parker, & Konečni, 1977). Some additional solutions to the outfielder problem that are derived from this analysis are provided in the Appendix.

Experiment 1

The analysis presented in the preceding section deals with properties of visual information that are available to a perceiving organism *in principle*. The remaining sections of this paper will investigate whether human observers are actually sensitive to that information. Experiment 1, for example, was an attempt to demonstrate that human observers are visually sensitive to time until collision in the special case of rectilinear motion where the value of acceleration is zero (see Equation 12).

Method

Stimuli were presented on a Tektronix 611 cathode ray tube (CRT), refreshed every 5 msec by a Nova minicomputer. The displays were viewed binocularly at a distance (H) of approximately 76.2 cm from a 16.5 × 21.6 cm display screen (see Figure 1). Head movements were not restricted.

A trial event consisted of 75 display frames presented within a 3-sec interval. This appeared as two gradually approaching square objects, each composed of 24 dots (see Figure 2). Observers were instructed to select the object that would contact the point of observation first. A left-hand response key was pressed to designate the object on the left, and a right-hand response key was pressed to designate the object on the right. Observers were informed that no responses would be recorded after a display had terminated. They were asked to respond as quickly as possible without sacrificing accuracy.

Immediate feedback was provided after each trial on an alpha-numeric display terminal located several feet to the left of the primary display screen. However, the observers reported that they did not bother to look at the feedback message on many trials because it was inconvenient to turn their heads and locate the appropriate line of text.

Each event was a mathematically accurate simulation of a pair of approaching objects. The widths (R) of these objects were selected at random prior to each trial from two possible values of 7.6 and 38.1 cm. Their velocities of approach (VX) were also selected at random from possible values of 6.1, 9.1, 12.2, 15.2, 18.3, 21.3, 24.4, and 27.4 m/sec. Once these parameters were determined, the initial starting distances (X) were computed automatically so that time until contact with the observation point would be 3 sec for one of the objects (selected at random) and greater than 3 sec for the other. Thus, there were 512 unique events, all of which had an equal probability of being presented on any given trial.

The width (R') of the image of an object was computed for each frame by the following equation: $R' = RH/X$. At some time prior to an expected collision this image would become too large to be completely contained on the display screen. The entire object would disappear as soon as this occurred, although the other object might still be visible if its image was small enough. This early visual termination was an essential aspect of the experimental design. Since the simulated objects were of varied sizes, it was often the case that the first object to disappear was not the first to collide with the observer. In other words, if the observers based their decisions on the relative size of the two images, they would be incorrect on a large number of trials.

The experiment consisted of 21 blocks of 50 trials each. The difference in collision time for the two simulated objects was held constant for a block of trials, but was systematically varied across blocks. The difficulty of the task was a direct function of the difference in time until collision. During pilot experiments it was discovered that performance was essentially perfect with a difference as large as 300 msec and that it gradually approached chance as the difference in collision time decreased. The differences that were actually used in the experiment were 300, 200, 150, 100, 50, 20, and 10 msec. Three blocks were run for each difference and the blocks were arranged in order of increasing difficulty. Since each block required about 15 min. to complete, the experiment was conducted over several sessions. The task demanded considerable concentration in order to obtain consistent performance; thus, to prevent fatigue an experimental session was usually limited to six blocks.

Five well-practiced observers participated in the experiment. One of the observers (the author) was thoroughly familiar with all aspects of the experimental design and was also aware of the optimal strategy for performing the task. The other observers were only vaguely aware of the purpose of the experiment and had no knowledge of the mathematical issues involved or the possible strategies for performing the task. All of the naive observers were paid $3 per hr. for their services.

Results and Discussion

Figure 3 shows the percentage of correct responses as a function of the difference in time until collision for the two simulated objects. It is evident from the figure that the observers were able to perform this task with surprising accuracy. Their responses were over 90% accurate for differences as small as 150 msec, and the level of performance did not reach chance until the difference in collision time was reduced to 10 msec.

Using the reaction time data, it was possible to compute for every trial the specific display frame that was being presented at the exact moment a response was recorded. Table 1 gives the percentage of trials in

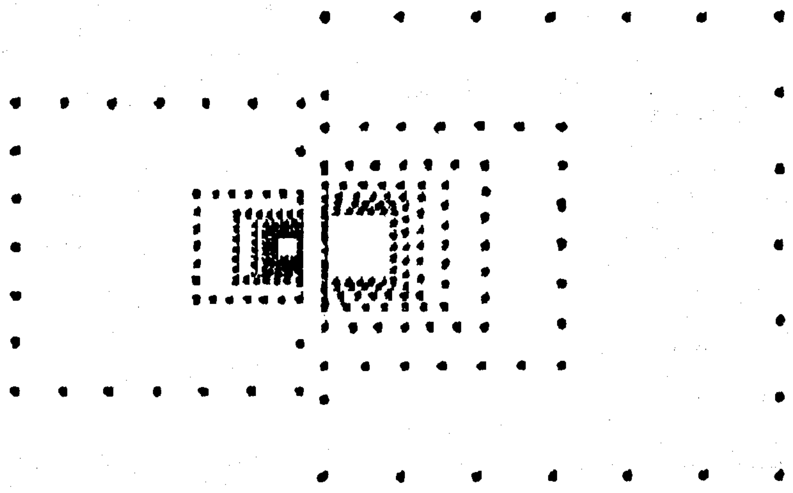

Figure 2. A static representation of a typical trial event in Experiment 1. (In a dynamic display, an observer would have no difficulty determining that the object on the left would be the first to reach the point of observation.)

which observers selected the object with the smallest image size, or the smallest rate of expansion. If the observers had responded to either of these variables in isolation, then all of the entries in the appropriate row of the table would be close to zero. Since this is not the case, it is reasonable to conclude that the observers were sensitive to the visual information described by Equation 12, which uniquely specifies time until collision for objects approaching at a constant velocity. It is interesting to note, however, that none of the naive observers were consciously aware of any particular strategy for performing the task. When asked to describe their decision criteria during a debriefing session, they most often responded, "I don't know."

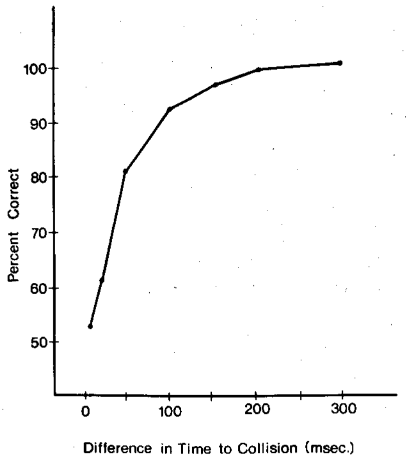

Figure 3. The percentage of correct responses for five observers as a function of the difference in time to collision between two simulated objects.

Experiment 2

Experiments 2 and 3 attempted to discover the visual information that allows an observer to perceive where a free-falling projectile will contact the ground. The method of inquiry was first to examine the various sources of information that are available in principle, and then, using computer simulations, to discover empirically the specific aspects of the available information that are actually used by human observers.

A detailed analysis of the optic projection of a free-falling projectile is provided in the Appendix. The analysis assumes that horizontal acceleration is zero and that vertical acceleration is constant, both of which are guaranteed in a natural environment if the coordinate system is fixed to the direction of gravity. Within this frame of reference, there are several sources of information that uniquely specify whether a free-falling pro-

Table 1
The Percentage of Trials in Experiment 1 for Which Observers Selected the Object With the Smallest Projected Size or the Smallest Projected Rate of Expansion

	Difference in time to collision (msec)						
Selection	300	200	150	100	50	20	10
Smallest projected size (cm)	34.3	35.3	38.5	38.0	44.1	50.6	44.0
Smallest projected rate of expansion (cm/sec)	20.0	25.8	28.5	30.9	39.7	49.4	44.1

jectile will land in front or in back of the point of observation.

The simplest source of information is also the most restrictive. If an object is at the highest point in its trajectory (see Equations 7, 9, and 14), and TE is the amount of time since it first crossed the horizontal axis (see Figure 1), then it will always land in front of the observer if:

$$TE < R'/VR'. \tag{18}$$

The visual information described by Equation 18 is of limited value because it is only defined at the exact moment when an object is at the top of its trajectory. Some other explanation is obviously necessary to account for a baseball player's ability to accurately judge where a fly ball will land while it is still rising.

Other potential sources of information are more generally applicable. For example, it can also be demonstrated that an object will land in front of the observer if:

$$-AY/2R > VY' * VR'/(R'**2). \tag{19}$$

The visual information described by Equation 19 is available at all points in an object's trajectory, but the critical value that marks the boundary between landing in front or in back of the point of observation is dependent on the ratio between the size of the object and the acceleration due to gravity. To use such information an observer would have to discover the critical value through repeated observations. This might be appropriate in baseball, where the size of the ball and the acceleration due to gravity are always the same from one event to the next, but it cannot account for an observer's ability to catch unfamiliar objects.

Another condition that uniquely specifies that an object will land in front of an observer is:

$$0 > AY'. \tag{20}$$

The visual information described by Equation 20 is surprisingly general. Unlike other sources of information, it does not involve projected size (R') or rate of expansion (VR') and is fully defined for the motion of a single point (see Figure 4). It is available at all points in an object's trajectory and does not require any prior knowledge on the part of the observer. It is important to keep in mind, however, that Equation 20 is a second-order differential equation. It is reasonable to expect that human observers are sensitive to this information, since they are apparently unrestricted in their ability to judge where a free-falling projectile will land. However, since the visual information described by Equations 18 and 19 is of a lower order of complexity, an observer's performance might improve whenever that alternative information is also available.

Experiments 2 and 3 were specifically designed to test these hypotheses. Experiment 2 examined an observer's ability to judge where a simulated object will land when the size and acceleration of the object are either fixed or varied across trials and the displays are terminated before the object reaches the top of its trajectory. Experiment 3 examined performance under similar conditions when the top of the trajectory is included as part of each display.

Method

The apparatus and general procedure were roughly equivalent to those used to Experiment 1. A trial event

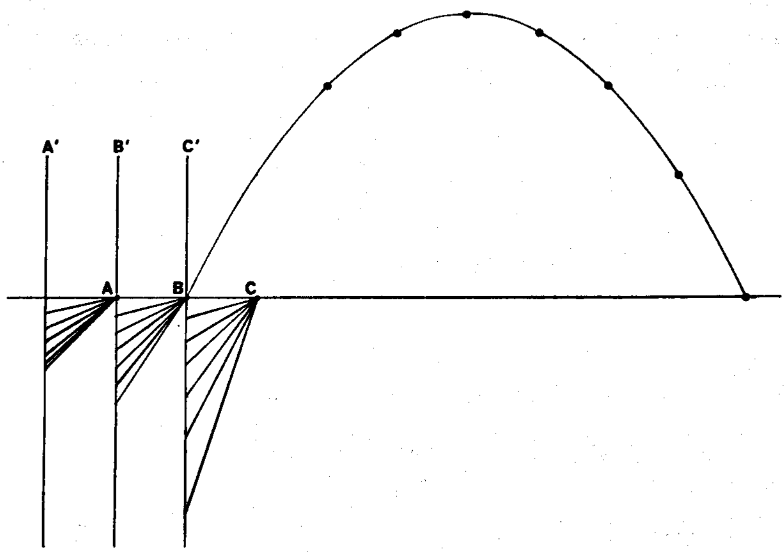

Figure 4. The optic projections defined over time for a free-falling projectile at three different points of observation. (Each of the three vantage points [A, B, C] is separated from its corresponding projection surface [A′, B′, C′] by a unit distance. The density gradient among points on the projection surface uniquely specifies whether the projectile will land in front or in back of the point of observation.)

consisted of 18 display frames presented within a .75-sec interval, which appeared to the observer as an object approaching the display screen on a parabolic path of motion (see Figure 5). In most cases the display was terminated before the object reached the top of its arc. The observer's task was to decide where the object would eventually land. A left-hand response key designated that the object would land in front of the observer, and a right-hand response key designated that the object would land at the exact point of observation.

Immediate feedback was provided after each response. Following a correct response, a + was presented on the display screen for 1 sec, and following an incorrect response, a − was presented.

As in Experiment 1, each event was a mathematically accurate simulation of an actual approaching object. There were three separate conditions for which the parameters of this simulation were systematically varied.

In Condition A, the width (R) of the approaching object was fixed at 38.1 cm on every trial and the downward accelaration (AY) was also fixed at 9.8 m/sec^2. This was intended to simulate the conditions for catching a baseball, where the size of the ball and the acceleration due to gravity are constant. The starting distance (X) was selected at random from possible values of 30.5, 33.5, 36.6, 39.6, 42.7, and 45.7 m, and the firing angle (ϕ) was selected at random from possible values of 10°, 15°, 20°, 25°, 30°, 35°, 40°, 45°, 50°, and 55°. These particular values were chosen so that the display would always be contained within the visible portion of the CRT screen. Once these parameters were selected, a firing velocity was computed so that the object would land at the point of observation on half of the trials (selected at random) and in front of the point of observation on the remaining trials. Considering all possible combinations of these parameters, there were 120 unique events, all of which had an equal probability of occurring on any given trial.

Condition B was included to examine whether prior knowledge of a moving object's size or acceleration can facilitate an observer's judgment about where the object will land. To eliminate such knowledge in Condition B, the width (R) of the simulated object was selected at random on each trial from possible values of 25.4, 38.1, and 50.8 cm. The possible values of downward acceleration (AY) were 6.8, 9.8, and 12.8 m/sec^2. The possible firing angles (ϕ) were 10°, 25°, 40°, and 55°, and the possible starting distances (X) were 30.5 and 45.7 m. With two possible landing distances for each combination of the above parameters, there were 144 unique events that could potentially occur on any given trial.

Condition C was included to examine whether changes in size provide necessary information for judging landing distance. The possible parameter values were identical to those used in Condition A, except that the width (R) of the simulated object was made so small that it appeared on the display screen as a single point throughout its entire trajectory.

The width (R') of the image of an object and its height (Y') on the display screen were computed for each frame by the following equations (see Figure 1): $R' = RH/X$; and $Y' = YH/X$.

The experiment consisted of 36 blocks of 50 trials each. As described above, the trial events were designed so that the simulated object would fall in front of the point of observation on half of the trials. The actual value of this landing distance was the same within a block of trials but was systematically varied across blocks from possible values of .6, 3.0, 6.1, and 9.1 m. Three blocks were run for each condition and each landing distance, and the blocks were arranged in order of increasing difficulty.

Six well-practiced observers including the author participated in the experiment. All of the observers except the author were unaware of the differences between the conditions and were paid $3/hr. for their services.

Results and Discussion

Figure 6 shows the percentage of correct responses as a function of the difference between the two possible landing distances on a block of trials.

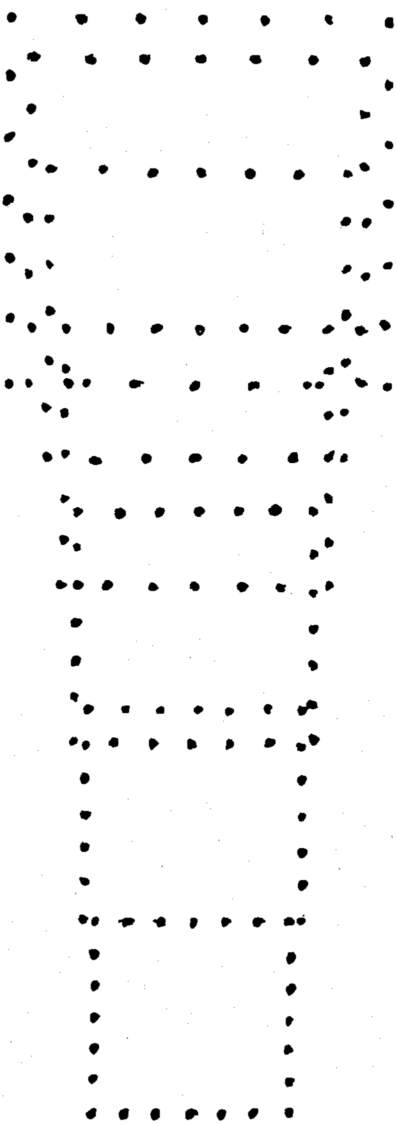

Figure 5. A static representation of a typical trial event in Experiments 2 and 3.

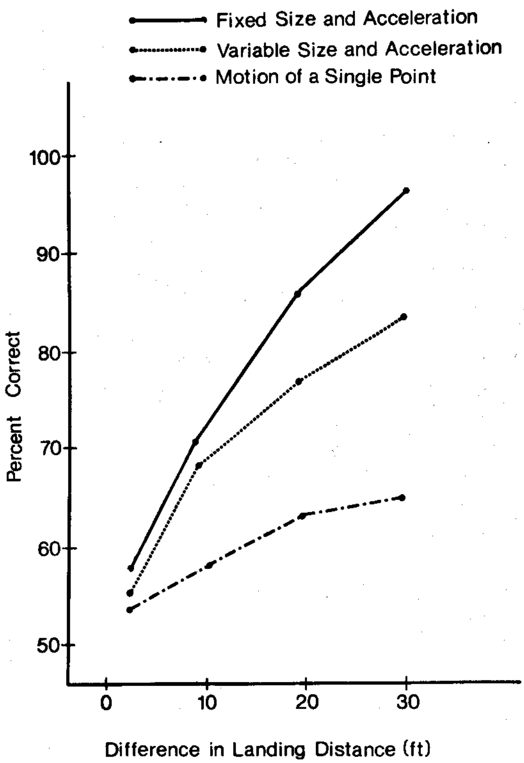

Figure 6. The percentage of correct responses for six observers as a function of the difference between the two possible landing distances for each block of Experiment 2. (1 ft. = .3048 m.)

An analysis of variance[1] revealed that the motion of an isolated point in Condition C produced significantly more errors than the motion of a square configuration of points in Conditions A and B, $F(1, 115) = 139.31$, $p < .001$. This finding suggests that the observers were unable to take advantage of the information described by Equation 20, which is fully defined by the motion of a single point (see Figure 4). It is reasonable to conclude that the necessary information for judging landing distance involves changes in the size of a configuration of many elements. This conclusion is supported by the fact that all of the observers reported that there was a sharp phenomenal distinction between the two viewing conditions. The simulated square objects were perceived as moving in depth, whereas the isolated points were perceived

[1] To make direct comparisons between Experiments 2 and 3, the error term of this analysis was determined from the combined data of both experiments.

as moving in two dimensions on the display screen. Moreover, when asked to describe their decision criteria for performing the task, the observers reported that their responses in Conditions A and B were based on whether the simulated square object "looked like it would fall short" of the observation point, and that their responses in Condition C were based on the amount of vertical displacement observed during each display. Other things being equal, the image of an object that would land at the point of observation would rise higher on the display screen than the image of an object that would land in front of the point of observation. This guessing strategy is only marginally effective, however, since vertical displacement was also affected by varying starting distance, firing velocity, and firing angle. This explains why the level of performance in Condition C did not rise above 64% accuracy.

Data analysis revealed that there were significantly fewer errors in Condition A than in Condition B, $F(1, 115) = 14.64$, $p < .001$. This general pattern of results was obtained for five of the six observers, and it is consistent with the mathematical analysis of free-falling projectiles presented in the Appendix. Because size and acceleration were assigned fixed values on every trial in Condition A, it was possible to exploit the visual information described by Equation 19, which was not available in Condition B because size and acceleration were varied randomly across trials. Apparently, all of the observers except one were able to take advantage of this information, although none of them, including the author, was consciously aware of doing so. The observers had no way of knowing prior to the experiment the particular values of size and acceleration that would be used. To take advantage of the visual information described by Equation 19, it was necessary to discover the critical value ($-AY/2R$) by repeated observations.

Experiment 3

Experiment 3 was designed to determine if human observers are sensitive to the visual information described by Equation 18. This information is only available at the moment an object is at the top of its trajectory, but unlike the information described by Equation 19, it does not depend on specific values of size and acceleration. Thus, if observers are able to take advantage of this information, the detrimental effect of varying size and acceleration should be significantly reduced when the highest point in a simulated object's trajectory is included in each display.

Method

The procedures were identical to those used in Experiment 2 with the following exceptions: A trial event consisted of a variable number of display frames presented at a rate of 25 per sec. A display was terminated when the simulated object reached the top of its trajectory. The possible firing angles were also changed to 5°, 10°, 15°, 20°, 25°, 30°, 35°, and 40° for Conditions A and C where size and acceleration were held constant, and 5°, 16°, 28°, and 40° for Condition B where size and acceleration were varied across trials. The same six observers who participated in Experiment 2 again volunteered their services.

Results and Discussion

Figure 7 shows the percentage of correct responses as a function of the difference between the two possible landing distances on a block of trials.

The results are equivalent to those obtained in Experiment 2, indicating that the highest point in a simulated object's trajectory has little effect on an observer's ability to judge landing distance. As in Experiment 2, the motion of an isolated point produced significantly more errors than the motion of a square configuration of points, $F(1, 115) = 139.00$, $p < .001$, and there was a significant detrimental effect from varying size and acceleration, $F(1, 115) = 4.78$, $p < .05$. This latter finding suggests that the observers did not take advantage of the available information described by Equation 18, which is independent of particular values of size and acceleration. Although the effect was smaller than the one obtained in Experiment 2, a post hoc comparison revealed that the difference was not significant ($p > .1$).

It is important to keep in mind when evaluating the results of Experiments 2 and 3 that much of the optic structure present in natural events was conspicuously absent

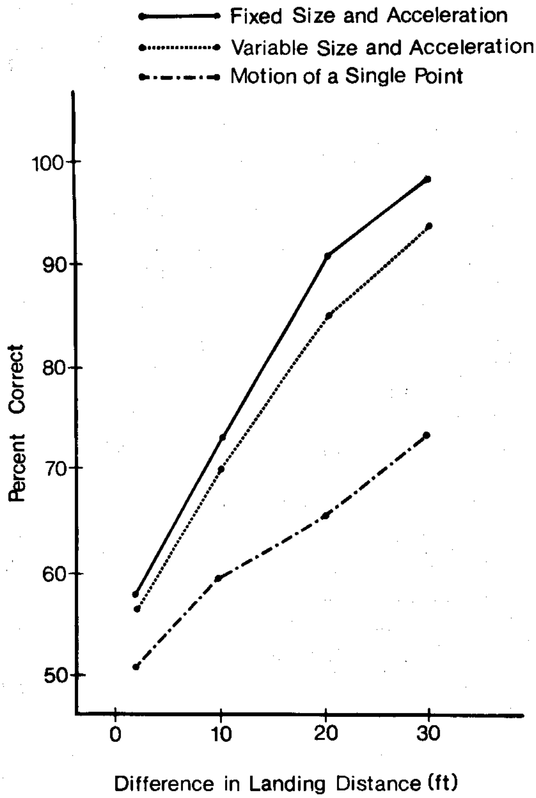

Figure 7. The percentage of correct responses for six observers as a function of the difference between the two possible landing distances for each block of Experiment 3. (1 ft. = .3048 m.)

from the computer-generated displays. Since the center of each approaching object was confined to the observer's saggital plane throughout its trajectory, there was no information provided by components of motion perpendicular to the line of sight. No information was provided by gradients of illumination on the surface of an object, and there was no ground plane, horizon, or clouds to form a background for the object's motion. In spite of this impoverished stimulus information, the observers were able to perform the task with surprising accuracy. Although each trial was terminated when the simulated object was 15.2 to 30.5 m from the point of observation, the observers could distinguish between a 0 and 9.1 m landing distance with almost 100% accuracy. They could make this distinction with over 90% accuracy even when size and acceleration were randomly varied across trials.

It is probably best to be circumspect in drawing conclusions about the specific source of information which served as a basis for the observers' judgments. The results seem to indicate that the observers relied on the visual information described by Equation 19, since they did not take advantage of the alternative information described by Equations 18 and 20. However, there is no reason to believe that these three equations provide a complete inventory of the information that was potentially available in the displays. This uncertainty is highlighted since none of the observers, including the author, were consciously aware of any particular strategy for performing the task. The ability to judge where a free-falling projectile will land generally requires a considerable amount of practice and concentration, but it does not involve cognitive skills such as mental arithmetic or logical reasoning. It is instead a predominantly perceptual ability that is often observed in subjects whose cognitive skills are quite limited, such as small children and dogs. Since the perceptual processes involved in judging landing distance are performed tacitly, an observer's introspections provide few insights about the visual information on which these processes are based.

Experiment 4

Experiment 4 examined observers' sensitivity to the visual information about acceleration described by Equations 10 and 13.

Method

The apparatus and general procedure were roughly equivalent to those used in Experiments 2 and 3. A trial event consisted of a variable number of display frames presented at a rate of 25 per sec, which appeared to the observer as an object approaching the display screen in a rectilinear path. An event was terminated when the image of the approaching object became too large to be completely contained on the display screen. The observer's task was to press a left-hand response key if the object appeared to be accelerating and a right-hand response key if the object appeared to be decelerating. Immediate feedback was provided after every trial with a 1-sec presentation of either a + or a − on the primary display screen.

There were two conditions in which the parameters of the simulation were systematically varied. In Condition A, the size (R) of the simulated object was always assigned a value of 15.2 cm, whereas the starting distance (X) and starting velocity (VX) were selected at random prior to each trial. The possible starting distances were 15.2, 16.8, 18.3, 19.8, 21.3, 22.9, and 24.4 m, and the possible starting velocities were 24.4, 25.9, 27.4, 29.0, 30.5, 32.0, and 33.5 m/sec. In Condition B, the size of the object was also varied across trials from possible values of 7.6, 15.2, and 22.8 cm. The possible starting distances were 15.2, 18.3, 21.3, and 24.4 m, and the possible starting velocities were 24.4, 27.4, 30.5, and 33.5 m/sec.

The experiment consisted of 30 blocks of 50 trials each. The absolute value of acceleration (*AX*) never varied within a block, but was assigned a positive value on half the trials (selected at random) and a negative value on the remaining half. Thus, there were 98 possible events for each block of Condition A, and 96 possible events for each block of Condition B. The absolute value of acceleration was systematically varied across blocks from possible values of .6, 3.0, 6.1, 9.1, and 12.2 m/sec^2. Three blocks were run for each condition combined with each value of acceleration, and the blocks were arranged in order of increasing difficulty. Three of the observers who participated in Experiments 2 and 3 (including the author) again volunteered their services.

Results and Discussion

Figure 8 shows the percentage of correct responses as a function of the difference between the two possible accelerations (positive and negative) on a block of trials.

The most obvious conclusion that can be drawn from these results is that the observers' sensitivity to acceleration was extremely poor. When the size of the simulated object was varied across trials, the highest level of performance was less than 80% for differences in acceleration almost two and a half times the acceleration due to gravity. This finding is in close agreement with other experiments on the perception of acceleration perpendicular to the line of sight (e.g. Gottsdanker, 1952a, 1952b, 1955; Gottsdanker, Frick, & Lockard, 1961; Runeson, 1974, 1975).

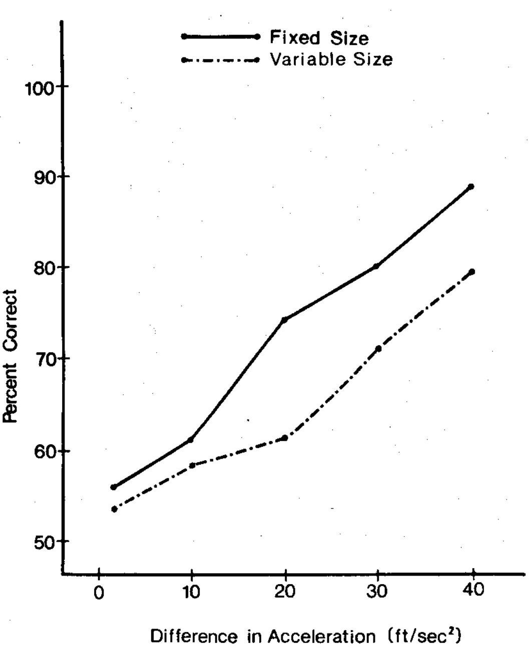

Figure 8. The percentage of correct responses for three observers as a function of the difference between the two possible accelerations for each block of Experiment 4. (1 ft. = .3048 m.)

Both naive observers reported at the conclusion of the experiment that their responses were not based on acceleration at all, but rather, on the rate of expansion just prior to the termination of a trial event. This is not an unreasonable strategy. As the experiment was designed, the maximum rate of expansion was generally greater than 6.1 m/sec for accelerating objects and less than 6.1 m/sec for decelerating objects. The percentage of trials outside of these categories was increased somewhat by varying starting distance, starting velocity, and size, but these controls were only effective for the smaller differences in acceleration. (This could explain why performance was facilitated when the size of the simulated object was assigned a fixed value on every trial.)

To verify the observers' verbal reports, the trials were divided into five different categories based on their maximum rates of expansion. Figure 9 shows the percentage of deceleration responses in each category for each difference in acceleration. As is evident in the figure, there was a high proportion of deceleration responses for low rates of expansion, and a low proportion of deceleration responses for high rates of expansion. This relationship holds even for small differences in acceleration, where the observers' ability to distinguish acceleration from deceleration was no greater than chance. These findings suggest that human observers are not sensitive to the available information about a moving object's acceleration or deceleration toward the point of observation (cf. Lee, 1976).

General Discussion

The present paper has attempted to develop a mathematical analysis of visual in-

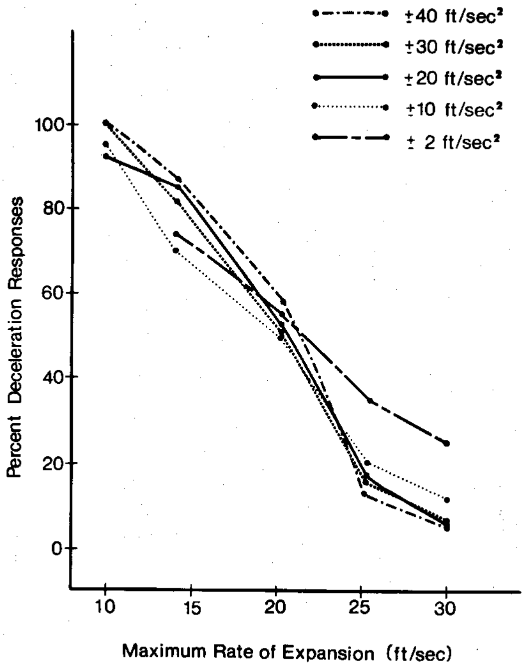

Figure 9. The percentage of deceleration responses for three observers as a function of maximum rate of expansion for each value of acceleration used in Experiment 4. (1 ft. = .3048 m.)

formation that can account for an observer's ability to effectively interact with moving objects. The analysis was based on previous demonstrations that rigid motion is uniquely specified in terms of optic variables, and that given rigidity, the three-dimensional form and orientation of a moving object are also specified (Ullman, 1977, 1979). Within this framework it was formally demonstrated that there is visual information available to an observer about a moving object's angle of approach, changes in its velocity and acceleration, whether its velocity and acceleration are positive or negative, its time until collision with both the horizontal and vertical axes, and whether it will cross the horizontal axis in front of or behind the point of observation.

In an attempt to demonstrate the perceptual validity of this analysis, several experiments were reported in which observers' sensitivities to various sources of information were examined using computer simulations. In Experiment 1, observers judged relative time to collision for two approaching objects in the special case of rectilinear motion where the value of acceleration was zero. In Experiments 2 and 3, observers judged whether a simulated, free-falling projectile would land in front of the point of observation. In Experiment 4, they judged whether an approaching object was accelerating or decelerating.

To perform the complex tasks required by these experiments, the observers had to be sensitive to a variety of individual optic variables including size, position, changes in size and position, and changes in the rate of change. They also had to be sensitive to invariant relationships defined over these variables that are normally in one-to-one correspondence with important properties of environmental events. In Experiment 1, for example, the observers made accurate judgements about relative time to collision based on a ratio between an object's projected size and its rate of expansion. Many of the relationships among optic variables had a critical value that separated an event into two distinct categories, such as whether an object was accelerating or decelerating or whether it would land in front or in back of the point of observation. In Experiments 2 and 3, the critical value for Equation 19 was dependent upon particular values of size and acceleration and had to be discovered by repeated observations.

Many important properties of environmental events are multiply specified. The outfielder problem provides a typical example, since there are at least three sources of visual information that can uniquely specify where a baseball will land. However, the results of these experiments indicate that some of the potentially informative relationships among optic variables, although available in principle, cannot be exploited by human observers. None of the participants in Experiments 2, 3, and 4 showed any indication that they were able to take advantage of second derivatives with respect to time, even when the detection of this higher order information was absolutely essential for performing the task. It was also concluded that observers cannot take advantage of the special case information for judging landing distance described by Equation 18. This information is only defined at the moment when an object is at the highest point in its trajectory, but the presence or absence of this information had no significant effect on performance.

Other perceptually salient properties of visual stimulation provide approximate information about environmental events. In Experiment 4, for example, the observers made judgments about a simulated object's acceleration based on its rate of expansion. These two variables are highly correlated, but they are not in perfect one-to-one correspondence. Thus, the observers could only achieve a consistently high level of performance when the differences in acceleration were extremely large.

All of the experiments were designed so that performance could not be mediated by an observer's expectations. This was accomplished by assigning random values to all of the relevant physical variables needed to generate a display. Although performance improved significantly when certain physical variables such as size and acceleration were fixed in Experiments 2, 3, and 4, the observers had no way of knowing prior to an experiment the particular values of the variables that would be used. The absence of cognitive mediation is also suggested by the fact that observers were generally unaware of the specific information on which their performance was based.

One important issue that the present research does not address is implementation. Describing the available information about moving objects is only a prerequisite to the potentially more difficult problem of describing the physical mechanisms that are able to detect that information. A workable solution to the problem of implementation would have many practical applications.It would allow the construction of optical control systems to prevent passenger vehicles from crashing into other objects, or of visually guided robots for exploring the moon. Developing techniques for visual analysis of moving objects is a growing area of research in the field of artificial intelligence. Prager (1979), for example, has written a computer program that can analyze visual input provided by a camera mounted on a moving automobile. The program can isolate the boundaries of different objects from properties of visual flow. It can determine the trajectory of an object relative to the plane of observation and can compute time until collision whenever appropriate. However, because of the inherent limitations of serially organized digital computers, the program cannot satisfy one of the most fundamental demands on biological organisms, namely, that any analysis of a changing optic array must be performed in real time in order to be useful for responding appropriately to environmental events.

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(*Appendix follows on next page*)

Appendix

Special Case Solutions Involving Time Until Collision

The following derivations are for the special case of parabolic motion where forward acclera-tion (AX) is zero and downward acceleration (AY) is constant. As described in the text, Equation 16 specifies the amount of time (TX) until $X = 0$; Equation 17 specifies the amount of time (TY) until $Y = 0$. A moving object will always cross the horizontal axis in front of the observer if $TY < TX$. For the special case where forward acceleration is zero, Equation 16 can be reduced by substituting Equation 12 to obtain $TX = R'/VR'$.

If the observed object is at the highest point in its trajectory and TE is the amount of time since it first crossed the horizontal axis, then it is easy to see that $TE = TY$ and that the object will land in front of the observer if $TE < R'/VR'$. Alternatively, by substituting TX for TY in Equation 17, we find that the object will also land in front of the observer if:

$$0 > ((R'/VR')**2)(AY/2Y) + (R'/VR')(VY/Y) + 1.$$

Substituting Equation 14, we get:

$$0 > ((R'/VR')**2)(AY/2Y) + (VY'/Y')(R'/VR').$$

Since Y', R', and VR' must be positive, this can be reduced by arranging terms and substituting Equations 4 and 1 to obtain:

$$-(VY' * VR')/(R'**2) > AY/2R.$$

Substituting Equation 11, we get $0 > AY'/2R'$. Finally, since R' must be positive, this reduces to $0 > AY'$.

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