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ORIGINAL ARTICLE

# Wide distribution of external local sign in the normal population

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**Abstract** The extent of the apparent visual field was determined for a group of 78 naïve visual observers. We find that there exists a minority (less than 10%) that is essentially veridical, but that the majority of the population experiences an apparent visual field of only about 90°, thus *much* narrower than the dioptrics of the eye would suggest (a little over 180°). This is in good accordance with available (albeit mainly anecdotal) evidence, though formal data have been lacking thus far. The finding is discussed in the context of metrical calibration of the topological structure of the visual field, an aspect of "local sign".

## Introduction

In haptics the orientation of rods is referred to the *hand orientation* of the (blindfolded) observer, rather than his/her *physical surroundings* (Kappers, 2004; Kappers & Koenderink, 2004). This leads to huge errors in the judgement of parallelity for rods that are located far apart in the left-right dimension on a horizontal table top in front of the observer requiring rotations in the shoulder joint.

Anecdotal evidence suggests that spatial surface attitude is likewise referenced to the *visual direction* (which changes due to rotations about the center of the eye-ball) rather than the *straight ahead* direction. Thus one expects a hemisphere with the eye at its center to appear as a frontoparallel plane, whereas frontoparallel planes should appear as convex towards the observer (see Fig. 1). That this might indeed be the case is suggested by intuitive (non-perspective) drawings of scenes of panoramic extent from various periods and cultures (Barre & Flocon, 1968; Dubery & Willats, 1983; Gombrich, 1960; Pirenne, 1970).

Early authors (Pirenne, 1970) describe the visual field as a cone with (full) top-angle of 90°. It is not clear whether the phenomenal or the anatomical extent is intended though there is no doubt that these authors were aware of the fact that human observers visually experience about a half-space in front of them. Later authors are more specific. For instance, Helmholtz (1892), after discussing the dioptrics of the eye and the nature of eye movements, remarks:

> ... the field of view (G. Sehfeld) of each eye, which in the geometrical sense measures from right to left about 180°, appears much narrower. For the most left and right lying objects that one can still see and whose straight connection is a line through our eye, still appear to us as lying in front of us, as if their visual directions made an obtuse or rather right angle with each other. If one looks at the sky, such that no terrestrial objects of known position or size intrude in the visual field, then the bright field one has in front of oneself has about the diameter of a right angle from right to left, perhaps even less from top to bottom. It is as if you looked into the external world with your head at a certain depth (Helmholtz, 1892, p. 698, our translation).

A similar remark was made much earlier by Keppler in his *Paralipomena* as he saw (Lindberg, 1976)

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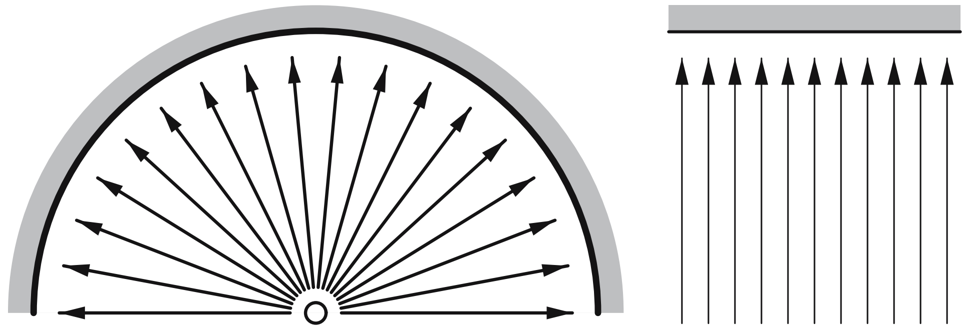

**Fig. 1** Two limiting cases. When the visual rays diverge (as they actually do in external space) a surface normal to all rays is a hemisphere centered at the vantage point (figure at *left*). In case the divergence of the visual rays is not recognized (figure at *right*) the surface normal to all (apparent) visual rays is a frontoparallel plane

> ... both the sun an my shadow as though they were not opposite but both were situated toward the front.

from which he concludes "*... you fall only a little short of being able to see your own ears*". Thus Keppler knew very well that the physical field of view is a hemisphere but he perceived it all as "situated toward the front", just as Helmholtz did. As of to date there appears to exist no formal investigation of this issue.

We describe an experiment in which many (78) naïve, monocular observers reported on the phenomenal shape of a hemispherical surface centered on their vantage point.

## Design of the experiment

The experiment is conceptually simple though there are many possible pitfalls. The essential idea is to have a large number of naïve, monocular observers report on the phenomenal shape of a hemispherical surface about their vantage point. A large number of observers is desirable because there might conceivably exist variations in the normal population, but has the disadvantage that the experiment should be simple to do, non-ambiguous and take only little time. One needs to control for misunderstandings of the task, problems with unambiguously and uniformly reporting of even clear-cut phenomenal experiences and interference from a variety of unwanted cues and predispositions. The experimental paradigm and setup have been designed with these factors in mind. Some trade-offs had to be accepted, these are described here.

It is a requirement that the observers are unaware of the actual structure of the scene in front of them and have no possibilities to find out about it except by way of the ostensible cues. The design aims to offer compelling cues that any local surface element is perpendicular to the (local) line of sight. A texture cue (Gibson, 1950; Gårding, 1992) via a random spatial distribution of identical circular discs was chosen. With the texture elements being uniformly, though randomly distributed over a spherical surface concentric with the vantage point, this is an almost ideal choice. The uniform distribution yields zero texture density gradient (thus "global local frontoparallelity"), there is no anisotropy (thus again "global local frontoparallelity") and the individual texture elements strongly indicate local frontoparallelity.

A large number of black polka dots were pasted on the inside of a translucent hemisphere to form a panoramic pattern. The distribution was uniform, though overlaps of the polka dots were avoided. The hemisphere had a diameter of 120 cm, the polka dots subtended visual angles of about 1.5°, their average separation was *ca*. 6°. The hemisphere was illuminated from behind, thus the polka dots appeared a uniform black against a uniformly white background. The open cross section of the hemisphere was covered by a large wall with a peep hole at the center of the hemisphere (see Fig. 2).

The observers were confronted with a peep hole at eye-height in an otherwise featureless wall. The peep hole was 4 cm diameter forcing monocular viewing. Since the anatomy of many human faces makes it hard to place the eye at the nominal vantage point—the nose being in the way—the wall near the peephole was made slightly pliable. We estimate that the observers manage to put their eye at the intended location give or take about 2 cm in any direction whereas their field of view was the maximum possible. They were encouraged to move their heads and eyes—always constrained by the peep hole—to take in as much of the display as possible. This is desirable since naïve observers have a hard time to maintain head position, eye fixation, and so forth. Moreover, the research question does not imply such constraints.

This approximates the natural condition for "*seeing the scene in front of you*", the visual field in a head-centric frame [G.: *Blickfeld* (Helmholtz, 1892)]. It is different from the artificial case of strict fixation of the straight ahead direction which is preferred for many formal psychophysical or ophthalmological studies. In the latter case one studies the "peripheral visual field" [G.: *Sehfeld* (Helmholtz, 1892)] in an oculocentric frame, which is not our objective.

Although by no means ideal, we believe it would be hard to improve on the present design. The set up should amply suffice for the problem at hand.

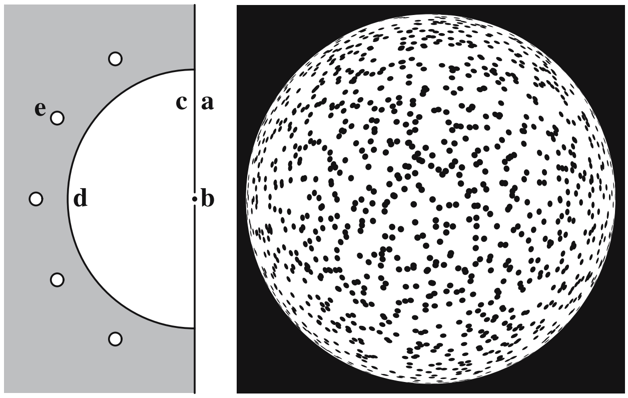


**Fig. 2** Left: a schematic cross-section of the set up. A hemispherical shell (*d*) of translucent, milky plexiglass is illuminated from behind through a bank of light sources (*e*). The shell is covered with a screen (*a*), containing a peep hole at the center *b* of the hemisphere. The interior side of the cover (*c*) is black. As seen from the peep hole the plexiglass hemishere appears uniform, luminous white. The interior surface of the shell is covered with a large number of opaque black occluding circular disks. Seen from the peep hole these appear as black circular polka dots. The dots subtend *ca.* 1.5° of visual angle. They are arranged in a uniform random pattern, avoiding overlaps or tangencies, with an average separation of 6°. Thus, as seen from the peep hole, one sees a uniform random polka dot pattern in any (forward) direction. The frontal side of the cover (*a*) appears as the wall of a room with a peep hole *b* at eye height. The hemispherical structure is completely hidden as seen from the room. The peep hole is 4 cm diameter, thus vision of the interior is constrained to be monocular. The wall near the peep hole is slightly pliable, yielding to the nose as the observer uses eye and head movements to see as much of the hemispherical dome as possible. Most observers are aware of a white, luminous surface, covered with polka dots, of indefinite extent (no apparent boundaries). Right: the interior of the hemisphere in an orthographic frontal view

A 4 cm lateral movement will change the aspect ratio of a polka dot by at most a factor 1.001, which is certainly subliminal, overall size changes are likewise subliminal. Since all polka dots are on a spherical surface concentric with the vantage point any parallactic effects may safely be neglected. One undesirable cue that is not under our control is the accommodation of the eye lens (Ciuffreda, Wang & Vasudevan, 2007). For young observers (about half of our observers were presbyopic though) this is a marginal possibility, though an influence on our results is rather unlikely judging from the (scarce) available data on the topic (Graham, 1966; Palmer, 1999).

Observers were not permitted to see the actual surface in any way but through the peep hole. They were made to promise solemnly not to discuss their experiences with others before the conclusion of the experiment.

A total of 78 volunteers of both genders recruited from the Utrecht University Department of Physics and Astronomy (administrative and technical personel, students, some staff) participated in the experiment, about one-third of them female. All were fully naïve with respect to the task, very few had previously participated in any perceptual experiment at all. Having volunteer observers made it virtually impossible to subject them to a battery of tests for visual functions or do a comprehensive ophthalmological check.

Prior to the experiment we had observers perform a number of dummy tasks in order to familiarize them with some simple geometrical concepts ("horizontal cross section", "depth", "apparent size") and with the (to many naïve people shocking) notion that visual percepts may differ from the physical stimulus. Observers viewed an illuminated sphere, a face mask and an inverted face mask. They were free to look at these objects from all sides, but were instructed to draw the horizontal cross section as seen from a fixed peep hole position. Especially the inverted mask came as a shock to many and forcefully brought down the notion that vision need not be "veridical". This served to prepare the observers to rely on their eye-measure (at least for the purpose of this experiment) and to convince them that "*the observer is always right*". This conviction is crucial in the actual experiment. After this warming up session the observers read a short formal instruction and viewed the actual display.

The observers were handed an A3 sized paper on which the eye position and frontal pole of the surface were indicated, leaving so much free space that virtually any profile might be drawn in without feeling constrained by the edges of the paper (see Fig. 3). Observers were instructed to sketch a horizontal cross section, to scale, using a pencil and doing as many retries as deemed necessary. When satisfied they were asked to trace their best bet with a black marker. Typically this procedure took less than 2 min. After the trial observers were free to (verbally) volunteer their impressions. This evidently served a function though we ignore these remarks.

Observers are not asked to estimate the *apparent distance* to the surface. This is totally ambiguous because not optically specified (except—perhaps—for a minor

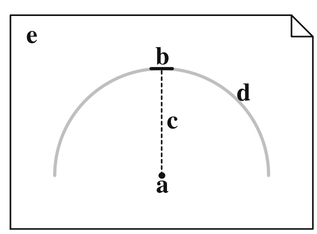


**Fig. 3** The observers were handed a sheet of white paper *e* in A3 format (420 × 297 mm) on which a configuration *abc* was printed. They were told that the dot *a* represented their eye position, the short line *b* the location of the surface they perceived through the peep hole. It was suggested that this surface was frontoparallel to the straight ahead direction *c* at *b*, but otherwise might have any shape or size. The curve *d* (absent in the sheet of paper handed to the observers!) shows the predicted response of a perfectly veridical observer. Notice that the paper had wide enough margins that it would not significantly constrain the responses of the observers

accommodation cue) and irrelevant to the research question. Not knowing the distance is no hindrance to performing the task in the way it is posed. In view of Helmholtz's remark that "everything appears as lying in front of us" a (perhaps unlikely) limiting case appears to be an orthographic projection with parallel (apparent) visual rays (see Fig. 1). Asking for distance might confuse the observer.

It is a time consuming task to run an experiment with naïve volunteers, requiring a great deal of tactfulness and comforting by the experimenter yet making sure that the observers are suitably aware or unaware of various issues and overcoming the natural feeling of "not being able to draw". We used formal, printed instructions and the task of coaching the observers was handled by a single person (coauthor Andrea van Doorn) to ensure that the data are homogeneous.

## Experiment

Among the 78 observers, there were three that failed to obtain the impression of a white surface covered with black polka dots. Even trained observers often require a few seconds to obtain the impression of a stable, integral surface. Initially one has the impression of a luminous, misty space, with black balls floating about in random configuration. The majority of the naïve observers felt convinced of being confronted with a solid surface though.

We digitized all traces and converted them to a standard format. The traces were symmetrized (apparent deviations from bilateral symmetry being obviously unintentional in all cases), the extent measured and subsequently scaled to standard size. This final curve was fit with an eighth order combination of (even) Legendre polynomials (Legendre, 1785; Abramowitz & Stegun, 1972). This representation by a mere five numbers (visual extent and order two, four, six and eight Legendre polynomial coefficients) provided a more than satisfactory fit in all cases. These numbers are considered to be the data collected in the experiment.

An overview of all data is presented in Fig. 4.

## Analysis

After applying several statistical methods (regression analysis, cluster analysis (for up to four clusters), principal components analysis and a factor analysis on the basis of speculative models) it was decided to stick to the simplest possible analysis. All methods point at two major clusters and these are already evident from the elementary analysis. No doubt a more sophisticated method of analysis would yield additional insights when combined with further data descriptive of the visual functions of the observers, but this remains for future work.

The data are very robustly summarized through only two parameters (see Fig. 5): the "apparent field of view" $\lambda$ (expressed in degrees) and the overall shape as indicated by the "shape angle" $\tau$ (also expressed in degrees).

The veridical field of view is 180° whereas the veridical shape angle is 90°. A frontoparallel plane would imply a shape angle of 180°, whereas a convexity turned towards the observer would imply a shape angle in excess of 180°.

In Fig. 6 we show histograms of the apparent fields of view $\lambda$ and the shape angles $\tau$ over all observers:

- The histogram of the apparent fields of view $\lambda$ is clearly *bimodal*. One (minor) mode is centered at the veridical value of 180° whereas the major mode is much broader and roughly centered at 90°. Values near $\lambda = 0°$ (formally indicating true orthographic projection) are absent, though this is perhaps not surprising since the observers were required to draw *something*. There exist outliers up to 225° at the other side. The latter cases correspond to apparent field of views "extending beyond the ears", evidently exceeding the veridical value and even the width of the physiological field of view;
- The histogram of shape angles $\tau$ is very broad. Perhaps there is a (minor) mode at 180°, which is the case of the frontoparallel plane. However, the major mode is broadly distributed over the (roughly) 90°–150° range. Thus the shapes are typically much flatter than the veridical value of 90°.

There exists no significant correlation between these parameters ($R^2 = 0.085$). For a simple test we split the data in sets $\lambda < 135°$ or $\lambda > 135°$. From the whisker box plots for $\tau$ (Fig. 7) it seems that smaller apparent fields of view imply flatter curves. In any case, for $\lambda > 135°$ the curves always turn a concave side to the observer.

Only a minor, though not unsubstantial, group of naïve observers has a roughly veridical visual experience (only 20% of the observers are in a 30° range about the veridical value $\lambda = 180°$). For most observers the apparent field of view is more like 90° or even less (down to 45°). Likewise, the shape angles are only in a veridical range (a 30° range about the veridical value $\tau = 90°$) for a minor (though again substantial) group of observers (less than 23%). For the majority of observers the surface is experienced as much flatter than it actually is, even totally flat impressions not being rare.

## Discussion

### The empirical findings

About 4% of the observers failed to see a surface in front of them. Their reports suggest that they perceived the kind of indefinite "misty" space as described by observers

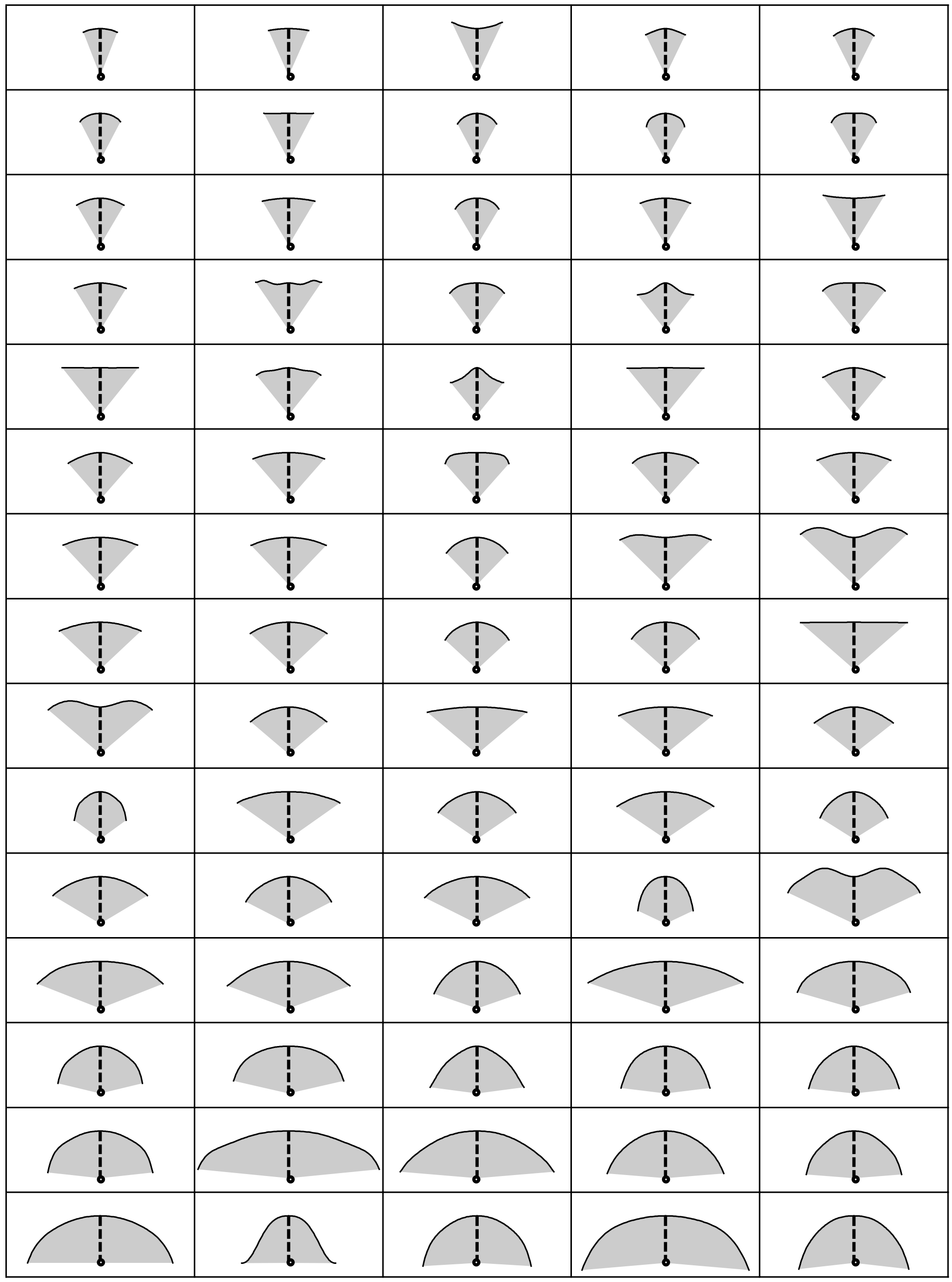

**Fig. 4** All data collected in the experiment (total of 75 observers since three of the 78 observers failed to perceive a surface at all). These data have been sorted with respect to the value of the parameter $\lambda$, that is the apparent width of the visual field as formally defined in Fig. 5

confronted with a Ganzfeld (Metzger, 1975). Apparently most people experience the blue sky as a *surface* (Minnaert, 1942) (the "vault of heaven"), although it is equally featureless as the Ganzfeld. Most of the reports on the apparent geometry of Ganzfelds or the blue sky are anecdotal and involve only a small number of—usually highly trained—observers though. This is an issue on which little can be said at this stage. We discuss only the results obtained with the 75 observers who experienced a surface in front of them.

The most salient facts are:

- Apparently the normal population is far from being homogeneous, there exist both qualitative and quantitative variations that are surprisingly large;
- The median apparent field of view of the 75 observers is only 56% of the veridical value, close to the top angle of the "cone of vision" according to the Greek authors and close to the value estimated by Helmholtz;
- The median shape angle is 1.44 times the veridical value, thus the spherical shape is typically perceived as much flatter than it is, though certainly more curved than a frontoparallel plane. It is similar to the "vault of heaven" as experienced by normal observers when looking at a large extent of open blue sky (Minnaert, 1942);
- Somewhat fewer than 10% of the observers had veridical apparent visual fields (defined as $|\lambda - 180°| \leq 15°$) and veridical apparent shape (defined as $|\tau - 90°| \leq 15°$)

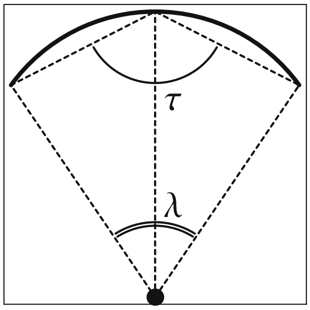


**Fig. 5** Definition of the parameters $\lambda$ and $\tau$. The parameter $\lambda$ denotes the extent of the apparent field of view whereas the parameter $\tau$ is a robust measure of the shape (*global curvature*) of the apparent surface

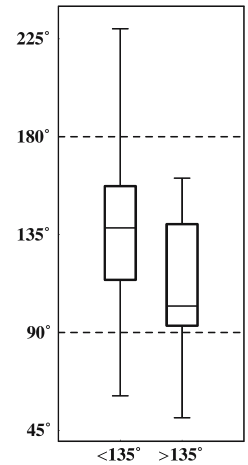


**Fig. 7** Whisker box plots for the shape angle (parameter $\tau$) for two groups of observers. The groups are those with apparent visual field (parameter $\lambda$) smaller or larger than 135°. The horizontal line in the boxes denote the medians, the box limits the 25 and 75% quantiles. The *lower and upper limits* denote the most extreme outliers (total range). The *upper horizontal broken line* denotes the case of a flat frontoparallel plane, the *lower broken line* denotes the veridical case (*semicircular shape*)

experiences. Though only a minor group, the very existence of such observers is conceptually important.

These facts may be interpreted in terms of the "local sign" of retinal locations. However, in order to do so it is necessary to distinguish between two conceptually distinct meanings related to the notion of "local sign". The conventional notion was introduced by the philosopher Hermann Lotze (1881) and—though in general use today—fails to recognize the distinction. There being no generally recognized terms we will refer to the two meanings as the "internal" and "external" local signs.

## Local sign

### Internal local sign

The *internal local sign* has to do with the *topological structure* of the visual field. The visual field appears as a two-dimensional manifold, topologically equivalent to the two-dimensional disc. Apparently the mind treats some neural activities as due to close (or overlapping) causes whereas others are treated as remote from each other. Here the notion of "overlap" or "distance" refers to the "visual field", a mental entity. The notion does not explicitly depend on the (neurophysiological) concept of "somatotopy" (Hubel, 1955).

A likely mechanism [perhaps "linking hypothesis" (Brindley, 1970) is a better term] was suggested by Helmholtz, 1977 (1878): two neurons are treated as representing overlapping regions of the visual field if their signals are significantly correlated. If the neurons are able to provide mutually uncorrelated signals they represent disjunct regions of the visual field. Helmholtz conceived of the notion as he noticed that patients with toothache cannot localize the bad tooth as being in the upper or lower jaw, evidently due to the fact that teeth at corresponding locations in the upper and lower jaw necessarily send correlated signals to the brain when chewing food. The idea yields a purely functional basis for the topology of the visual field (Koenderink, 1984) that requires no reference to the external world, thus is purely "internal".

Patients with *tarachopia* (Hess, 1982) apparently have normal retinas but "scrambled visual fields" (see Fig. 8). The physiological basis for this impairment is unknown.

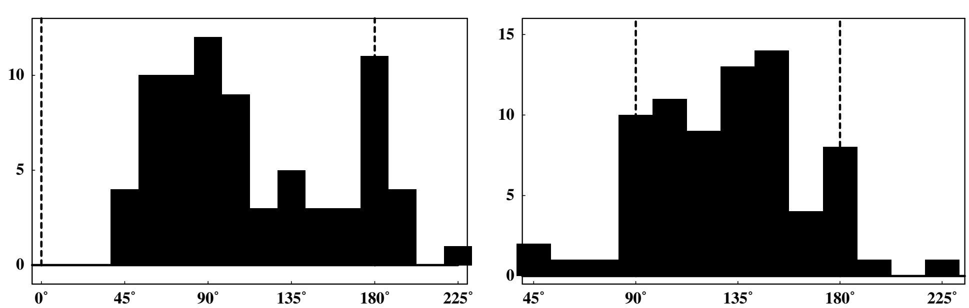


**Fig. 6** Left: histogram of the $\lambda$ parameter (the apparent field of view) for the 75 observers. The veridical value is 180° (*rightmost broken line*), whereas "orthographic projection" (parallelity of all apparent visual rays) would imply a value of 0° (*leftmost broken curve*). Right: Histogram of the $\tau$ parameter (the "shape angle") for the 75 observers. The veridical value is 90° (*leftmost broken curve*) whereas a frontoparallel *plane* (implied for the case of "orthographic projection", or parallelity of all apparent visual rays) obtains for a value of 180°

**Fig. 8** An original (*left*) and a replication with minor (*center*) and fairly strong (*right*) amount of local scrambling of image elements. This simulates the effects of "tarachopia", a form of amblyopia that apparently involves an insufficiently developed *internal* local sign. All the image elements are present (the intensity histograms of the three renderings are identical), but their spatial order has been partly lost in tarachopia. The tarachopic visual field is like a scrambled jig-saw puzzle

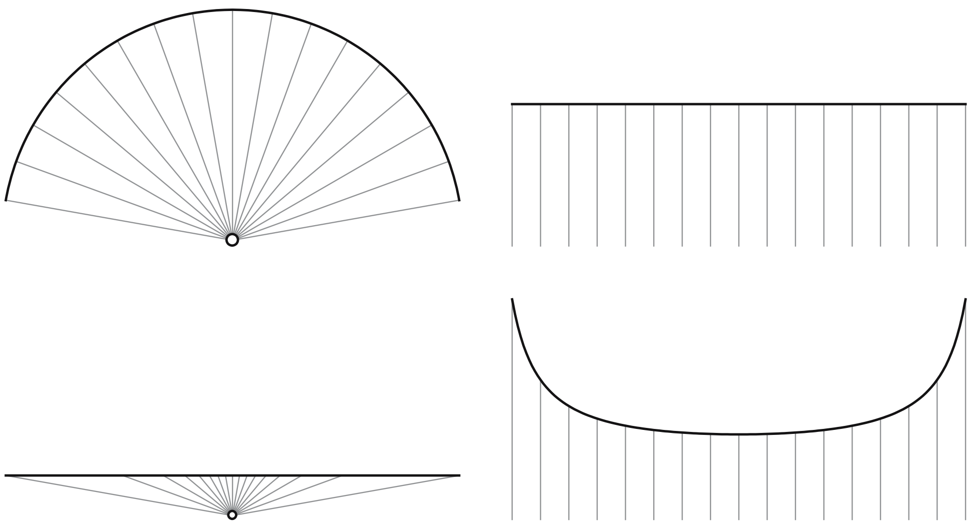

**Fig. 9** At left top an equidistant and at left bottom a frontoparallel section of a surface. At right the representation for a hypothetical observer with perfect internal local sign but totally lacking external local sign, under the assumption that the texture cue perfectly specifies local slant. Local slant is zero for all visual directions in the case of the equidistant surface, but increases monotonically with eccentric angle for the frontoparallel surface. Due to lack of external local sign all visual directions are taken identical, due to internal local sign they are equally spaced (as the true angular spacing is). The surfaces drawn at right have the same local slant as those at the left. Due to lack of external local sign the slant is simply referred to the local "visual ray". Thus the equidistant surface must appear frontoparallel to such an observer, whereas the actual frontoparallel surface appears as curved with the convexity turned towards the observer

These cases perhaps provide evidence for the functional (rather than anatomical, e.g., due to somatotopy) origin of internal local sign.

To summarize, one might say that an intact internal local sign allows the observer to see the topological and possibly metrical relations of places in the visual field *in relation to each other*. Thus internal local sign has nothing to do with the functional relation between places in the visual field and directions in extrapersonal space from the egocenter. This does not imply that internal local sign might not be developed taking eye movements and external configurations into account. For instance, the mechanisms discussed by Platt (1960) are exactly of this type. Internal local sign is all that is needed in discussions of, e.g., the classical geometrical illusions.

## External local sign

The *external local sign* has to do with the relation of retinal locations with directions in the external world relative to the frame of the eye-ball. These directions correspond to the "visual rays" of Euclid (in Burton, 1945). Because such relations have to be acquired via experience in the world we refer to them as "external". The "local signs" as introduced by Lotze (1881) have both aspects, they are both "internal" and "external".

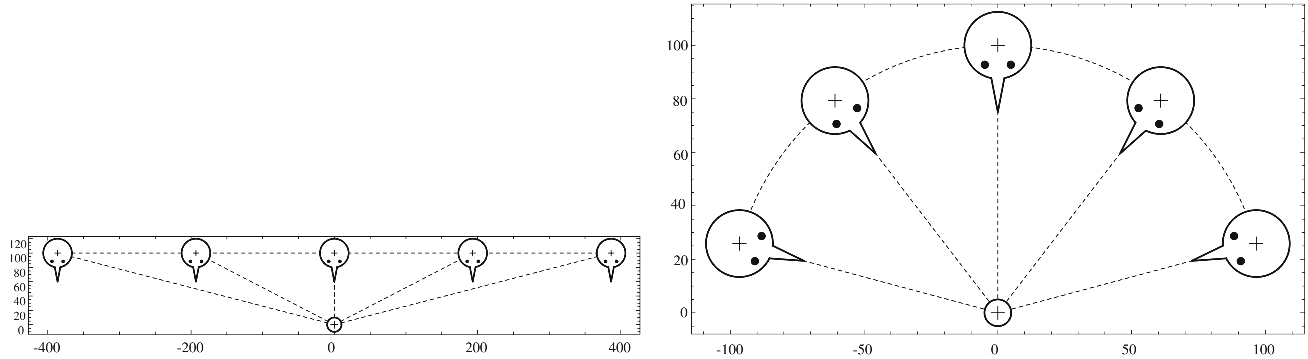


**Fig. 10** At left five persons in strict military order: Equidistant, collinear configuration, all noses parallel. The 120° view makes that the observer sees the left profile of the leftmost person and the right profile of the rightmost person, wheres the person at center is seen *en face*. The distances to the persons are very different. At right the five persons are positioned in a circular arc with the observer at center. Moreover, all directly face the observer. In this case all persons are seen *en face* and with equal apparent sizes. The field of view is again 120°

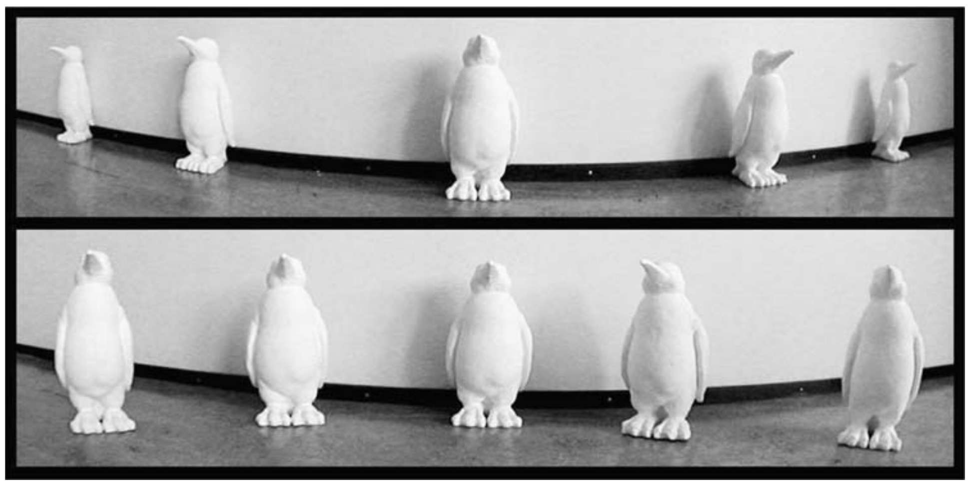

**Fig. 11** The top photograph is of a configuration as shown in Fig. 10 *left*, the bottom photograph of a configuration as shown in Fig. 10 *right*. The representation is in Riemann normal coordinates (distance from the center proportional to eccentric angle). Notice that the circular configuration *looks* like a neat military lineup, whereas the true military lineup looks like an arc with its convexity towards the observer. Such problems are discussed in the book by Barre and Flocon (1968). Compare Fig. 9

In our interpretation the external local sign may be understood as a projective and metric calibration of the internal local sign, that is to say, the topological structure (Klein, 1872). For instance, an eye-movement may shift the retinal image of an objective straight line in itself, thus establishing *collinearity* (Platt, 1960). This suffices to establish the *projective* structure. Likewise, an eye-movement may rigidly shift a certain stretch along a straight line, thus establishing *affine* structure. Such mechanisms were already envisioned by Lotze (1881) and Helmholtz, 1977 (1878) and their fundamental importance is not in doubt.

It is less clear whether there exists a firm basis for a metrical calibration of the visual field that matches the geometrical metric in the bundle of "visual rays". It is certainly possible to think of viable ways to attain such a calibration (Berkeley, 1709). The evolutionary advantage provided by such a calibration is not obvious though, since projective-affine calibration would amply suffice to sustain efficacious perception–action cycles.

It is not difficult to predict the likely visual experiences of observers with perfect internal local sign, but lacking external local sign for the simple case of a perfect texture cue. The texture cue would allow the observer to judge the local slant of external surface elements with respect to the local visual direction (Fig. 9).

## Overall conclusion

Our results imply that a full metrical external local sign is lacking in a major part of the population. The existence of a minority that is essentially veridical shows that this lack of metrical calibration is by no means *necessary* though. Thus the generic condition raises an intriguing conceptual question.

The observers that (partly or wholly) lack an external local sign are expected to experience the effects described by Barre and Flocon (1968), (see Figs. 10, 11). Such experiences are likely to be quite common in view of the fact that visual artists have frequently made use of them (documented by Barre & Flocon, 1968). The fact that many cultures prefer (pseudo-)orthographic rendering, rather than linear perspective (e.g., the east and far east) may also be related.

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