International Journal of Industrial Organization 27 (2009) 474–487

Contents lists available at ScienceDirect

International Journal of Industrial Organization

journal homepage: www.elsevier.com/locate/ijio

Dynamic entry and exit with uncertain cost positions☆

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ARTICLE INFO

Article history:
Received 22 September 2008
Received in revised form 18 November 2008
Accepted 10 December 2008
Available online 24 December 2008

JEL classification:
D82
D83
L13

Keywords:
Entry
Shakeout
Learning
Cost positions

ABSTRACT

We study the dynamics of entry and exit based on firms' learning about their relative cost positions. Each firm's marginal cost of production is its own private information, thereby facing ex ante uncertainty about its cost position. The (inelastic) market demand can accommodate only a fraction of firms to operate, and thus only firms with relatively lower costs are viable in the long run. Some firms in the market will exit if excessive entry (or overshooting) occurs. We derive the unique symmetric sequential equilibrium. The equilibrium properties are consistent with empirical observations: (i) entry occurs gradually over time with lower cost firms entering earlier than higher cost firms, (ii) exiting firms are among the ones that entered later (indeed in the last period). Moreover, equilibrium overshooting probability is shown to always be positive and decreasing over time.

1. Introduction

Empirical evidence suggests the following three features of industry dynamics: (1) entry occurs over time or in waves; (2) mass-exit or "shakeout" follows mass-entry; (3) during the shakeout firms that entered just before shakeout are more likely to exit than earlier entrants. This pattern has been documented, for example, by Jovanovic and MacDonald (1994) and Klepper and Simons (2000) for the US tire industry, by Klepper and Simons (1997) for the US automobile industry, and by Horvath et al. (2001) for the US beer brewing industry. Moreover, among the 42 industries that are studied by Gort and Klepper (1982), the evolution of most industries also exhibits the above mentioned patterns.

This paper aims to account for the aforementioned three features of industry dynamics based on firms' learning about their cost positions, which are uncertain ex ante. Specifically, at the beginning a new market opens up, and it is known to be able to accommodate exactly N firms. There are $N+L$ potential entrants, and entry involves some amount of sunk cost. Though the sunk cost of entry is the same among all firms, their marginal costs of production are different. Before entry each firm's marginal cost is its own private information, but it becomes public information after entry. The time horizon is infinite. In each period, upon observing the history of entry, the remaining firms simultaneously decide whether to enter. If there are strictly more than N incumbents in a period, all the incumbent firms simultaneously decide whether to exit.

The dynamic game goes through the following three phases in order: an entry phase in which there are strictly less than N incumbents, a possible exit phase in which there are strictly more than N firms in the market, and a long run state in which there are exactly N incumbents. We show that their is a unique symmetric equilibrium in the dynamic game, which is characterized by a strictly increasing sequence of cost cutoffs, with lower cost firms entering earlier than higher cost firms. Though the evolution of the equilibrium cost cutoffs depends on the realized history, they can be traced recursively. The reason behind the cutoff strategy is that higher cost firms have stronger incentive to wait than lower cost firms. Intuitively, when a firm makes the entry decision it faces the following trade-off. Waiting entails that the firm forgoes the potential profit in the current period, which we call the cost of waiting. On the other hand, by waiting one more period, the firm may avoid wrong entry in case that the firm is not among the N lowest cost firms, which we call the benefit of waiting. The cost of waiting is decreasing in marginal cost, since the current period profit forgone is lower for a higher cost firm. On the other hand, the benefit of waiting is increasing in marginal cost. This is because lower cost firms are more likely to be

☆ We would like to thank Dirk Bergemann, Luis Cabral, Johannes Horner, Dan Levin, Matt Lewis, Ichiro Obara, James Peck, Tadashi Sekiguchi, an anonymous referee, and the participants of the Fall 2007 Midwest Economic Theory Meetings at Michigan, the 2007 Workshop on Mathematical Economics at Keio, the 2008 International Industrial Organization Conference at Washington D.C., and the seminars at ARISH, Nihon University, Ohio State University and Nagoya University for helpful discussions and comments. Hanazono gratefully acknowledges the financial support from Japan Economic Research Foundation.

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0167-7187/$ – see front matter
doi:10.1016/j.ijindorg.2008.12.002

among the N lowest cost firms, thus the probability of wrong entry is smaller. Combining these two effects, lower cost firms have less incentive to wait than higher cost firms.

Under the cutoff strategy entry occurs gradually over time, with the length of the entry phase being uncertain. The key is that firms are uncertain regarding their relative cost positions since the marginal costs are private information ex ante. If firms' relative cost positions were common knowledge, then entry would always be completed in the first period, with only the N lowest cost firms entering immediately, and no subsequent exit would occur. In equilibrium, as the cost cutoff increases over time, the uncertainty regarding the relative cost positions for the remaining firms is gradually resolved. However, the remaining firms still face the uncertainty regarding the relative cost positions among themselves as long as they are in the entry phase, which implies that the probability of overshooting (when strictly more than N firms have entered) is always strictly positive.

If there is excessive entry in a period, leading to strictly more than N incumbents, exit will follow. Naturally, more entries in a period lead to more exits in the following period. Since lower cost firms enter earlier than higher cost firms, exit occurs only among the firms entering in the last period of the entry phase. This is because some of those firms are not among the N lowest cost firms, while firms entering earlier are among the N lowest cost firms. This explains the empirical pattern that firms that entered later are more likely to exit.

In the entry phase, both the expected number of entry and the probability of overshooting are shown to decrease over time (robust for any continuous distribution of marginal cost). This is due to the equilibrium feature that higher cost firms have stronger incentive to wait, hence they enter more cautiously. In later periods of the entry phase, the remaining firms have higher costs. As a result, the probability of entry for each remaining firm decreases over time, which reduces the expected number of entry and lowers the probability of overshooting. This prediction is consistent with some empirical evidence. Klepper and Graddy (1990) and Klepper and Miller (1995) found that industries that have a longer entry phase are less likely to experience a severe shakeout.

In terms of comparative statics, we show that, fixing a history of entry, an increase in the discount factor, a decrease in the sunk cost, an increase in the market size N, or a decrease in the number of extra firms L, all lead to more aggressive entry among the remaining firms. However, no definite comparative statics results can be shown over the whole equilibrium path, since different parameter values in general lead to different histories. We do provide examples showing that the actual length of the entry phase is nonmonotonic in any parameter values. We also study a limiting case in which the length of each period approaches zero. In the limiting case, entry still occurs gradually over time while the possibility of overshooting vanishes.

In an extension to the basic model we consider the setting in which the market price in a period is a decreasing function of the number of active firms in the market. We spell out how we construct the on-path cutoffs analogous to the ones in the basic model, which confirms that a symmetric cutoff strategy equilibrium exists in this setting, with lower cost firms entering earlier than higher cost firms. The new feature is that the number of firms in the long run is uncertain, and it depends on the realized marginal cost profile.

The rest of the paper is organized as follows. The next subsection reviews the related literature. Section 2 sets up the model. The symmetric equilibrium in the dynamic game is characterized in Section 3, and Section 4 presents equilibrium properties. Section 5 extends the basic model and Section 6 concludes. All the technical proofs are contained in the Appendix.

1.1. Related literature

A strand of literature (e.g., Klepper and Graddy, 1990; Jovanovic and MacDonald, 1994; Klepper, 1996a; Klepper and Simons, 2000) focuses on technology innovation or improvement as the driving force behind industry dynamics. In contrast, our paper focuses on informational learning as the driving force for industry dynamics. The papers mentioned above typically cannot explain why later entrants are more likely to exit during shakeout. For example, in Klepper (1996a) and Klepper and Simons (2000) initially both innovators and imitators enter. Later on as market price decreases due to output expansion, only innovators enter. When shakeout occurs, later entrants (innovators) and early imitators are more likely to exit. But it is not clear whether later entrants are more likely to exit than early entrants as a whole.

Jovanovic and Lach (1989) study industry dynamics with learning-by-doing. Later entrants have lower costs of production than earlier entrants due to the spillover from learning-by-doing. However, their model implies that old firms are more likely to exit than new entrants when shakeout occurs. Cabral (1993) incorporates experience advantage in studying entry dynamics. Specifically, earlier entrants' production costs gradually decrease as they gain more experience by operating in the market. In both papers, firms are homogenous ex ante and the informational aspect is absent. In contrast, in our model firms are heterogenous and informational learning plays a key role in driving industry dynamics. Jovanovic (1982) builds a model of selection to explain firm dynamics. Firms learn their "true" production costs over time: the efficient grow and survive while the inefficient decline and exit. While in his model firms learn their production costs, in our model firms know their production costs but learn their relative cost positions among all potential entrants. His model is able to explain why young firms have higher and more variable growth rates. However, since all the firms enter in the first period, his model does not account for entry dynamics and later entrants are more likely to exit during shakeout. On the other hand, our focus in on how firms wait for the right time to enter and the possibility of shakeout.[1]

Rob (1991) studies entry dynamics in a setting where firms learn the market size over time.[2] In particular, the size of the market is revealed only if the total capacity of the industry overshoots it. Entry is shown to occur over time and exit follows when overshooting occurs. Firms that entered in different times are equally likely to exit, however, since firms are homogenous in his model. Horvath et al. (2001) present a model in which firms learn the profitability of entry over time. Specifically, the post-entry performance of incumbents provides data from which firms learn the profitability of entry, and ex ante identical firms draw different production costs upon entry. Using numerical simulations, they provide an example that generates the empirical patterns (1)-(3) mentioned above for some parameter values.[3] To sum up, both papers focus on learning about a *common* value as the driving force behind the entry dynamics, while we stress learning about *individual* values (relative cost positions). One empirical implication differentiates our model from the above two papers: in our model more efficient firms enter earlier, while in their models the average efficiency of entrants is invariant over time since firms are ex ante identical.

Levin and Peck (2003) consider a two-firm dynamic entry game with each firm's entry cost being heterogenous and private

[1] Abbring and Campbell (2007) study the entry and exit dynamics in oligopolistic markets with sunk costs and demand uncertainty. They assume the feature of last-in first-out: an entrant expects to produce no longer than any incumbent. Our paper provides a theoretical foundation for this feature in their model.

[2] See also Vettas (1997). Vettas (2000b) studies the entry dynamics when the initial demand is unknown and demand is an increasing function of past sales.

[3] In essence, in their model delay in entry and mass entry before shakeout result from learning the profitability of entry. And the implication that later entrants are more likely to exit in shakeout is due to the fact later entrants have the mean cost in expectation, while incumbents have lower cost than the mean since earlier entrants that have higher costs would have exited before already.

information, and both firms have the same cost of production.[4] The eventual market structure can either be monopoly, or duopoly in which each firm's gross profit is lower than that in monopoly. In equilibrium, entry occurs over time with lower cost firms entering earlier. Firms' entry decisions balance the following trade-off: entering earlier increases the chance of being the monopolist but also increases the chance of simultaneously entry (coordination failure). In contrast, in our model firms have the same fixed cost, but marginal costs are heterogenous and private information, so entering early has a different tradeoff: earning profits early versus the risk of wrong entry. A more important difference is that they do not consider the possibility of exit. Also, their model only considers the case with two firms, while we consider a more general case with finite number of firms.[5]

Among all the papers mentioned above, only Cabral (1993) and Horvath et al. (2001) are able to simultaneously account for the empirical patterns (1)–(3) mentioned before.[6] Besides empirical patterns (1)–(3), our model also generates a prediction that is consistent with empirical evidence, for which other extant papers are not able to explain: industries that have a longer entry phase are less likely to experience a severe shakeout.

Bulow and Klemperer (1999) analyze a generalized war of attrition with firms' winning prizes being private information. In equilibrium, firms with lower winning prizes exit earlier. Our model differs from theirs in that we consider both entry and exit. The option value of waiting in the entry phase of our model resembles that in Chamley and Gale (1994). In their model waiting can lead to more accurate information about a common investment return, while in our model waiting can potentially avoid wrong entry. To some extent, our model is also related to Bulow and Klemperer (1994), which studies a dynamic auction game with N items being auctioned off to N buyers among $N+L$ potential buyers. Generally, buyers with higher values bid earlier. As information regarding higher value bidders gradually revealed, the bidding behavior of the remaining agents are affected accordingly.

2. Model setup

A new market just opened up, or the existing market size increased with new consumers born. The demand (or demand increase) can accommodate $N \geq 1$ firms. There are $N+L$ potential entrants to meet the market demand, with $L \geq 1$. Each entrant incurs a sunk cost K upon entry, which is common for all firms. For simplicity, we assume that each incumbent firm produces a single unit of output in each period. This assumption can be interpreted as there being a unique efficient size of the firm, which might arise from pure technological reasons. The market size N is known at the beginning and fixed over time. Time is discrete, which is indexed by $t=1,2,\ldots$, and the horizon is infinite. All firms share the same discount factor $\delta \in (0,1)$.

Firms are heterogeneous in marginal costs of production. Specifically, each firm's marginal cost c_i is an independent and random draw from a distribution function $F(c)$ on $[\underline{c},\bar{c}]$, with $\underline{c}<\bar{c}<1$. We assume that $F(c)$ is common knowledge and it is continuously increasing on its support without any mass points. A firm's c_i is its own private information before it enters. However, after a firm enters the market, its c_i becomes public information. We adopt this assumption mainly for tractability.[7] We think this assumption is not unrealistic. Before entry, though all potential entrants have an incentive to learn each other's marginal costs so as to infer its cost position, there is very limited source to learn such information. When a firm enters, however, it needs to choose a specific production technology or process, and these are (at least partially) observable to other firms and provide good information about the entering firm's marginal cost.[8] Moreover, an entering firm needs to hire employees, who could leak some information about the firm's cost, say, with bribery by other firms. To sum up, we believe that entry transforms a potential entrant into a real/physical existence, and as a result other firms have more sources to learn that firm's marginal cost.

We assume the following (reduced form) market price that only depends on the number of operating firms in the market: (i) if there are less than or equal to N firms, then the market price is 1 (after normalization). (ii) if there are more than N firms, then due to over capacity, the market price in that period will be driven down to the $(N+1)$th lowest marginal cost among the operating firms. To justify this particular pricing behavior, one can think of a market with N homogenous consumers, each of whom has a unit demand with reservation value 1. If the number of operating firms is less than N, each firm can charge a price up to the reservation value without worrying about finding consumers. On the other hand, if the number of operating firms is greater than N, (Bertrand) competition drives down the market price to the marginal cost of the marginally efficient firm (the $(N+1)$th lowest).

Note that a firm with marginal cost c can at most earn a gross lifetime return $(1-c)/(1-\delta)$ upon entry, where $1-c$ is the highest period payoff that the firm can earn. To ensure entry is profitable, we assume that

$$\frac{1-\bar{c}}{1-\delta} > K. \qquad (1)$$

Thus entry is potentially profitable even for the firm with the highest possible marginal cost. Assumption (1) ensures that there are $N+L$ potentially viable entrants.

In each period, entry and exit occur according to the timing specified below. We assume that exit involves no cost. The history of entering and exiting up to the previous period is perfectly observable to all firms. The timing of events in a period is summarized in Fig. 1.

At the beginning of a period, each remaining entrant makes the entry decision simultaneously. The marginal costs of the newly entered firms then become public information. All the firms in the market (including those having just entered) then decide simultaneously whether to exit. Finally, the firms staying in the market produce goods and set prices.

Given the structure of the game, in the long run only the N or $N+1$ lowest cost firms will operate in the industry. To simplify matters, we assume that each firm has to pay a very small amount ε to maintain its machines even if it does not produce any goods in a period. Consequently, the number of operating firms (in equilibrium) must

[4] Dixit and Shapiro (1986) study a dynamic entry game with homogenous cost and complete information. The symmetric equilibrium in their model involves mixed strategy. See also Vettas (2000a) for the features of the symmetric equilibrium in the model of Dixit and Shapiro. Bolton and Farrell (1990) introduce private information about entry costs. In their model there are only two firms, and they focus on the comparison between centralized and decentralized coordination. All these papers share the feature that firms have the same cost of production.

[5] In an extension they do consider a general model with n firms. However, with the assumption by which the game always ends immediately after a firm enters, the entry dynamics mainly exhibit similar properties to those in the two-firm model.

[6] Another difference between Cabral's and our model lies in the on-path equilibrium patterns of entry-exit dynamics. Specifically, the typical process of entry, overshooting, and shakeout may not be clearly identified for some equilibrium paths in his model. This is because firms play mixed strategies in his model. When "overshooting" occurs, firms that have least experience randomize between staying and exiting. If too many firms exit, firms that have just exited randomize between entering and staying out. It is thus possible that some firms alternate between entering and exiting over finite but long periods.

[7] If firms' cost were private information after entry, then in the exiting phase following overshooting incumbent firms will play a war of attrition game with incomplete information about costs. This will significantly complicates the analysis for the entry phase, as firms' value functions will become very complicated.

[8] In addition, after entry a firm is usually required to provide some tax documents annually to the government, which would reveal information about its cost structure.

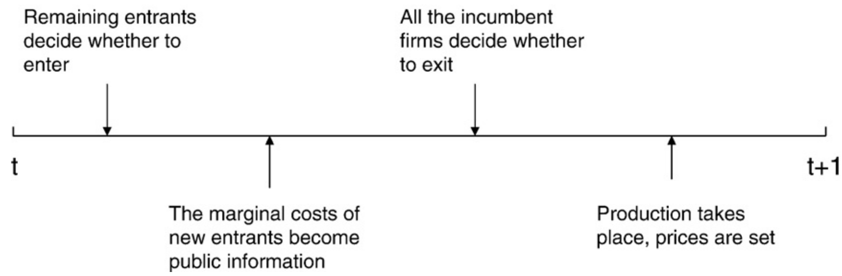

Fig. 1. Timing.

be N in the long-run, since the $(N+1)$th lowest cost firm will lose money if it stays in the market.[9]

The dynamic game can thus be divided into three phases corresponding to the number of firms in the market. In the first phase, there are strictly less than N firms in the market, thus further entry will occur in the future. We call this phase the *entry* or *expansion* phase. In the second phase, there are strictly more than N firms in the market, and some firms have to exit eventually. This phase is thus termed as the *exit* or *shakeout* phase. Finally, in the third phase exactly N firms are operating in the market, and we call this the *long-run state*. Note that, if exactly N firms in total have entered in the entry phase, the long-run state directly follows and the exit phase does not arise.[10]

The key factor for the entry dynamics is the uncertainty regrading the relative cost positions. If each firm's marginal cost were publicly known from the very beginning, efficient entry would have been completed in the first period: the N lowest cost firms would enter, and there will be no exit and the long-run state is reached immediately. However, given cost uncertainty each firm is unsure whether and when to enter. Naturally, in this scenario entry occurs through time, with firms learning their relative cost positions along the way.

3. Symmetric equilibrium

Each firm's strategy has two components: an entry decision for remaining entrants and an exit decision for incumbents. In a symmetric equilibrium each incumbent's optimal exit decision is straightforward. When there are strictly more than N firms in the market, the incumbent firms that are not among the N lowest cost firms will exit immediately, otherwise those firms will incur a loss. This implies that on the equilibrium path the market price is always 1, since there are always N or fewer incumbent firms in the competition stage in each period.

We thus focus on the entry phase of the dynamic game. Naturally, one would think that a lower cost firm will enter earlier than a higher cost firm does, and each firm thus adopts a cutoff strategy: each potential entrant enters in period t if and only if its cost is below the cutoff cost, for each history at period t. The underlying reason is that if a firm with cost c earns a positive expected return by entering at period t, a firm with cost $c'<c$ earns more by entering at the same period. To see this, we only need to compare the expected life-time gross returns since both firms have the same entry cost K. Conditional on both firms surviving in the long-run state (i.e., both are among the N lowest cost firms), the firm with c' has a higher gross return, since its per period profit after entry is higher. Moreover, the firm with c' is more likely to survive in the long-run state than the other firm does. The lower cost firm therefore has a higher expected return. Given the possibility of waiting, however, not every potential entrant with a positive expected return in the current period will enter. To justify the cutoff strategy, we need to show that a lower cost firm gains less by waiting than a higher cost firm does, which will be shown later.

Formally, a potential entrant's (behavioral) strategy is a mapping from its cost c_i and the history of previous entry to whether or not to enter in period t. Let h_t denote the cost realizations of entrants entering in period t (Ø if no firm enters in period t). Let $H_t=(h_1,\ldots,h_{t-1})$ denote a history of previous entry at the beginning of period t. A pure strategy of a potential entrant is therefore a mapping from $c_i \times H_t \mapsto \{enter, wait\}$.

We focus on symmetric pure strategy sequential equilibria (SE). A symmetric equilibrium can be defined by the following system of value functions and a corresponding belief system. Let $V_t^I(c_i|H_t)$ denote the expected life-time payoff of an incumbent i with cost c_i (evaluated at the beginning of period t) given history H_t.[11] Similarly, let $V_t(c_i|H_t)$ denote the value of a new entrant i with c_i that enters in period t, and $W_t(c_i|H_t)$ the value of a potential entrant with c_i that waits in period t. The value functions are written as follows:

$$V_t^I(c_i|H_t) = E_{h_t^{-i}}\left[\pi_t\left(c_i|H_t\times h_t^{-i}\right) + \delta V_{t+1}^I\left(c_i|H_t\times h_t^{-i}\right)\right],$$
$$V_t(c_i|H_t) = E_{h_t^{-i}}\left[\pi_t\left(c_i|H_t\times\left(c_i,h_t^{-i}\right)\right) + \delta V_{t+1}^I\left(c_i|H_t\times\left(c_i,h_t^{-i}\right)\right)\right] - K,$$
$$W_t(c_i|H_t) = \delta E_{h_t^{-i}}\left[\max\left\{W_{t+1}\left(c_i|H_t\times h_t^{-i}\right), V_{t+1}\left(c_i|H_t\times h_t^{-i}\right)\right\}\right].$$

In the above expressions, $\pi_t(\cdot|\cdot)$ is the gross payoff in period t. In particular,

$$\pi_t\left(c_i|H_t\times h_t^{-i}\right) = \begin{cases} 1-c_i & \text{if there are } N-1 \text{ or fewer firms in the market whose cost are less than } c_i, \text{ given } H_t\times h_t^{-i}, \\ 0 & \text{otherwise.}\end{cases}$$

The equilibrium strategy is to enter if and only if $V_t(c_i|H_t)\geq W_t(c_i|H_t)$. History h_t^{-i} is a realized cost profile of entrants (excluding firm i) at period t. The expectation is taken over h_t^{-i}, which arises according to each remaining entrant's equilibrium strategy and to firm i's belief about the remaining entrants' cost types.[12]

[9] Allowing immediate exit of newly entered firms and introducing ε cost of maintaining machines simplify the computation. The qualitative results of this paper do not depend on these two assumptions. The avoidable fixed cost ε needs not being small. But a negligible ε can simplify the algebra.

[10] Conceivably, a firm could wait until a shakeout phase or long-run state arises, and afterwards it would enter given that this firm is among the N lowest cost firms. We ignore this case since it would never arise in equilibrium.

[11] If it is certain that a firm with c_i is not among the N lowest cost firms given H_t, $V_t^I(c_i|H_t)=0$ since this firm will optimally exit.

[12] Firm i's belief at an off-the-equilibrium-path information set must indeed be such that the remaining entrants have followed the equilibrium strategy so far, as the following argument shows. First, recall that off-the-equilibrium-path beliefs in a sequential equilibrium need to be consistent in the sense that they are the limit of the beliefs derived from a completely mixed strategy profile converging to the equilibrium strategy profile. In our model, an off-the-equilibrium-path information set of firm i at t describes either of the following two situations (or the intersection of both); (i) firm i itself has deviated before (waiting too long), or (ii) firm i has ever observed a firm entering too early or too late. Neither situation has to do with the type profile of the remaining entrants other than i that have never entered until t. Recall that firm i's belief at t is about the other remaining entrants' cost types. Possibly, firm i might suspect that some of the remaining entrants have waited too long. However, the consistency requirement rules out this possibility, since the probability attached to waiting too long in a remaining entrant's completely mixed strategy must vanish in the limiting argument.

3.1. Cutoff strategy equilibrium

We first construct a cutoff strategy equilibrium, and then show that it is indeed a unique symmetric SE. A cutoff strategy is defined as follows: for any history H_t, there is $\alpha_{t+1}(H_t)$ such that a firm enters in period t if and only if its cost $c \le \alpha_{t+1}(H_t)$. Thus a candidate equilibrium is characterized by a sequence of cutoffs $\alpha_t(H_{t-1})$. To abuse notation, we write $\alpha_t(H_{t-1})$ as α_t when there is no confusion.

In a symmetric cutoff strategy equilibrium, an on-the-equilibrium-path history H_t can be summarized by two state variables, n_t and α_t, where n_t denotes the number of incumbents at the beginning of period t and α_t is the cutoff cost in the previous period. This is because, given the state variables, the continuation game is exactly the same regardless of the cost realizations of the incumbent firms, as the incumbents have lower costs than the remaining entrants and the number and the cost distribution of remaining entrants are always the same. Note that $\alpha_1 = \underline{c}$ and $n_1 = 0$.

Information updating in a symmetric cutoff strategy equilibrium works as a truncated operator. In particular, denote the belief about the cost distribution of each remaining entrant at the beginning of period t as F_t, which has the support $[\alpha_t, \bar{c}]$. If all the remaining entrants with cost within the interval $(\alpha_t, \alpha_{t+1}]$ invest in period t, firms' posterior belief about the cost distribution of each remaining entrant, F_{t+1}, becomes

$$F_{t+1}(c) = \frac{F_t(c) - F_t(\alpha_{t+1})}{1 - F_t(\alpha_{t+1})} = \frac{F(c) - F(\alpha_{t+1})}{1 - F(\alpha_{t+1})} \text{ with the support } [\alpha_{t+1}, \bar{c}].$$

The on-path cutoffs can be derived in a recursive manner. Consider a system of the value functions associated with a cutoff strategy equilibrium. The on-path cutoff α_{t+1}, given $n_t < N$ and α_t, must satisfy the following indifference condition:

$$V_t(\alpha_{t+1} | n_t, \alpha_t) = W_t(\alpha_{t+1} | n_t, \alpha_t). \tag{2}$$

That is, given n_t and α_t, a firm with cost α_{t+1} should be indifferent between entering and waiting in period t. Note that, in a symmetric equilibrium, each firm expects that the other firms follow the strategy with cutoff α_{t+1} in period t. Let $A^i(c)$ denote the event that firm i with cost c is among the N lowest cost firms (the winning group). By information updating, for $c > \alpha_t$, we have

$$Pr\left[A^i(c) | \alpha_t, n_t\right] = \sum_{j=0}^{N-n_t-1} \binom{N - n_t + L - 1}{j} \left[\frac{F(c) - F(\alpha_t)}{1 - F(\alpha_t)}\right]^j \left[\frac{1 - F(c)}{1 - F(\alpha_t)}\right]^{N + L - n_t - j - 1}.$$

The value functions for the on-path cutoff type α_{t+1} are

$$\begin{aligned} V_t(\alpha_{t+1} | n_t, \alpha_t) &= Pr\left[A^i(\alpha_{t+1}) | \alpha_t, n_t\right] \frac{1 - \alpha_{t+1}}{1 - \delta} - K; \\ W_t(\alpha_{t+1} | n_t, \alpha_t) &= \delta Pr\left[A^i(\alpha_{t+1}) | \alpha_t, n_t\right] \left[\frac{1 - \alpha_{t+1}}{1 - \delta} - K\right]. \end{aligned}$$

If firm i with cost α_{t+1} enters in period t, with probability $\Pr[A^i(\alpha_{t+1}) | \alpha_t, n_t]$ it is among the winning group and it will earn a gross lifetime return $(1 - \alpha_{t+1}) / (1 - \delta)$. If a firm with cost α_{t+1} waits in period t, then, given that all the other firms follow cutoff strategy α_{t+1}, it will enter if and only if it is among the winning group.[13] The indifference condition (2) is then rewritten as

$$Pr\left[A^i(\alpha_{t+1}) | \alpha_t, n_t\right] = \frac{K}{1 - \alpha_{t+1} + \delta K}. \tag{3}$$

Note that the right hand side of (3) is between (0,1), following assumption (1). Thus Eq. (3) is well defined.

Lemma 1. *Given $n_t < N$ and α_t, there is a unique $\alpha_{t+1} \in (\alpha_t, \bar{c})$ satisfying (3).*

Proof. The LHS of (3), the probability of being among the winning group $Pr[A^i(\alpha_{t+1}) | \alpha_t, n_t]$, is strictly decreasing in α_{t+1}. On the other hand, the RHS of (3) is increasing in α_{t+1}. Therefore, α_{t+1} must be unique if it exists. For $\alpha_{t+1} = \alpha_t$, the LHS of (3) equals 1, while the RHS of (3) is strictly less than 1 by assumption (1), therefore LHS > RHS. For the other extreme $\alpha_{t+1} = \bar{c}$, the LHS of (3) equals 0, while the RHS of (3) is strictly greater than 0, therefore LHS < RHS. By continuity of the two sides of (3), there is a unique $\alpha_{t+1}(n_t, \alpha_t) \in (\alpha_t, \bar{c})$ that satisfies (3). □

By Lemma 1, condition (3) recursively defines a strictly increasing sequence of on-path cutoffs $\{\alpha_t\}$. In the first period, the state variables are $n_1 = 0$ and $\alpha_1 = \underline{c}$. The first period cutoff, α_2, can thus be uniquely calculated from (3). Depending on the realized n_2 (suppose it is strictly less than N), the second period cutoff $\alpha_3(n_2, \alpha_2)$ again can be uniquely determined by (3). This procedure can be used recursively to pin down the on-path cost cutoffs $\{\alpha_t\}$. Note that $\alpha_t (t \ge 3)$ depends on the realized history.

To completely define a symmetric cutoff strategy equilibrium, we need to specify off-the-equilibrium-path cutoffs and the associated beliefs. In the sequential equilibrium of our model, indeed, each firm at any information set has the belief that all the other remaining entrants have followed the equilibrium strategy so far (see footnote 12). The associated beliefs for a cutoff strategy equilibrium are therefore straightforward.

There are two kinds of deviations characterizing the off-path information sets: entering too early or entering too late. Entering too early refers to the case that a firm with cost $c > \alpha_{t+1}$ enters in period t or before, and entering too late occurs if a firm with cost $c \in (\alpha_t, \alpha_{t+1})$ waits in period t. Note that entering too late by a firm is detected by other firms only after the deviating firm enters in a later period. Off-path strategies related to entering too late are easy to define. For the very firm that has been entering too late, its belief is that no remaining entrants have a lower cost, and thus it should enter immediately. For a remaining entrant that has ever observed entering too late by another firm, the situation is just the same as in the on-path history given n_t and α_t, and thus the cutoff strategy is defined by the indifference condition (3) accordingly.

On the other hand, entering too early is immediately detected. This implies that some of the remaining entrants might have lower costs than the deviator while others have higher costs, and thus each remaining entrant's cutoff depends on its cost, in principle. For instance, consider the case that all the other remaining entrants adopt a cutoff α_t in period $t-1$, but that a firm with cost $c' > \alpha_t$ entered in period $t-1$. Suppose in addition that the total number of firms in the market is $n_t' \le N$. Let H_t' denote this particular history. Importantly, among the remaining firms, firms with different costs view the intensity of competing for remaining slots in different ways. Specifically, for firms with cost $c \in (\alpha_t, c')$ there are $N - n_t' + 1$ slots available in the market, since these firms have lower costs than the deviator. On the other hand, for firms with cost $c > c'$ there are only $N - n_t' + 1$ slots available. For $c \in (\alpha_t, c')$, therefore, the associated indifference condition needs to be modified from (3); there are $N - n_t' + 1$ available slots sought after by $N + L - n_t'$ firms.[14] The cutoff derived from this associated indifference condition for $c \in (\alpha_t, c')$, denoted by α'_{t+1}, is higher than $\alpha_{t+1}(n_t', \alpha_t)$. This is because more slots available implies more aggressive entry for remaining entrants. Now the actual cutoff $\alpha_{t+1}(H_t')$ is defined as follows:

$$\alpha_{t+1}(H_t') = \begin{cases} \alpha'_{t+1} & \text{if } \alpha'_{t+1} < c' \\ c' & \text{if } \alpha_{t+1}(n_t', \alpha_t) < c' \le \alpha'_{t+1} \\ \alpha_{t+1}(n_t', \alpha_t) & \text{if } \alpha_{t+1}(n_t', \alpha_t) \ge c' \end{cases}.$$

[13] All the uncertainty regarding whether a firm with cost α_{t+1} should enter in period $t+1$ is resolved. If strictly less than $N - n_t$ firms enter in period t, then the firm in question is definitely among the winning group, thus should enter in period $t+1$. Otherwise, it is definitely not in the winning group and should not enter later.

[14] Recall that condition (3) is associated with $N - n_t$ available slots sought after by $N + L - n_t$ firms.

In the top case, the associated indifference condition holds for $\alpha'_{t+1} \in (\alpha_t, c')$. In the bottom case, condition (3) holds for $\alpha_{t+1}(n'_t, \alpha_t) \geq c'$. In the middle case, neither indifference condition holds, and the actual cutoff is c'.

It is straightforward to specify the cutoffs for other off-path histories associated with entering too early. Given the previous period's cutoff α_t, type distribution F_{t-1}, and the observed deviation (s), we can define the cutoff α_{t+1}, accordingly.

Proposition 1. *There is a unique symmetric equilibrium in cutoff strategies, with the evolution of on-the-equilibrium cutoffs $\{\alpha_t\}$ governed by* (3).

Proof. See Appendix A. □

The intuition behind the cutoff strategy equilibrium is that higher cost firms have stronger incentives to wait. Intuitively, waiting one more period has both a cost and a benefit. The cost is that a firm forgoes its profit in period t if it is indeed among the N lowest cost firms (correct entry). The benefit is that it avoids paying the entry cost K if it is not among the N lowest cost firms (wrong entry). The cost of waiting is decreasing in c for two reasons. First, the profit forgone in period t is higher for a lower cost firm. The second reason is that the probability that a firm is among the N lowest cost firms is higher for a lower cost firm. On the other hand, the benefit of waiting is increasing in c, since a higher cost firm is less likely among the N lowest cost firms and thus wrong entry is more likely. Therefore, firms with lower costs have less incentives to wait. The equilibrium cutoff α_{t+1} defined in indifference condition (3) balances the cost and benefit of waiting.

Indeed, the equilibrium identified in Proposition 1 is the unique symmetric equilibrium, which is shown in the following proposition.

Proposition 2. *There is no symmetric equilibrium with non-cutoff strategies.*

Proof. See Appendix B. □

The intuition behind Proposition 2 is similar to that behind Proposition 1. If there were a non-cutoff strategy equilibrium, in some history a higher cost type enters whereas a lower cost type waits. Since the cost and benefit of waiting are monotonic in type as discussed above, such an entry decision cannot be supported in any sequential equilibrium. Therefore, we can conclude that a unique symmetric equilibrium exists and consists of cutoff strategies.

The on-path equilibrium behavior in the entry phase exhibits several features. First, unlike the complete information setting in which efficient entry is completed in the first period, entry occurs over time and it may take a long time to reach the long-run state. Second, lower cost firms enter (weakly) earlier than higher cost firms do. Higher cost firms enter when the uncertainty regarding the number of lower cost firms gradually resolves over time.

3.2. A numerical example

Consider the case where $F(c)$ is uniform on [0.3, 0.8] with density 2. $N=3$, $L=3$, $\delta=0.9$, and $K=1$. For each $n_t=0, 1, 2$ Eq. (3) can be explicitly written as

$$\sum_{j=0}^{2-n_t} \binom{5-n_t}{j} \left[\frac{2\alpha_{t+1}-2\alpha_t}{1-2(\alpha_t-.3)}\right]^j \left[1-\frac{2\alpha_{t+1}-2\alpha_t}{1-2(a_t-.3)}\right]^{5-n_t-j} = \frac{1}{1-\alpha_{t+1}+.9}.$$

Note that the above equation is highly nonlinear, and thus it is quite hard to generate the closed-form solution for $\alpha_{t+1}(n_t, \alpha_t)$. Therefore, we use numerical methods to compute $\alpha_{t+1}(n_t, \alpha_t)$. First, since $\alpha_1=0.3$ and $n_1=0$, we have $\alpha_2=0.49189$. Cutoff α_3 depends on n_2, which we denote as $\alpha_3(n_2)$. We numerically obtain the period 2 cutoffs: $\alpha_3(0)=.59897$, $\alpha_3(1)=.56676$, $\alpha_3(2)=.52305$. The cutoff in period 3 α_4 implicitly depends on n_2 and n_3, which we denote as $\alpha_4(n_2, n_3)$. Our calculation shows that $\alpha_4(0,0)=.66341$, $\alpha_4(0,1)=.64227$, $\alpha_4(0,2)=.61509$, $\alpha_4(1,1)=.61909$, $\alpha_4(1,2)=.58700$, and $\alpha_4(2,2)=.54945$. The equilibrium cutoffs in later periods can be computed accordingly.

4. Equilibrium properties

4.1. The identity of exiting firms

Since in equilibrium lower cost firms enter earlier than higher cost firms, once n_t reaches or overshoots N in period t, the long-run state is reached. All the remaining entrants have higher costs than the incumbents, thus entry and exit will not occur after period t. Note that the exit phase is reached if and only if n_t overshoots N in some period. This might not occur if n_t exactly reaches N in some period.[15] However, in the entry phase the probability of overshooting is always positive, which is shown in the following lemma.

Lemma 2. *Given any on-the-equilibrium-path history n_t and α_t, with $n_t<N$, the equilibrium probability of capacity overshooting is always strictly positive.*

Proof. Suppose in the entry phase the overshooting probability is 0 given history n_t and α_t. Since the distribution of the number of new entries in period t is binomial, we must have $\alpha_{t+1}=\alpha_t$. This means that the LHS of (3) is 1. However, by Assumption (1) $K<1-\alpha_{t+1}+\delta K$, hence the RHS of (3) is strictly less than 1. A contradiction. Therefore, the overshooting probability must be strictly positive. □

Actually, condition (3) means that in equilibrium the marginal type α_{t+1} is balancing the expected loss from overshooting, in which case it loses K, and the current period expected profits by entering. Since waiting entails forgoing the current period's expected profit, to make the marginal type indifferent the overshooting probability must be strictly positive.

Lemma 2 shows that overshooting occurs with positive probability in any period in the entry phase, which implies that the exit phase arises with positive probability on the equilibrium path. Recall that in equilibrium it is always the case that lower cost firms enter earlier while higher cost firms enter later. Thus in the exit phase, it is always the firms that entered later (actually entered in the last period of the entry phase) will possibly exit, and the firms that entered earlier do not exit. This result is summarized in the following proposition.

Proposition 3. *Exit occurs with a positive probability on the equilibrium path. When exit occurs, only firms that entered in the last period of the entry phase will possibly exit. Firms that entered earlier than the last period of the entry phase will not exit.*

Proposition 3 implies that later entering firms are more likely to exit than firms that entered earlier. This is due to the equilibrium feature that firms that entered earlier have lower costs thus are more efficient.[16] This implication is consistent with some empirical evidence. According to Horvath et al. (2001), the US beer brewing industry experienced shakeout during 1880–1890. The sharp decline in the total number of firms during this period is almost entirely accounted for by the exit of firms that entered between 1874 and 1878. A similar pattern is found in the automobile industry. Based on the study of Horvath et al. (2001), the automobile industry experienced a massive wave of entry in 1906–1907 and a shakeout period in 1909–1912. Roughly 40% of the exits during the shakeout period are from firms that entered between 1906 and 1907, the years just prior to shakeout. Though a weaker pattern is found for the tire industry (Klepper and Simons, 2000; Horvath et al., 2001), a large portion of

[15] Among the 16 products (industries) studied by Klepper and Miller (1995), 9 experienced shakeouts and 7 did not have conspicuous shakeouts.

[16] This feature is absent in Rob (1991), since in his model firms are homogenous, thus the identity of exiting firms can not be determined.

the exiting firms during the shakeout period (1921–1930) come from the cohorts that entered in 1919–1921.

4.2. Intertemporal properties

Define p_t as the probability that each remaining entrant enters in period t, and $E[y_t]$ as the expected number of new entries in period t, conditional on period t being in the entry phase. We are interested in how p_t and $E[y_t]$ change over time along each equilibrium path. Specifically,

$$p_t = \frac{F(\alpha_{t+1}) - F(\alpha_t)}{1 - F(\alpha_t)}; \tag{4}$$

$$E[y_t] = (N + L - n_t)p_t. \tag{5}$$

To simplify notation, we define

$$B(j; N, p) = \binom{N}{j} p^j (1-p)^{N-j}.$$

Using Eq. (4) and $B(j;N,p)$, indifference condition (3) can be re-written as:

$$\sum_{j=0}^{N-n_t-1} B(j; N - n_t + L - 1, p_t) = \frac{K}{1 - \alpha_{t+1} + \delta K} \tag{6}$$

To show the intertemporal properties of p_t and $E[y_t]$, we first prove a useful lemma.

Lemma 3. *For any $p \in (0,1)$, and integers N_1, N_2 and L that satisfy $N_1 < N_2$ and $L \geq 1$,*

$$\sum_{j=0}^{N_1} B(j; N_1 + L, p) < \sum_{j=0}^{N_2} B(j; N_2 + L, p). \tag{7}$$

Proof. See Appendix C. □

In statistical terminology, Lemma 3 says that, given that each experiment succeeds with the same independent probability p, the probability of less than N successes out of $N+L$ trials is increasing in N. Intuitively, adding one more slot and one more trial will reduce the probability of shooting the upper bound of successes.

Proposition 4. *Both the probability of entry, p_t, and the expected number of entries, $E[y_t]$, are strictly decreasing in t.*

Proof. First note that n_t is (weakly) increasing in t. Now by (5), $E[y_t]$ is strictly decreasing in t if p_t is strictly decreasing in t. Thus it is sufficient to show p_t is strictly decreasing in t.

Let $t' > t$. Hence $n_{t'} \geq n_t$, and $\alpha_{t'+1} > \alpha_{t+1}$. Suppose to the contrary, $p_{t'} \geq p_t$. Then

$$\begin{aligned}\sum_{j=0}^{N-n_t-1} B(j; N - n_t + L - 1, p_t) &\geq \sum_{j=0}^{N-n_t-1} B(j; N - n_t + L - 1, p_{t'}) \\ &\geq \sum_{j=0}^{N-n_{t'}-1} B(j; N - n_{t'} + L - 1, p_{t'}).\end{aligned} \tag{8}$$

The first inequality is implied by $p_{t'} \geq p_t$ (the probability that less than $N - n_t - 1$ firms enter decreases if each remaining firm enters with a higher probability), while the second inequality follows Lemma 3 and the fact that $n_t \leq n_{t'}$. On the other hand, since $\alpha_{t+1} < \alpha_{t'+1}$, the RHS of Eq. (6) satisfies

$$\frac{K}{1 - \alpha_{t+1} + \delta K} < \frac{K}{1 - \alpha_{t'+1} + \delta K}.$$

Now by Eq. (6), the LHS must exhibits

$$\sum_{j=0}^{N-n_t-1} B(j; N - n_t + L - 1, p_t) < \sum_{j=0}^{N-n_{t'}-1} B(j; N - n_{t'} + L - 1, p_{t'}),$$

which contradicts inequality (8). Therefore, it must be the case that $p_{t'} < p_t$. □

Proposition 4 indicates that expected entry decreases monotonically over time. Two effects are responsible for this intertemporal pattern. First, since n_t is increasing in t, less viable slots are available in later periods. This makes remaining entrants enter more cautiously. The second effect comes from the fact that higher cost firms have stronger incentives to wait. As time goes by, the remaining entrants are revealed to having higher costs, and their stronger incentives to wait naturally lead to more cautious entry.[17]

Note that the above intertemporal pattern does not depend on the distribution function of costs, $F(c)$. In Rob (1991), in order to derive intertemporal properties of entry, a certain property on the distribution function of the demand size needs to be imposed.[18] In our model, the monotonic decreasing pattern of expected entry arises naturally: higher cost firms enter more cautiously. One may wonder why in our model the monotonic pattern of expected entry holds for any $F(c)$ that is strictly increasing and continuous. This is because in setting the equilibrium cutoff α_{t+1}, the distribution function $F(c)$ has been taken into account. If the density from α_t to α_{t+1} is high (i.e., many of remaining entrants' costs are expected to lie in this range), then α_{t+1} will be low, and vice versa. Thus p_t is more or less the same regardless of the distribution function.

Let us get back to the uniform distribution example presented in the last section. We can numerically compute the conditional probability of entry given history. Denote $p_t(n_2, n_3, \ldots, n_t)$ as the equilibrium probability of entry in period t conditional on history $(n_2, n_3, \ldots, n_t)$. Table 1 shows the evolution of $p_t(\cdot)$ up to period 3. We can clearly see that $p_t(\cdot)$ is decreasing in t.

We are interested in how the probability of capacity overshooting changes over time. Denote this probability as P_t^o, conditional on period t being in the entry phase, i.e.,

$$P_t^o = 1 - \sum_{j=0}^{N-n_t} B(j; N - n_t + L, p_t).$$

Proposition 5. The probability of capacity overshooting, P_t^o, *is decreasing in time period t.*

Proof. See Appendix D. □

Proposition 5 implies that excessive entry or overshooting is more likely to happen in the very beginning. As time goes by, if the market is still in the entry phase, then overshooting becomes less likely. This intertemporal pattern arises because higher cost firms have stronger incentive to wait; they are less willing to take the risk of overshooting. Since the remaining entrants' costs are higher as time goes by, the equilibrium probability of overshooting decreases over time. This result is stronger than Proposition 4 in the following sense: the expected entry not only decreases over time, but it decreases fast enough such that the overshooting probability also decreases.[19] Another implication of Proposition 5 is that there is a trade-off between delay and overshooting: a shorter entry phase implies less delay to reach

[17] Note that this prediction is consistent with the empirical fact that shakeout usually follows massive entry. Proposition 4 predicts that the expected number of entry decreases over time. But the realized number of entry might not decrease over time. Shakeout is triggered if the realized number of entry in one period is surprisingly high.

[18] In Rob's model firms are homogenous, thus the intertemporal properties of entry rates depends on the distribution function of the demand size.

[19] Note that a mere decrease in expected entry is not enough to generate a decrease in overshooting probability, since there are fewer slots available thus fewer entries are needed to generate overshooting in later periods.

the long run state, but increases the probability and severity of overshooting.

Denote $P_t^o(n_2, n_3,\ldots, n_t)$ as the equilibrium probability of overshooting conditional on history $(n_2, n_3,\ldots,n_t)$. Using the same specific example as before, Table 2 shows the evolution of $P_t^o(\cdot)$ up to period 3. We can clearly see that $P_t^o(\cdot)$ is decreasing in t.

Though no existing empirical studies directly tested this empirical implication, some evidence is consistent with it. Specifically, it seems that there is an inverse relationship between the severity of net exit during shakeouts and the length of the actual entry phase. Table 3 is constructed from Klepper and Graddy (1990) (combining their Table 3 and the corresponding industries in their Tables 1 and 2).

Based on the data in Table 3, we run a simple regression with the severity of shakeout as the dependent variable and the length of entry phase as the independent variable. It turns out that the severity of shakeout and the length of entry phase is negatively correlated: the coefficient is −0.056 and different from zero at a 95% significance level. The absolute value of the t-statistics is bigger than 2, which verifies that the relationship is statistically significant (the R^2 of the regression is 0.1844, indicating that there are other significant unexplained variations). To sum up, the general pattern is that products with a shorter entry phase experienced severe net exit during the shakeout, while those with a longer entry phase have mild net exit during the shakeout. Essentially, a longer entry phase means that the ascent to the peak number of firms is more gradual, which reduces the chance and severity of overshooting. Klepper and Miller (1995) found empirical support for this pattern. For 16 major products, they calculated the fraction of total pre-peak entries in the seven years immediately preceding the peak. For the 7 products that did not experience a severe shakeout the average of this statistic is .39, in contrast to .59 for the 9 products that experienced severe shakeouts.

4.3. Comparative statics

Now we study how changes in exogenous parameters affect the speed of entry. Among others, it would be highly desirable to see how the expected time needed to reach the long run state changes when parameters vary. However, such results are hard to obtain, as we will discuss later. Instead, we focus on the comparative static results that can be derived holding history constant, which are shown in the following proposition.

Proposition 6. *(i) Holding other parameters constant and fixing the history n_t and α_t, the probability of entry in the current period, p_t, is increasing in δ and decreasing in K. (ii) Fixing α_t and other parameter values, an increase in n_t reduces the probability of entry p_t. (iii) Holding other parameters constant and fixing the history n_t and α_t, the probability of entry p_t is increasing in N and decreasing in L.*

Proof. We start with changes in δ. Suppose $\delta'>\delta$. To show the probability of entry is higher under δ' than under δ, it is sufficient to show $\alpha_{t+1}(\delta') > \alpha_{t+1}(\delta)$. Suppose the opposite is true, that is, $\alpha_{t+1}(\delta')\leq\alpha_{t+1}(\delta)$. Denote the RHS of Eq. (6) under δ as RHS (δ). By $\delta'>\delta$ and $\alpha_{t+1}(\delta')\leq\alpha_{t+1}(\delta)$, RHS($\delta$)>RHS($\delta'$). On the other hand, $\alpha_{t+1}(\delta')\leq\alpha_{t+1}(\delta)$ implies that $p_t(\delta')\leq p_t(\delta)$. Thus the LHS of Eq. (6) is greater under δ', that is, LHS(δ')≥LHS(δ). A contradiction. Therefore, it must be the case that $\alpha_{t+1}(\delta')>\alpha_{t+1}(\delta)$. This implies that $p_t(\delta')>p_t(\delta)$. By a similar argument, we can show that if $K'>K$, then $p_t(K')\leq p_t(K)$. This proves part (i).

Next, consider $n_t'<n_t$. Suppose to the contrary that, $p_t'\leq p_t$. This implies that $\alpha_{t+1}'\leq\alpha_{t+1}$, since α_t is fixed. Now the RHS of Eq. (6) is greater for n_t than the RHS for n_t'. Consider the LHS of Eq. (6)

$$\sum_{j=0}^{N-n_t-1} B(j;N-n_t+L-1,p_t)\leq \sum_{j=0}^{N-n_t-1} B(j;N-n_t+L-1,p_t') < \sum_{j=0}^{N-n_t'-1} B(j;N-n_t'+L-1,p_t'),$$

where the first inequality follows from $p_t'\leq p_t$ and the second inequality follows Lemma 3 and $n_t'<n_t$. Thus the LHS of Eq. (6) is smaller for n_t than the LHS for n_t'. A contradiction. Therefore, we must have $\alpha_{t+1}'>\alpha_{t+1}$ and $p_t'>p_t$. This proves part (ii).

Part (iii) is implied by part (ii). Fixing other parameter values and α_t, an increase in N or a decrease in L is equivalent to a decrease in n_t, which increases the probability of p_t. □

Intuitively, an increase in K leads to a higher benefit of waiting, since by waiting an entrant now avoids a bigger loss in the case of wrong entry. Stronger incentives to wait naturally lead to a smaller probability of entry. To see the effect of an increase in δ, we rewrite Eq. (6) as follows

$$(1-\alpha_{t+1})\sum_{j=0}^{N-n_t-1} B(j;N-n_t+L-1,p_t) = K\left[1-\delta\sum_{j=0}^{N-n_t-1} B(j;N-n_t+L-1,p_t)\right]. \tag{9}$$

The LHS of Eq. (9) is the expected flow profit in period t from entering, which represents the cost of waiting. The RHS of Eq. (9) is

Table 1
The evolution of entry probabilities.

$t=1$	$t=2$	$t=3$
$p_1=.38378$	$p_2(0)=.34754$	$p_3(0,0)=.32055$
		$p_3(0,1)=.21539$
		$p_3(0,2)=.08018$
	$p_2(1)=.24300$	$p_3(1,1)=.22436$
		$p_3(1,2)=.08678$
	$p_2(2)=.10113$	$p_3(2,2)=.09532$

Table 2
The evolution of overshooting probabilities.

$t=1$	$t=2$	$t=3$
$P_1^o=.15754$	$P_2^o(0)=.11477$	$P_3^o(0,0)=.08799$
		$P_3^o(0,1)=.07042$
		$P_3^o(0,2)=.03458$
	$P_2^o(1)=.09627$	$P_3^o(1,1)=.07834$
		$P_3^o(1,2)=.04012$
	$P_2^o(2)=.05341$	$P_3^o(2,2)=.04784$

Table 3
The length of the entry phase and the severity of net exit during shakeouts.

Product name	Length of the entry phase (Years)	Net decrease / peak in the shakeout
Crystals, piezo	31	.38
DDT	9	.87
Electric blankets	51	.65
Electric shavers	8	.56
Engines, jet-propelled	21	.31
Fluorescent	2	.41
Freezers, home and farm	25	.62
Machinery, adding	38	.51
Motors, outboard	9	.38
Penicillin	7	.80
Photocopy machines	25	.53
Polariscopes	50	.38
Radio transmitters	40	.72
Records, phonograph	36	.61
Saccharin	12	.72
Shampoo	51	.04
Streptomycin	8	.85
Tanks, cryogenic	8	.35
Tires, automobile	26	.77
Tubes, cathode ray	37	.28
Windshield wipers	11	.59
Zippers	55	.18

Net decrease/peak is the ratio of the net number of exiting firms during the shakeout to the total number of firms right before the shakeout.

the saving in entry cost by waiting, which measures the benefit of waiting. As δ increases, the cost of waiting remains the same while the benefit of waiting decreases. Thus firms enter more aggressively.[20] An increase in N or a decrease in n_t means that there are more slots available, thus firms enter more aggressively. On the other hand, an increase in L implies that the competition for slots becomes more fierce, which naturally reduces the probability of entry for each firm.

Note that the comparative static results in Proposition 6 are derived by fixing the previous history. In the dynamic game, changes in parameter values would naturally lead to different histories. This means that the comparative static results in Proposition 6 only hold in the first period. Given different histories in later periods, it is very hard to derive the comparative static results over the whole equilibrium path, for example how an increase in K affects the *expected* time of reaching the long run state. What we can show is that, for some realizations of firms' cost profiles, the *actual* time of reaching the long run state is not monotonic in any parameter values.

For concreteness, consider two parameter values $K'>K$ with K' being very close to K. According to Proposition 6, in the first period we have the cutoffs α_2' slightly less than α_2. Now suppose there is a firm whose cost is in between (α_2', α_2). Then in the second period $n_2=n_2'+1$. If n_2 were equal to n_2', by continuity and the fact that K and K' are very close to each other, α'_3 should be slightly less than α_3. However, given that $n_2=n_2'+1$, by part (ii) of Proposition 6, α_3' will be (relatively significantly) higher than α_3. For some realization of firms' cost profiles, this results in a shorter time to reach the long run state for K' than for K. Since an increase in K basically discourages the remaining entrants from entering, fewer firms typically enter in the early periods for $K'>K$. This implies that more slots are available in the later period, leading to more aggressive entry by the remaining entrants. The actual time of reaching the long run state can thus be shorter for K' than for K.

The following example shows how the nonmonotonicity works. Consider the previous numerical example with $F(c)$ being uniform on [0.3, 0.8], $\delta=0.9$, $N=3$ and $L=3$. Suppose $K'=1.001$ instead of $K=1$. Numerically, $\alpha_2'=0.49183<\alpha_2$. Cutoff α_3' depends on n_2'. We numerically obtain

$$\alpha_3'(0,\alpha_2') = 0.59890; \alpha_3'(1,\alpha_2') = 0.56668; \alpha'_3(2,\alpha_2') = 0.52297.$$

Recall that for $K=1$, $\alpha_2=0.49189$ and

$$\alpha_3(0,\alpha_2) = 0.59897; \alpha_3(1,\alpha_2) = 0.56676; \alpha_3(2,\alpha_2) = 0.52305.$$

Now consider the following profile of realized costs. The lowest cost firm has $c\in$ (0.49183, 0.49189), and the second and the third lowest cost firms' costs are within the interval (0.56676, 0.59890) (within $(\alpha_3(1, \alpha_2), \alpha_3'(0, \alpha_2'))$). The lowest cost firm will enter in period 1 under K but not under K'. Thus $n_2=1$ but $n_2'=0$. As a result, under K' the long run state is reached in the second period since there are three firms whose costs are below $\alpha_3'(0, \alpha_2')$. However, the long run state is not reached in the second period under K, since the next two firms' costs are above $\alpha_3(1, \alpha_2)=0.56676$.

The above discussion also implies that the actual time of reaching the long run state is nonmonotonic in realized cost profile. It is not the case that, say, reducing some firms' costs while keeping the others' unchanged necessarily shortens the actual time of reaching the long run state. It is true that (weakly) more firms enter in the first period for the profile of lower cost realizations. However, more entry in the first period will slow down the entry later, which might lead to a longer entry phase for the profile of lower cost realizations.

To see this, consider the same numerical example above with $F(c)$ being uniform on [0.3, 0.8], $\delta=0.9$, $N=3$, $L=3$, and again $K=1$. In the first profile of realized costs, suppose all six firms' costs lie in the interval $(\alpha_3(1, \alpha_2), \alpha_3(0, \alpha_2))=(0.56676, 0.59897)$. In this case, at period 1 all firms wait, and at period 2 all firms enter (actually overshooting). Now consider the second profile of costs, with all the costs of other five firms being the same as in the original profile, and one firm's cost is smaller than $\alpha_2=0.49189$. Now one firm enters at period 1. The new threshold $\alpha_3(1)=0.56676$, which is below any cost of remaining firms. Thus no firm enters in period 2. Therefore, the entry phase will continue at least until period 3.[21]

5. A limiting case

Here we consider how the length of each period affects the equilibrium entry behavior. Let the length of each period be $\Delta>0$, and $\delta=e^{-r\Delta}$, with r being the interest rate. Holding r constant, as Δ approaches zero, δ converges to 1. Let us reinterpret $1-c$ to be an instantaneous profit of a firm with cost c. For a fixed Δ, a firm with cost c thus earns per period profits of

$$(1-c)\int_0^{\Delta} e^{-rt}dt = (1-c)\frac{1-e^{-r\Delta}}{r} = (1-c)(1-\delta)/r.$$

And hence the lifetime profit for a successful entrant after entry is $(1-c)/r$. The indifference condition (6), given n_t and α_t, is then rewritten as

$$\sum_{j=0}^{N-n_t-1} B(j; N-n_t+L-1, p_t) = \frac{K}{(1-e^{-r\Delta})(1-\alpha_{t+1})/r + e^{-r\Delta}K}. \quad (10)$$

Holding r constant, as $\Delta\to 0$, the RHS of Eq. (10) converges to 1, and thus on the LHS of Eq. (10) p_t must converge to zero. This implies that α_{t+1} converges to α_t as $\Delta\to 0$.

Intuitively, as the period length Δ goes to 0, the cost of waiting one more period (the expected profit forgone in one period) goes to zero. As a result, the benefit of waiting also goes to zero, implying that the information revealed in one period regarding remaining entrants' cost positions converges to zero as well $(\alpha_{t+1}\to\alpha_t)$.[22]

That the probability of entry p_t converges to zero implies that the probability of overshooting converges to zero as well. Thus the possibility of overshooting, and hence the exit phase, disappears as the period length goes to zero. Actually, as Δ goes to 0, the model becomes a continuous time setup, and the remaining entrants choose the optimal time (a continuous variable) to enter given the number of incumbents. The period length Δ is usually interpreted as the time lag to observe the actions of the other firms. The limiting case suggests that overshooting and the exit phase are possible only if there is a positive time lag to observe other firms' actions.

In the limit, though the possibility of overshooting disappears, the inefficiency resulting from delay in entry remains. These results are in contrast to those in Levin and Peck (2003). In their limiting case, as the period length Δ goes to zero, the probability of entry in each period still converges to a positive limit, and delay in entry disappears. The differences come from the fact that, in their model, each firm (regardless of entry costs) has the incentive to preempt the other firm by entering earlier, since firms are equally efficient ex post (have the same marginal cost). If the probability of entry becomes zero in one period, then one firm can profitably deviate by entering in that period. In our model, firms are different in efficiency both ex ante and ex post, as they have different marginal costs. A (high cost) firm that

[20] One may think that an increase in δ would encourage waiting, as in Chamley and Gale (1994). However, in our model an incumbent firm gets flow payoff in every period, while in Chamley and Gale firms only get investment return once (in the end). In our model, waiting cost is measured by the current period payoff $1-c$, which is independent of the discount factor.

[21] Related to this, in the setting of war of attrition, Bulow and Klemperer (1999) show that the equilibrium time of ending the game is not monotonic in players' valuations.

[22] In a previous exercise of comparative statics, we show that p_t increases as δ increases. There we hold the period length constant. In the current comparative statics, we fix the underlying time preference r and vary the period length Δ.

enters in an early period will be driven out of the market later if there are enough firms with lower costs.

6. An extension

In the basic model we have assumed that the number of firms that can be accommodated by the market is fixed. Moreover, the market price as a function of the number of active firms in the market takes a special form: it is invariant to the number of firms in the market up to some point, and then drops to the marginal cost of the marginal firm if additional firms enter. In this section, we relax this assumption regarding the discontinuity of the market price.

Specifically, we assume that there are $N>1$ potential entrants in total. Let n be the number of active firms in a period. Then the market price in that period is denoted as $P(n)$, which is a strictly decreasing function of n. That is, the market price decreases as more firms are operating in the market. Except for assumption (1), all the other assumptions are maintained as in the basic model.

Each firm's marginal cost c is again distributed independently on $[\underline{c},\bar{c}]$ with distribution function $F(c)$. We make the following two assumptions:

$$\frac{P(1)-\bar{c}}{1-\delta}>K; \tag{11}$$

$$P(N)\equiv\tilde{c}\in\left(\underline{c},\bar{c}\right). \tag{12}$$

Assumption (11) implies that it is potentially profitable for a firm with cost very close to $\bar{c}$ to enter. Assumption (12) means that exit is possible: if there are too many firms in the market, then firms with costs above $\tilde{c}$ might exit since the market price might not even recover their marginal costs.

Unlike the basic model in which the number of firms in the long run is always fixed, in this setting the number of firms in the long run is uncertain. Another difference is that in the basic model firms can always make correct entry decisions if they know their ranking in terms of the marginal costs. In this setting the information about the ranking is not sufficient to ensure correct entry. This is because, without knowing the costs of firms whose costs are above the firm in question, that firm is still uncertain how many firms will enter and remain in the long run.

Despite those differences, the symmetric equilibrium in this setting is still characterized by a sequence of strictly increasing cutoffs. Though a formal proof is not attempted, as it is similar to that in the basic model, we spell out the underlying intuition. By deciding to wait for one more period, a firm faces the following trade-off. It forgoes the current period payoff, but at the same time avoids wrong entry in case that there are many firms whose costs are lower than its own cost. The forgone current period payoff is decreasing in c, while the benefit of avoiding wrong entry is increasing in c since a lower cost firm has a lower probability of wrong entry. As a result, lower cost firms have less incentive to wait and thus enter earlier than higher cost firms.

We again denote $\{\alpha_t\}$ as the cost cutoffs, with firms whose costs lie between $(\alpha_t,\alpha_t+1]$ entering in period t. Let h_t be the cost realizations of entrants entering in period t, and H_t be the history at the beginning of period t. Denote m_t as the number of new entry in period t, and n_t as the number of incumbents at the beginning of period t. Thus, $n_{t+1}=n_t+m_t$, and $n_1=0$.

Unlike the basic model in which it is straightforward to show when the entry phase ends, in the extended model it is a little more complicated. In the following we specify when the entry phase ends. It can end under two scenarios. In the first scenario, no exit occurs in the current period t. However, in the next period the expected payoff of entry even for the lowest cost firm (among the remaining entrants) is negative. The condition can be written as

$$\frac{P(n_t+m_t+1)-\alpha_{t+1}}{1-\delta}<K.$$

The above condition means that even in the best scenario (the number of firms in the long run state is n_t+m_t+1), the lowest cost firm among the remaining entrants has no incentive to enter.

In the second scenario, exit occurs in the current period t. Let c_j be the cost of the j^{th} lowest cost firm that entered in period t. Of course, $j\le m_t$ by the definition of m_t. Exit occurs for a firm with c_j if $P(n_t+j)-c_j<0$. This is because this firm cannot recover its marginal cost.[23] By assumption (12), a necessary condition for this to happen is that $c_j>\tilde{c}$. Note that if a firm with c_j exits, then all the firms with $c>c_j$ that have already entered exit as well. This is because $P(n_t+j)$ is decreasing in j, and c_j is increasing in j. To be precise, no exit occurs in period t if $P(n_t+m_t)-c_{m_t}\ge 0$ (even the highest cost firm can recover its marginal cost). And exit occurs if $P(n_t+m_t)-c_{m_t}<0$. In this case, the total number of exits is m_t-j^*, where j^* is the largest number that satisfies $P(n_t+j)-c_j\ge 0$.[24] If exit occurs in period t, the long run state is reached. This is because no further entry will occur as the potential entrants' costs are higher than the exiting firm(s).

Now we characterize the evolution of the equilibrium cutoffs $\{\alpha_t\}$ in a symmetric equilibrium. Again, on-path histories at the beginning of period t can be summarized by n_t and α_t. Let k_t^{-i} denote the number of firms other than firm i that enter in period t, given that firms other than i follow cutoff strategy α_{t+1}. Denote the probability of k_t^{-i} as $\Pr(k_t^{-i})$. Given that other firms adopt the symmetric cutoff strategy,

$$Pr\left(k_t^{-i}\right)=\binom{N-n_t-1}{k_t^{-i}}\left[\frac{F(\alpha_{t+1})-F(\alpha_t)}{1-F(\alpha_t)}\right]^{k_t^{-i}}\left[\frac{1-F(\alpha_{t+1})}{1-F(\alpha_t)}\right]^{N-n_t-1-k_t^{-i}}.$$

The value functions of $c=\alpha_{t+1}$ can be expressed as:

$$\begin{aligned}V_t(\alpha_{t+1}|n_t,\alpha_t)=&\sum_{k_t^{-i}=0}^{N-1-n_t}Pr\left(k_t^{-i}\right)\max\left\{P\left(n_t+1+k_t^{-i}\right)-\alpha_{t+1},0\right\}-K\\&+\delta\sum_{k_t^{-i}=0}^{N-1-n_t}Pr\left(k_t^{-i}\right)V_{t+1}^I\left(\alpha_{t+1}|n_t+1+k_t^{-i},\alpha_{t+1}\right),\end{aligned}\tag{13}$$

$$W_t(\alpha_{t+1}|n_t,\alpha_t)=\delta\sum_{k_t^{-i}=0}^{N-1-n_t}Pr\left(k_t^{-i}\right)\max\left\{0,V_{t+1}\left(\alpha_{t+1}|n_t+k_t^{-i},\alpha_{t+1}\right)\right\},\tag{14}$$

where on the equilibrium path for $c>\alpha_t$ the value of $V_t^I(c|n_t,\alpha_t)$ *is*

$$V_t^I(c|n_t,\alpha_t)=E\left[\max\left\{P\left(n_t+\overline{k_t^{-i}}\right)-c,0\right\}|n_t,\alpha_t\right]+\delta EV_{t+1}^I\left(c|n_t+\overline{k_t^{-i}},\alpha_{t+1}\right).$$

In the above equation, $\overline{k_t^{-i}}$ might be different from k_t^{-i} because of the possibility of exit.

Given history α_t and n_t, the marginal type or the cutoff α_{t+1} is determined by the following indifference condition: $V(\alpha_{t+1}|n_t,\alpha_t)-W(\alpha_{t+1}|n_t,\alpha_t)=0$. More explicitly, following Eqs. (13) and (14), the condition can be expressed as:

$$\begin{aligned}&\sum_{k_t^{-i}=0}^{N-1-n_t}Pr\left(k_t^{-i}\right)\left[\max\left\{P\left(n_t+1+k_t^{-i}\right)-\alpha_{t+1},0\right\}+\delta V_{t+1}^I\left(\alpha_{t+1}|n_t+1+k_t^{-i},\alpha_{t+1}\right)\right]-K\\&-\delta\sum_{k_t^{-i}=0}^{N-1-n_t}Pr\left(k_t^{-i}\right)\max\left\{0,V_{t+1}\left(\alpha_{t+1}|n_t+k_t^{-i},\alpha_{t+1}\right)\right\}=0.\end{aligned}\tag{15}$$

[23] If a firm can recover its marginal cost, then it will stay in the market, since the entry cost K is sunk. This is the right condition since high cost firms will exit immediately if the market price is below their marginal costs. Consider the case where two firms entered in the current period and the market price $P(n_t+2)$ is below both firms' marginal costs. Essentially these two firms will play a war of attrition game. To simplify matters, we assume that the firm with the higher cost will exit immediately, for the reason that it will lose more money if it plays a war of attraction game with the other firm.

[24] Note that it is impossible for all the firms that entered in period t to exit. To see this, notice that occurs only if $P(n_t+1)-c_1<0$. But if this is the case, this c_1 firm would have not entered in the first place, as it has a negative expected payoff in the best scenario (it has the lowest cost among the remaining firms).

We can show that such an α_{t+1} exists, which satisfies Eq. (15) given n_t and α_t. When $\alpha_{t+1}=\alpha_t$, $\Pr(k_t^{-i}=0)=1$. By the fact that α_t enters in period t we have $V_t(\alpha_t|n_t+1,\alpha_t)-K>0$. Note that $V_t(\alpha_t|n_t+1,\alpha_t)\leq\frac{P(n_t+1)-\alpha_t}{1-\delta}$ since the number of active firms in any later period is greater than or equal to n_t+1, therefore we have $P(n_t+1)-\alpha_t>(1-\delta)K$. Then the LHS of Eq. (15) can be simplified as

$$P(n_t+1)-\alpha_t-(1-\delta)K+\delta\left[V^I_{t+1}(\alpha_t|n_t+1,\alpha_t)-K-V_{t+1}(\alpha_t|n_t,\alpha_t)\right]$$

Note that this term is strictly positive since $P(n_t+1)-\alpha_t>(1-\delta)K$, and

$$V^I_{t+1}(\alpha_t|n_t+1,\alpha_t)-K-V_{t+1}(\alpha_t|n_t,\alpha_t)\geq 0$$

because entering one period early by type α_t decreases the remaining firms' probability and speed of entry on the equilibrium path (those firms have higher costs than α_t). Therefore, the LHS is positive for Eq. (15) when $\alpha_{t+1}=\alpha_t$. On the other hand, when $\alpha_{t+1}=\bar{c}$, $\Pr(k_t^{-i}=N-n_t-1)=1$. By assumption (12), a firm with cost $\bar{c}$ cannot earn any flow profit in the market given that all potential entrants enter in period t. The LHS of Eq. (15) is hence just $-K$, the entry cost. Therefore, the LHS of Eq. (15) is negative when $\alpha_{t+1}=\bar{c}$. Combining the above results, by the continuity of the LHS there must be some $\alpha_{t+1}\in(\alpha_t,\bar{c}]$ satisfying Eq. (15).

It would be desirable to establish the uniqueness of α_{t+1} given α_t and n_t by showing that the LHS of Eq. (15) is strictly decreasing in α_{t+1}. It turns out that it is very hard to establish, though we conjecture it is true. The difficulty lies in the fact that it is hard to compare the magnitudes of changes in $V^I_{t+1}(\alpha_{t+1}|n_t+1+k_t^{-i},\alpha_{t+1})$ and $V_{t+1}(\alpha_{t+1}|n_t+k_t^{-i},\alpha_{t+1})$ as α_{t+1} varies, since two value functions involve with different histories. This means that we cannot rule out the possibility of multiple equilibria among symmetric equilibria.

Similar to the non-monotonicity results in the basic model, here the number of firms in the long run is not monotonic in firms' cost realizations. Reducing one firm's cost realization might lead this firm to enter early, thus discouraging remaining entrants from entering. This effect in general tends to reduce the number of firms in the long run.

7. Concluding remarks

This paper studies industry dynamics based on firms' learning about their relative cost positions. The industry experiences three phases in order: an entry phase with an uncertain length, a possible exit phase which lasts for one period, and the long run state with no entry or exit. In the unique symmetric equilibrium, lower cost firms enter earlier than higher cost firms, which leads to gradual entry over time. The exit phase due to overshooting arises with positive probability on the equilibrium path, in which only firms that entered immediately before will possibly exit. Both expected entry and the probability of overshooting decreases in the length of the entry phase, leading to a trade-off between delay and overshooting. These features of industry dynamics are largely consistent with empirical evidence.

We have to admit that our model cannot capture all the stylized facts of industry dynamics, as the evolution of technologies of a specific industry certainly impacts the evolution of that industry. However, we believe that our model captures an important aspect of industry dynamics: how learning about relative cost positions affects the dynamics of entry and exit.

Our model can also incorporate learning about an uncertain demand size. Specifically, the demand size N can be distributed according to some distribution $G(N)$, and it can be learned only if total entry overshoots it. The equilibrium will still have the feature that lower cost firms enter earlier than higher cost firms. The difference is that now learning about the demand size will play a role in determining the equilibrium cost cutoffs. This implies that expected massive entry might occur later in the entry phase if the demand size has the "right" distribution.[25] Moreover, overshooting and the exit phase now will certainly occur on the equilibrium path.

Appendix A. Proof of Proposition 1

Proof. We first show that on the equilibrium path no firm has incentive to deviate from the above cutoff strategy. Consider a continuation game on the equilibrium path (in the entry phase) with history n_t and α_t. Given that the other firms follow the cutoff strategy α_{t+1}, we need to show that it is optimal for the firm in question to follow the cutoff strategy α_{t+1}, i.e., for any $c\in(\alpha_t,\alpha_{t+1})$, $V_t(c|n_t,\alpha_t)-W_t(c|n_t,\alpha_t)>0$, and for any $c>\alpha_{t+1}$, $V_t(c|n_t,\alpha_t)-W_t(c|n_t,\alpha_t)<0$.

First consider the case $c\in(\alpha_t,\alpha_{t+1})$. Recall that, given the cutoff strategy, all the uncertainty about whether this firm should enter is resolved in period t. Hence,

$$V_t(c|n_t,\alpha_t)=Pr\left[A^i(c)|\alpha_t,n_t\right]\frac{1-c}{1-\delta}-K$$
$$W_t(c|n_t,\alpha_t)=\delta Pr\left[A^i(c)|\alpha_t,n_t\right]\left[\frac{1-c}{1-\delta}-K\right].$$

Then we have

$$\begin{aligned}V_t(c|n_t,\alpha_t)-W_t(c|n_t,\alpha_t)&=Pr\left[A^i(c)|\alpha_t,n_t\right][(1-c)+\delta K]-K\\&=Pr\left[A^i(c)|\alpha_t,n_t\right][(1-c)+\delta K]-K-\left\{Pr\left[A^i(\alpha_{t+1})|\alpha_t,n_t\right][(1-\alpha_{t+1})+\delta K]-K\right\}\\&=Pr\left[A^i(c)|\alpha_t,n_t\right][(1-c)+\delta K]-Pr\left[A^i(\alpha_{t+1})|\alpha_t,n_t\right][(1-\alpha_{t+1})+\delta K]>0.\end{aligned}$$

The second equality follows because $\Pr[A^i(\alpha_{t+1})|\alpha_t,n_t][(1-\alpha_{t+1})+\delta K]-K=0$ by Eq. (3), while the inequality holds since $\Pr[A^i(c)|\alpha_t,n_t]>\Pr[A^i(\alpha_{t+1})|\alpha_t,n_t]$, which follows from $c<\alpha_{t+1}$.

Next consider the case $c>\alpha_{t+1}$. Since the uncertainty regarding the firm with cost c is not completely resolved at the beginning of period $t+1$, the value functions can only be written recursively. Let y_t^{-i} denote the number of firms other than firm i entering in period t. The value functions are

$$\begin{aligned}V_t(c|n_t,\alpha_t)&=Pr\left[y_t^{-i}<N-n_t|\alpha_t,n_t\right]\left[(1-c)+\delta E_{h_t^{-i}}\left[V^I_{t+1}\left(c|(n_t,\alpha_t)\times\left(c,h_t^{-i}\right)\right)|y_t^{-i}<N-n_t\right]\right]\\&\quad-K,\\W_t(c|n_t,\alpha_t)&=Pr\left[y_t^{-i}<N-n_t|\alpha_t,n_t\right]\\&\quad\times\delta E_{h_t^{-i}}\left[\max\left\{V_{t+1}\left(c|(n_t,\alpha_t)\times h_t^{-i}\right),W_{t+1}\left(c|(n_t,\alpha_t)\times h_t^{-i}\right)\right\}|y_t^{-i}<N-n_t\right].\end{aligned}$$

The expressions follow since the firm with cost c earns no positive flow profit from period t on in the event when $y_t^{-i}\geq N-n_t$. With these value functions, we have

$$\begin{aligned}&V_t(c|n_t,\alpha_t)-W_t(c|n_t,\alpha_t)\\&=Pr\left[y_t^{-i}<N-n_t|\alpha_t,n_t\right]\{(1-c)+\delta E_{h_t^{-i}}\left[V^I_{t+1}\left(c|(n_t,\alpha_t)\times\left(c,h_t^{-i}\right)\right)|y_t^{-i}<N-n_t\right]\\&\quad-\delta E_{h_t^{-i}}\left[\max\left\{V_{t+1}\left(c|(n_t,\alpha_t)\times h_t^{-i}\right),W_{t+1}\left(c|(n_t,\alpha_t)\times h_t^{-i}\right)\right\}|y_t^{-i}<N-n_t\right]\}-K\\&\leq Pr\left[y_t^{-i}<N-n_t|\alpha_t,n_t\right]\{(1-c)\\&\quad+\delta E_{h_t^{-i}}\left[V^I_{t+1}\left(c|(n_t,\alpha_t)\times\left(c,h_t^{-i}\right)\right)-V_{t+1}\left(c|(n_t,\alpha_t)\times h_t^{-i}\right)|y_t^{-i}<N-n_t\right]\}-K\end{aligned}\tag{16}$$

To proceed, we show that the following important inequality holds

$$V^I_{t+1}\left(c\middle|(n_t,\alpha_t)\times\left(c,h_t^{-i}\right)\right)-V_{t+1}\left(c\middle|(n_t,\alpha_t)\times h_t^{-i}\right)\leq K.\tag{17}$$

To see this, first note that in the event of $A^i(c)$ (c is in the winning group), $V^I_{t+1}(c|\cdot)-V_{t+1}(c|\cdot)=K$ since the flow payoffs for both values are the same. In the other event that c is not among the winning group, the flow payoff is weakly lower for V^I_{t+1}. This is because knowing $c>\alpha_{t+1}$ will induce other remaining firms with cost below c to enter

[25] For instance, suppose the unknown demand size is either high or low, as examined in Horvath et al. (2001). Firms must be cautious to enter in earlier periods for fear of low demand, and thus the probability of entry must be lower. Once the number of firms in the market exceeds that for a low demand without overshooting, firms become more aggressive in entry. Thus massive entry is more likely in later periods.

earlier, thus the deviating firm gets flow payoffs for weakly fewer periods. Therefore, the difference between the two values is less than K.

Now applying inequality (17) to (16), we have

$$\begin{aligned}V_t(c|n_t,\alpha_t) - W_t(c|n_t,\alpha_t) &\le Pr\left(y_t^{-i}<N-n_t|\alpha_t,n_t\right)[(1-c)+\delta K]-K\\ &= Pr\left(y_t^{-i}<N-n_t|\alpha_t,n_t\right)[(1-c)+\delta K]-K-[V_t(\alpha_{t+1}|n_t,\alpha_t)-W_t(\alpha_{t+1}|n_t,\alpha_t)]\\ &= Pr\left(y_t^{-i}<N-n_t|\alpha_t,n_t\right)[(1-c)+\delta K]-Pr\left(y_t^{-i}<N-n_t|\alpha_t,n_t\right)\left[(1-\alpha_{t+1})+\delta K\right]\\ &= Pr\left(y_t^{-i}<N-n_t|\alpha_t,n_t\right)\left[(1-c)-(1-\alpha_{t+1})\right]<0.\end{aligned}$$

To see that the second equality holds, note that $\Pr(y_t^{-i}<N-n_t|\alpha_t,n_t)=\Pr[A^i(\alpha_{t+1})|\alpha_t,n_t]$.

We also need to show the optimality of the cutoff strategies following any off the equilibrium path. Indeed the basic logic of the proof is exactly the same, so the proof is omitted.

As to uniqueness, first, on the equilibrium path the cutoffs are uniquely determined by Lemma 1. Also, we have seen that each off-path belief is uniquely determined given the cutoff and the type distribution in the previous period, so that there is no freedom in varying the off-path cutoffs either. This shows the uniqueness of a symmetric cutoff strategy equilibrium. □

Appendix B. Proof of Proposition 2

Proof. Suppose there is a symmetric equilibrium with non-cutoff strategies. In particular, suppose that in a continuation game with history H_t a firm with cost c' enters in period t, while firms with some cost type(s) lower than c' do not enter in period t. Let $\tilde{c}$ be the lowest cost type that does not enter in period t.[26] Clearly, $\tilde{c}<c'$.

Note that type $\tilde{c}$ will enter at $t+1$ if period $t+1$ is still an entry phase, since there is no other potential entrant whose cost is less than $\tilde{c}$, and thus waiting is unprofitable. The values at period t for $\tilde{c}$ can thus be written as

$$\begin{aligned}V_t(\tilde{c}|H_t) &= E_{h_t^{-i}}\left[\pi_t\left(\tilde{c}|H_t\times\left(\tilde{c},h_t^{-i}\right)\right)+\delta V_{t+1}^I\left(\tilde{c}|H_t\times\left(\tilde{c},h_t^{-i}\right)\right)\right]-K,\\ W_t(\tilde{c},|H_t) &= \delta E_{h_t^{-i}}\left[\max\left\{0,V_{t+1}\left(\tilde{c}|H_t\times h_t^{-i}\right)\right\}\right].\end{aligned}$$

Since c' enters and $\tilde{c}$ waits at t, we have

$$V_t(c'|H_t)-W_t(c'|H_t)\ge 0\ge V_t(\tilde{c}|H_t)-W_t(\tilde{c}|H_t). \tag{18}$$

Note that, since $W_t(c'|H_t\times h_t^{-i})$ can never be negative,

$$\begin{aligned}V_t(c'|H_t)-W_t(c'|H_t) &= V_t(c'|H_t)-\delta_t E_{h_t^{-i}}\left[\max\left\{W_{t+1}\left(c'|H_t\times h_t^{-i}\right),V_{t+1}\left(c'|H_t\times h_t^{-i}\right)\right\}\right]\\ &\le V_t(c'|H_t)-\delta E_{h_t^{-i}}\left[\max\left\{0,V_{t+1}\left(c'|H_t\times h_t^{-i}\right)\right\}\right].\end{aligned} \tag{19}$$

Then Eqs. (18) and (19) imply

$$\begin{aligned}&E_{h_t^{-i}}\left[\pi_t\left(c'|H_t\times\left(c',h_t^{-i}\right)\right)-\pi_t\left(\tilde{c}|H_t\times\left(\tilde{c},h_t^{-i}\right)\right)\right]\\ &\ge\delta E_{h_t^{-i}}\left[\max\left\{0,V_{t+1}\left(c'|H_t\times h_t^{-i}\right)\right\}-V_{t+1}^I\left(c'|H_t\times\left(c',h_t^{-i}\right)\right)\right]\\ &-\delta E_{h_t^{-i}}\left[\max\left\{0,V_{t+1}\left(\tilde{c}|H_t\times h_t^{-i}\right)\right\}-V_{t+1}^I\left(\tilde{c}|H_t\times\left(\tilde{c},h_t^{-i}\right)\right)\right]\end{aligned} \tag{20}$$

The LHS of Eq. (20) is negative since $c'>\tilde{c}$; the flow profit at each period must be higher for a lower cost firm, and firm i with cost $\tilde{c}$ is more likely to be among the N lowest cost firms.

Recall that $A^i(c)$ denotes the event that firm i with cost c is among the N lowest cost firms. Since the uncertainty regarding whether $A^i(\tilde{c})$ occurs is completely resolved at the beginning of period $t+1$, we have $E_{h_t^{-i}}[-V_{t+1}^I(\tilde{c}|H_t\times(c',h_t^{-i}))|A^i(\tilde{c})^c]=0$,[27] and $E_{h_t^{-i}}[V_{t+1}(\tilde{c}|H_t\times h_t^{-i})-V_{t+1}^I(\tilde{c}|H_t\times(\tilde{c},h_t^{-i}))|A^i(\tilde{c})]=-K$. As a result,

$$\begin{aligned}&E_{h_t^{-i}}\left[\max\left\{0,V_{t+1}\left(\tilde{c}|H_t\times h_t^{-i}\right)\right\}-V_{t+1}^I\left(\tilde{c}|H_t\times\left(\tilde{c},h_t^{-i}\right)\right)\right]\\ &=E_{h_t^{-i}}\left[V_{t+1}\left(\tilde{c}|H_t\times h_t^{-i}\right)-V_{t+1}^I\left(\tilde{c}|H_t\times\left(\tilde{c},h_t^{-i}\right)\right)|A^i(\tilde{c})\right]Pr\left(A^i(\tilde{c})\right)=-KPr\left(A^i(\tilde{c})\right).\end{aligned} \tag{21}$$

Moreover, since $c'>\tilde{c}$, $A^i(\tilde{c})^c\subset A^i(c')^c$. Thus we have

$$E_{h_t^{-i}}\left[-V_{t+1}^I\left(c'|H_t\times\left(c',h_t^{-i}\right)\right)|A^i(\tilde{c})^c\right]=0. \tag{22}$$

Using the above equalities (21) and (22), the RHS of Eq. (20) becomes

$$\begin{aligned}&E_{h_t^{-i}}\left[\max\left\{0,V_{t+1}\left(c'|H_t\times h_t^{-i}\right)\right\}-V_{t+1}^I\left(c'|H_t\times\left(c',h_t^{-i}\right)\right)|A^i(\tilde{c})\right]Pr\left(A^i(\tilde{c})\right)\\ &+E_{h_t^{-i}}\left[-V_{t+1}^I\left(c'|H_t\times\left(c',h_t^{-i}\right)\right)|A^i(\tilde{c})^c\right]Pr\left(A^i(\tilde{c})^c\right)\\ &-E_{h_t^{-i}}\left[\max\left\{0,V_{t+1}\left(\tilde{c}|H_t\times h_t^{-i}\right)\right\}-V_{t+1}^I\left(\tilde{c}|H_t\times\left(\tilde{c},h_t^{-i}\right)\right)|A^i(\tilde{c})\right]Pr\left(A^i(\tilde{c})\right)\\ &-E_{h_t^{-i}}\left[-V_{t+1}^I\left(\tilde{c}|H_t\times\left(\tilde{c},h_t^{-i}\right)\right)|A^i(\tilde{c})^c\right]Pr\left(A^i(\tilde{c})^c\right)\\ &=E_{h_t^{-i}}\left[\max\left\{0,V_{t+1}\left(c'|H_t\times h_t^{-i}\right)\right\}-V_{t+1}^I\left(c'|H_t\times\left(c',h_t^{-i}\right)\right)|A^i(\tilde{c})\right]Pr\left(A^i(\tilde{c})\right)+KPr\left(A^i(\tilde{c})\right)\\ &\ge E_{h_t^{-i}}\left[V_{t+1}\left(c'|H_t\times h_t^{-i}\right)-V_{t+1}^I\left(c'|H_t\times\left(c',h_t^{-i}\right)\right)|A^i(\tilde{c})\right]Pr\left(A^i(\tilde{c})\right)+KPr\left(A^i(\tilde{c})\right)\\ &\ge -KPr\left(A^i(\tilde{c})\right)+KPr\left(A^i(\tilde{c})\right)=0.\end{aligned}$$

The last inequality follows from the fact that $V_{t+1}^I(c'|H_t\times(c',h_t^{-i}))-V_{t+1}(c'|H_t\times h_t^{-i})\le K$, similar to inequality (17). This shows that $0>$LHS of Eq. (20)$\ge$RHS of Eq. (20)≥ 0, a contradiction. Therefore, there is no symmetric equilibrium in non-cutoff strategies. □

Appendix C. Proof of Lemma 3

Proof. To show that Eq. (7) holds, we only need to show that the following one-step property holds:

$$\sum_{j=0}^{N_1}B(j;N_1+L,p)<\sum_{j=0}^{N_1+1}B(j;N_1+L+1,p). \tag{23}$$

We proceed with the following algebra:

$$\begin{aligned}\sum_{j=0}^{N_1}B(j;N_1+L,p) &= [p+(1-p)]\sum_{j=0}^{N_1}B(j;N_1+L,p)\\ &=(1-p)^{N_1+L+1}+\sum_{j=1}^{N_1}\left[\binom{N_1+L}{j-1}+\binom{N_1+L}{j}\right]p^j(1-p)^{N_1+L+1-j}\\ &+\binom{N_1+L}{N_1}p^{N_1+1}(1-p)^L=(1-p)^{N_1+L+1}\\ &+\sum_{j=1}^{N_1}\binom{N_1+L+1}{j}p^j(1-p)^{N_1+L+1-j}+\binom{N_1+L}{N_1}p^{N_1+1}(1-p)^L\\ &<\binom{N_1+L+1}{0}(1-p)^{N_1+L+1}+\sum_{j=1}^{N_1}\binom{N_1+L+1}{j}p^j(1-p)^{N_1+L+1-j}\\ &+\binom{N_1+L+1}{N_1+1}p^{N_1+1}(1-p)^L=\sum_{j=0}^{N_1+1}B(j;N_1+L+1,p)\end{aligned}$$

The inequality holds since $\binom{N_1+L}{N_1}<\binom{N_1+L+1}{N_1+1}$. Therefore, Eq. (23) is valid. □

Appendix D. Proof of Proposition 4

Proof. For notational simplicity, denote

$$\Gamma(k;N,p)\equiv\sum_{j=0}^{k}B(j;N,p).$$

That is, $\Gamma(k;N,p)$ is the cumulative probability. The probability of overshooting is then $P_t^o=1-\Gamma(N-n_t;N+L-n_t,p_t)$.

Fix t and t' such that $t'>t$. Note that $n_t\le n_{t'}$. Denote

$$N_1=N-n_t+L-1, N_2=N-n_{t'}+L-1.$$

By the above definitions, $N_1\ge N_2$. We wish to show $P_t^o>P_{t'}^o$, or equivalently

$$\Gamma(N_1-L+1;N_1+1,p_t)<\Gamma(N_2-L+1;N_2+1,p_{t'}) \tag{24}$$

Recall that $\alpha_{t'+1}>\alpha_{t+1}$ for $t'>t$. By Eq. (6), the following inequality holds:

$$\Gamma(N_1-L;N_1,p_t)<\Gamma(N_2-L;N_2,p_{t'}) \tag{25}$$

[26] If such $\tilde{c}$ does not exit, it would suffice to take an infinmum type to wait at t instead, and use approximation arguments when necessary.

[27] The expression $A^i(c)^c$ is the complement of $A^i(c)$.

We will use the following equality in later derivations:[28]

$$\Gamma(N_1-L+1;N_1+1,p_t)=\Gamma(N_1-L;N_1,p_t)+(1-p_t)B(N_1-L+1;N_1,p_t). \quad (26)$$

We consider the following two mutually exclusive cases in turns.

Case A: $B(N_1-L+1;N_1,p_t)\le B(N_2-L+1;N_2,p_{t'})$.

By Proposition 4, $p_{t'}<p_t \Leftrightarrow 1\text{-}p_{t'}>1\text{-}p_t$, thereby

$$(1-p_t)B(N_1-L+1;N_1,p_t)\le(1-p_{t'})B(N_2-L+1;N_2,p_{t'}). \quad (27)$$

By Eqs. (25), (26) and, (27) we have

$$\begin{aligned}&\Gamma(N_1-L+1;N_1+1,p_t)=\Gamma(N_1-L;N_1,p_t)+(1-p_t)B(N_1-L+1;N_1,p_t)\\ &<\Gamma(N_2-L;N_2,p_{t'})+(1-p_{t'})B(N_2-L+1;N_2,p_{t'})=\Gamma(N_2-L+1;N_2+1,p_{t'}),\end{aligned}$$

which is the desired result (Eq. (24)).

Case B: $B(N_1-L+1;N_1,p_t)>B(N_2-L+1;N_2,p_{t'})$.

In this case, we use the following condition:

$$\Gamma(N_1-L+1;N_1,p_t)<\Gamma(N_2-L+1;N_2,p_{t'}), \quad (28)$$

which is later verified from Eq. (25). Inequality Eq. (28) implies

$$\Gamma(N_2-L;N_2,p_{t'})-\Gamma(N_1-L;N_1,p_t)>B(N_1-L+1;N_1,p_t)-B(N_2-L+1;N_2,p_{t'}).$$

By $1>1-p_{t'}>1-p_t$, we obtain

$$\begin{aligned}&B(N_1-L+1;N_1,p_t)-B(N_2-L+1;N_2,p_{t'})\\ &>(1-p_t)B(N_1-L+1;N_1,p_t)-(1-p_{t'})B(N_2-L+1;N_2,p_{t'}).\end{aligned}$$

Therefore,

$$\begin{aligned}&\Gamma(N_2-L;N_2,p_{t'})-\Gamma(N_1-L;N_1,p_t)\\ &>(1-p_t)B(N_1-L+1;N_1,p_t)-(1-p_{t'})B(N_2-L+1;N_2,p_{t'})\\ &\Leftrightarrow\Gamma(N_2-L;N_2,p_{t'})+(1-p_{t'})B(N_2-L+1;N_2,p_{t'})\\ &>\Gamma(N_1-L;N_1,p_t)+(1-p_t)B(N_1-L+1;N_1,p_t)\\ &\Leftrightarrow\Gamma(N_1-L+1;N_1+1,p_t)<\Gamma(N_2-L+1;N_2+1,p_{t'}).\end{aligned}$$

This again yields the desired result (Eq. (24)). □

The rest of the proof is devoted to showing Eq. (28) holds given Eq. (25). More explicitly, we want to show

$$\Gamma(N_1-L;N_1,p_t)<\Gamma(N_2-L;N_2,p_{t'})\Rightarrow\Gamma(N_1-L+1;N_1,p_t)<\Gamma(N_2-L+1;N_2,p_{t'}). \quad (29)$$

By changing variables (N_1-$j=j'$) and using the fact that $\binom{N_1}{j}=\binom{N_1}{N_{1-i}}$ we have

$$\begin{aligned}\Gamma(N_1-L;N_1,p_t)&=\sum_{j=0}^{N_1-L}\binom{N_1}{j}p_t^j(1-p_t)^{N_1-j}=\sum_{j'=N_1}^{L}\binom{N_1}{N_1-j'}p_t^{N_1-j'}(1-p_t)^{j'}\\ &=1-\sum_{j'=0}^{L-1}\binom{N_1}{j'}p_t^{N_1-j'}(1-p_t)^{j'}=1-\Gamma(L-1;N_1,1-p_t).\end{aligned}$$

Then Eq. (29) can be further rewritten as

$$\begin{aligned}&\Gamma(L-1;N_1,1-p_t)>\Gamma(L-1;N_2,1-p_{t'})\\ &\Rightarrow\Gamma(L-2;N_1,1-p_t)>\Gamma(L-2;N_2,1-p_t').\end{aligned}$$

We first simplify notation further. Denote $q_1=1-p_t$, $q_2=1-p_{t'}$, with $q_1<q_2$, and

$$\begin{aligned}&B_1(j)=B(j;N_1,q_1),\Gamma_1(j)=\textstyle\sum_{k=0}^{j}B_1(k),\\ &B_2(j)=B(j;N_2,q_2),\Gamma_2(j)=\textstyle\sum_{k=0}^{j}B_2(k).\end{aligned}$$

By the above definitions, we have $\Gamma_1(N_1)=1$, $\Gamma_2(N_2)=1$, and both $\Gamma_1(j)$ and $\Gamma_2(j)$ are strictly increasing in j. Under the simplified notations, (Eq. (29)) becomes

$$\Gamma_1(L-1)>\Gamma_2(L-1)\Rightarrow\Gamma_1(L-2)>\Gamma_2(L-2).$$

Thus it is sufficient to show that

$$\Gamma_1(j)>\Gamma_2(j)\Rightarrow\Gamma_1(j-1)>\Gamma_2(j-1). \quad (30)$$

To proceed, consider the following likelihood ratio $\zeta(j)=\ln\{B_1(j)/B_2(j)\}$ for each $j=0,\ldots,N_2$. Then

$$\begin{aligned}\zeta(j)&=\ln\frac{N_1!}{j!(N_1-j)!}\frac{j!(N_2-j)!}{N_2!}\frac{q_1^j(1-q_1)^{N_1-j}}{q_2^j(1-q_2)^{N_2-j}}\\ &=\ln\frac{N_1!}{N_2!}+\ln\frac{(N_2-j)!}{(N_1-j)!}+j\ln\frac{q_1/(1-q_1)}{q_2/(1-q_2)}+\ln\frac{(1-q_1)^{N_1}}{(1-q_2)^{N_2}}\\ &=-\sum_{l=N_2-j+1}^{N_1-j}\ln l+j\ln\frac{q_1/(1-q_1)}{q_2/(1-q_2)}+const,\end{aligned}$$

where *const* is some constant. Thus,

$$\zeta(j+1)-\zeta(j)=\ln\frac{N_1-j}{N_2-j}+\ln\frac{q_1/(1-q_1)}{q_2/(1-q_2)}.$$

It is easy to see that $\zeta(j+1)-\zeta(j)$ weakly increases in j (strictly when $N_1>N_2$). This is because $N_1\ge N_2$ implies that $(N_1-j)/(N_2-j)$ is weakly increasing in j (strictly, when $N_1>N_2$), and the other term $\frac{q_1/(1-q_1)}{q_2/(1-q_2)}$ is constant.

Lemma 4. *(i) $\zeta(j)$ is either monotonic or U-shape (i.e., first decreasing but then increasing); (ii) there is some $j\le N_2$ such that $\zeta(j)<0$.*

Proof. First consider the case $N_1=N_2$. Then $\zeta(j+1)$-$\zeta(j)$ is constant and negative since by $q_1<q_2$

$$\frac{q_1/(1-q_1)}{q_2/(1-q_2)}<1\Rightarrow\ln\frac{q_1/(1-q_1)}{q_2/(1-q_2)}<0.$$

Thus we have $\zeta(j)$ being strictly decreasing.

Next consider the case that $N_1>N_2$. Note that in this case $\zeta(j+1)-\zeta(j)$ is strictly increasing in j. We consider two subcases. In the first subcase, $\zeta(1)-\zeta(0)\ge0$. Then we have $\zeta(j)$ being strictly increasing. Next consider the other subcase, $\zeta(1)<\zeta(0)$. Then if $\zeta(j')-\zeta(j'-1)\ge0$ for some $j'\ge2$, $\zeta(j)$ must be increasing for any $j\ge j'$. Therefore, either $\zeta(j)$ must be decreasing for all j, or it is of U-shape. This proves part (i).

To show part (ii), suppose $\zeta(j)\ge0$, i.e., $B_1(j)\ge B_2(j)$ for all $j\le N_2$. First, $q_1<q_2$ implies that $\zeta(j)\ne0$ for some j. This means that $B_1(j)>B_2(j)$ for some j. Now summing up $B_1(j)$ and $B_2(j)$, we have $\Gamma_1(N_2)>\Gamma_2(N_2)$. But by $N_2\le N_1$,

$$1=\Gamma_1(N_1)\ge\Gamma_1(N_2)>\Gamma_2(N_2)=1,$$

a contradiction. □

Lemma 4 implies that we only need to consider the following two cases.

CASE I If $\zeta(0)\le0$, either $\zeta(j)$ is always nonpositive, or crosses the line of $\zeta=0$ from below only once. Equivalently,[29]

$$\begin{aligned}&B_1(j)\le B_2(j),\quad for\, j=0,\ldots,\bar{j},\\ &B_1(j)\ge B_2(j), for\, j=\bar{j}+1,\ldots,N_2,\left(0\le\bar{j}\le N_2\right),\end{aligned}$$

[28] This is because, the event that at most N-n_t entrants out of $N+L$-n_t is the disjoint union of {at most N-n_t-1 entrants out of $N+L$-n_t-1 firms} and {exactly N-n_t entrants out of $N+L$-n_t-1 firms and the remaining firm waits}.

[29] It is straightforward that an equality may hold only for $j=0,\bar{j},\bar{j}+1$, or N_2.

CASE II If $\zeta(0)>0$, $\zeta(j)$ must cross the line of $\zeta=0$ from above (by part (ii) of Lemma 4), and might cross it from below later at most once. Equivalently,[30]

$$\begin{aligned}&B_1(j)>B_2(j), \quad \text{for } j=0,\ldots,j_1,\\ &B_1(j)\le B_2(j), \quad \text{for } j=j_1+1,\ldots,j_2,\\ &B_1(j)\ge B_2(j), \quad \text{for } j=j_2+1,\ldots,N_2.(0\le j_1<j_2\le N_2).\end{aligned}$$

We consider Case I and Case II in turns.

Lemma 5. *In CASE I*, $\Gamma_1(j)\le\Gamma_2(j)$ for $j=0,\ldots,N_2$.

Proof. Suppose on the contrary that, for some j'

$$\Gamma_1(j')>\Gamma_2(j'). \tag{31}$$

This j' must be greater than $\bar{j}$ since for $j=0,\ldots,\bar{j}$, $B_1(j)\le B_2(j)$, implying $\Gamma_1(j)\le\Gamma_2(j)$ for $j=0,\ldots,\bar{j}$. Given $j'>\bar{j}$, we have $B_1(j)\ge B_2(j)$, for $j=j'+1,\ldots,N_2$. This implies

$$\sum_{j=j'+1}^{N_2} B_1(j)\ge \sum_{j=j'+1}^{N_2} B_2(j) \tag{32}$$

Adding Eqs. (31) and (32), we get $\Gamma_1(N_2)>\Gamma_2(N_2)$. But

$$1=\Gamma_1(N_1)\ge\Gamma_1(N_2)>\Gamma_2(N_2)=1,$$

a contradiction. □

The above lemma indicates that Case I is irrelevant, since it fails to satisfy the presumption in Eq. (30). Thus we only need to consider Case II.

Lemma 6. *In CASE II, $\Gamma_1(j)>\Gamma_2(j)$ for $j=0,\ldots,j^*$, and $\Gamma_1(j)\le\Gamma_2(j)$ for $j=j^*+1,\ldots,N_2$, where j^* is between j_1 and j_2.*

Proof. Recall that $B_1(j)>B_2(j)$ for $j=0,\ldots,j_1$, thereby $\Gamma_1(j)>\Gamma_2(j)$ for $j=0,\ldots,j_1$.

Now we show that for $j=j_2,\ldots,N_2$, $\Gamma_1(j)\le\Gamma_2(j)$. Suppose there is $j'\in\{j_2,\ldots,N_2\}$ such that $\Gamma_1(j')>\Gamma_2(j')$.Recall that $B_1(j)\ge B_2(j)$ for $j=j_2+1,\ldots,N_2$. Summing up we get $\Gamma_1(N_2)>\Gamma_2(N_2)$. But this leads to a contradiction that $1=\Gamma_1(N_1)\ge\Gamma_1(N_2)>\Gamma_2(N_2)=1$.

Next we show that the cutoff j^* where $\Gamma_1(j)$-$\Gamma_2(j)$ changes sign is between j_1 and j_2. Recall that $B_1(j)\le B_2(j)$, for $j=j_1+1,\ldots,j_2$. Thus if $\Gamma_1(j')\le\Gamma_2(j')$ for some $j'\in\{j_1+1,\ldots,j_2\text{-}1\}$, then the same inequality must hold for $j=j'+1,\ldots,j_2$. Even if such j' does not exist in $\{j_1+1,\ldots,j_2\text{-}1\}$, we already know that the above inequality holds for $j=j_2$. This implies the cutoff j^* is within $\{j_1+1,\ldots,j_2\}$. □

By the above lemma, $\Gamma_1(j)>\Gamma_2(j)$ means that $j\in\{0,\ldots,j^*\}$. This further implies that $\Gamma_1(j\text{-}1)>\Gamma_2(j\text{-}1)$ since $j\text{-}1<j^*$. Therefore, (30) holds. This completes the whole proof.

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[30] Similarly, an equality may hold only for $j=0,j_1,j_1+1,j_2,j_2+1$ or N_2.