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# Cheap talk with two senders and complementary information ☆

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ABSTRACT

This paper studies a cheap talk model in which two senders having partial and non-overlapping private information simultaneously communicate with an uninformed receiver. The sensitivity of the receiver's ideal action to one sender's private information depends on the other sender's private information. We show that the senders' information transmissions exhibit strategic complementarity: more information transmitted by one sender leads to more information being transmitted by the other sender.



## 1. Introduction

Decision makers often seek advice from multiple experts. Consider a firm with two functional divisions: a marketing division and a production division. The firm's CEO must choose the size of a new factory to produce a new product. The optimal size of the new factory depends on the profitability of the new product, which further depends on the demand for and cost of production of the new product. Due to functional specialization, the marketing division manager knows only the demand for the product while the production division manager knows only the production cost. Thus the CEO must consult both managers regarding his decision, but the managers' interests may not be perfectly aligned with the CEO's. In particular, a manager might prefer a smaller or a bigger factory relative to the CEO's ideal size.

The above example has three distinguishing features: (i) a decision maker consults two experts regarding relevant information before making a decision; (ii) the experts' interests are not perfectly aligned with the decision maker's; and (iii) the experts observe different aspects of information that are relevant for the decision (i.e., the experts observe *non-overlapping information*). The purpose of the paper is to study communication or information transmission in the above setting with communication modeled as cheap talk (Crawford and Sobel, 1982, CS hereafter). While the first two features are standard in cheap talk models, the third feature is understudied in the literature despite its obvious relevance in the real world. Different experts observe non-overlapping information in many situations as a result of specialization. In organizations such

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as firms and governments, different divisions specialize in different functional areas. At the individual-level, experts often have expertise in only one field.

Real world situations sharing the above three features abound. For instance, a president deciding on a bailout plan for banks must determine the optimal size of the bailout, which depends on how deep the banking crisis is and the constraints of the federal budget. The president consults a banking expert who knows only how serious the banking crisis is and a budget expert who knows only the availability of bailout funds. Alternatively, a military leader who must decide how many troops to send into combat consults with an intelligence expert and a field commander. The optimal number of troops depends on both the strength of the enemy and the strength of his own army. While the intelligence expert may only know the strength of the enemy, the field commander may only know the strength of his own forces.[1]

We develop a cheap talk model capturing all three of the above features. To model senders (experts) having partial and non-overlapping private information, we assume that the state of the world has two dimensions, $\theta_1$ and $\theta_2$. Each expert $i$ perfectly observes the realized state in dimension $i$ ($\theta_i$) but does not observe the realized state in dimension $j$ ($\theta_j$). The receiver (decision maker) takes a single action and observes neither $\theta_1$ nor $\theta_2$. We focus on the situation in which the two experts send messages simultaneously. The receiver's ideal action, $y^*(\theta_1, \theta_2)$, is multiplicative in the realized states. That is, $y^*(\theta_1, \theta_2) = \theta_1\theta_2$. This formulation has the property that *the marginal impact of information in dimension $i$ on the ideal decision depends on the realized state in dimension $j$*. In particular, the larger the realized state in one dimension, the more *sensitive* the ideal action is to the information in the other dimension. Consider the CEO example in which $\theta_1$ is the demand size and $\theta_2$ is the efficiency of production. With $y^*(\theta_1, \theta_2) = \theta_1\theta_2$, the optimal factory size is more sensitive to production efficiency when the market size is larger and vice versa.[2] In Appendix A, we show that under monopoly pricing the optimal factory size (and, hence, output level) can indeed be expressed as $\theta_1\theta_2$.

Given multiple senders and multi-dimensional states, there are several ways to model experts' biases. This paper focuses on the situation in which each expert's ideal action differs from the receiver's ideal action by a constant independent of realized states. Of course, two experts can have different biases. This formulation of biases is realistic in situations in which biases are generated by different ideologies: both experts agree with the receiver about how the optimal action depends on realized states, but the experts prefer different actions due to their different ideologies.[3]

Equilibria are shown to be partition equilibrium in which each sender indicates only to which interval the realized state that he observes belongs as in standard CS cheap talk models. We focus on the most informative equilibrium. Interestingly, the senders' information transmissions exhibit strategic complementarity: the more information that one sender transmits, the less the incentive the other sender has to distort his report in the direction of his bias, and, hence, the more information he will transmit. As a result, a reduction in one sender's bias leads not only to more information being transmitted by himself but also induces the other agent to transmit more information. We also show that the equilibrium information transmissions depend only on the absolute values of the senders' biases and not whether the experts have like or opposing biases.[4]

The underlying reason for strategic complementarity is a *"variance" effect*. The rough intuition is as follows. Agent 1 ideally wants to distort the receiver's decision (relative to the receiver's ideal action) by certain amount. To achieve this end, however, agent 1 can only distort his own report. Given that the ideal decision is multiplicative, the impact of agent 1's distortion on the receiver's action depends on agent 2's report. As agent 2 transmits more information, the ex ante uncertainty about agent 2's report increases; for any given information distortion by agent 1, the resulting distortion in the receiver's action is more uncertain and has a bigger variance. Given that agent 1 has a quadratic-loss utility function, agent 1's expected utility loss will increase if the distortion in the receiver's action has a bigger variance. Therefore, agent 1 will have an incentive to reduce his information distortion in order to reduce the variance of the distortion in the receiver's action.

Following the original work of CS on cheap talk, there is a growing literature on cheap talk with multiple senders. Gilligan and Krehbiel (1989), Epstein (1998), and Krishna and Morgan (2001b) study models in which two experts with opposing biases simultaneously communicate by submitting bills to a decision-making legislature. Krishna and Morgan (2001a) study a more general model in which the two experts, who can have like or opposing biases, communicate sequentially.[5] A common feature of these models is that the state space is one-dimensional and both senders perfectly observe the same realized state. In contrast, in our model the two senders have partial and non-overlapping private information.[6]

[1] For another example, consider a dean who must decide how much to invest in a new interdisciplinary center involving both economics and psychology, the optimal size of which depends on the local conditions in the two departments. The dean consults with the chairs of both departments who only know the conditions of their own departments.

[2] In the military example, let $\theta_1$ be the weakness of one's own army and $\theta_2$ be the strength of the enemy. With $y^*(\theta_1, \theta_2) = \theta_1\theta_2$, the weaker one's own army is, the more sensitive is the optimal number of troops to the strength of the enemy and vice versa.

[3] In the bailout example, for instance, a Republican expert who believes in limited government may always prefer a smaller bailout plan than a Democratic expert. In the troop deployment example, a dovish expert may always prefer to deploy fewer troops than a hawkish expert.

[4] Having like (opposing) biases refers to the case in which the experts' biases have the same (opposite) sign.

[5] In an extension by Gick (2009), the receiver commits to not best responding to the second sender. Li (2010) studies a model in which two experts perfectly observe the realized state, but each expert's bias is his own private information.

[6] For cheap talk models with one sender and multiple receivers, see Farrell and Gibbons (1989) and Goltsman and Pavlov (2011). For cheap talk models in which both the receiver and sender have private information, see Harris and Raviv (2005), McGee (in press), and Chen (2009).

The most closely related papers to ours are Alonso et al. (2008) and Hori (2006). Alonso et al. (2008) study strategic communication with two senders having non-overlapping private information.[7] Our study differs from theirs in that in our model there is only one decision to make instead of two. Furthermore, the need to communicate in their model results from the need to coordinate two decisions, while the need to communicate arises in our model because the optimal decision for the receiver depends on the private information of both experts. While agents' information transmissions are strategic complements in our model, in their model agents' information transmissions are strategically independent. Hori (2006) considers a model similar to ours, but he focuses on the case in which the receiver's ideal action is additive in two states: $y^*(\theta_1, \theta_2) = \theta_1 + \theta_2$. With this formulation, agents' information transmissions are strategically independent.

Battaglini (2002) and Ambrus and Takahashi (2008) study multi-dimensional cheap talk models with multiple senders. One surprising result in those models is that generically information can be fully revealed in equilibrium communication. In their models, each sender observes the realized states in all dimensions and the decision is a two-dimensional vector. Our model differs in that senders observe non-overlapping information and the decision is a one-dimensional variable.

Austen-Smith (1993) and Morgan and Stocken (2008) study a multiple-sender cheap talk model with senders receiving correlated (conditionally independent) signals regarding the state. The correlation of signals (i.e., the overlapping of the agents' private information) leads to an information congestion effect: as other agents transmit more information the receiver's decision becomes less sensitive to the report of any individual agent, and thus individual agents have less incentive to transmit information. As a result, in both models agents' information transmissions are strategic substitutes.[8] Because in our model agents have non-overlapping private information, this information congestion effect is absent. Instead, the strategic interaction between the agents operates through the variance effect when the ideal decision is multiplicative in the states.

The rest of the paper is organized as follows. Section 2 lays out the model. In Section 3 we characterize equilibrium and examine the strategic interaction between the senders' information transmissions. Section 4 offers conclusions and discussions. All proofs can be found in Appendix A.

## 2. The model

Consider a decision maker (DM) who consults two experts $i = 1, 2$. Both experts and the DM are expected utility maximizers. The DM takes an action $y \in R$, and his utility depends on some underlying states of nature $\theta_1$ and $\theta_2$. Each $\theta_i$ is uniformly distributed on $[0, A_i]$ with density $1/A_i$, and $\theta_1$ and $\theta_2$ are independent from each other. The DM does not observe the realization of either $\theta_1$ or $\theta_2$, and expert $i$ observes only the realized value of $\theta_i$. This captures the fact that each expert is knowledgeable only in his own field. Note that both experts have private information, but this private information is not overlapping in the sense that $\theta_1$ and $\theta_2$ are independent. Expert $i$ offers advice to the DM by sending message $m_i$. We focus on the case of simultaneous communication in which the two experts send messages simultaneously. After receiving messages $m_1$ and $m_2$, the DM takes an action $y(m_1, m_2)$.

The utility function for the DM is

$$U^P(y, \theta_1, \theta_2) = -\big(y - y^*(\theta_1, \theta_2)\big)^2,$$

where $y^*(\theta_1, \theta_2)$ is the ideal action for the DM. Expert $i$'s ideal action is $y^*(\theta_1, \theta_2) + b_i$, where the constant $b_i$ is expert $i$'s bias relative to the DM. We assume that for $i = 1, 2$, $b_i \neq 0$. The bias $b_i$, which can be positive or negative, measures the degree to which the DM's and expert $i$'s interests are aligned. Specifically, the utility function for expert $i$ is

$$U^{A_i}(y, \theta_1, \theta_2, b_i) = -\big[y - \big(y^*(\theta_1, \theta_2) + b_i\big)\big]^2.$$

The biases are common knowledge. When $b_1$ and $b_2$ have the same sign, we say that the experts have like biases; otherwise, we say that they have opposing biases. Note that in the current setup the difference between agent $i$'s ideal action and the DM's ideal action, $b_i$, is independent of the realized states $\theta_1$ and $\theta_2$. In the rest of the paper, we focus on the case where the DM's ideal action is multiplicative in the realized states. That is,

$$y^*(\theta_1, \theta_2) = \theta_1\theta_2.$$

## 3. Equilibrium and equilibrium properties

### 3.1. Equilibrium characterization

Under simultaneous communication, a strategy for expert $i$ is a communication rule $\mu_i(m_i|\theta_i)$, which specifies the probability that agent $i$ sends message $m_i$ when the state $i$ is $\theta_i$. A strategy for the DM specifies an action $y$ for each message pair

[7] Specifically, there are two division managers and a CEO. Each manager has private information regarding the local conditions of his own division, and a decision needs to be made for each division. Furthermore, the decisions of the two divisions need to be coordinated. They compare two communication modes: vertical communication (centralization) versus horizontal communication (decentralization).

[8] Two recent papers study strategic communication in networks. In Hagenbach and Koessler (2010), the true state is additive in agents' private information. As a result, agents' information transmissions are strategically independent. In Galeotti et al. (2010), agents receive correlated (conditionally independent) signals about the state. This leads to an information congestion effect and less truthful communication in larger communities.

$(m_1, m_2)$, which is denoted by the decision rule $y(m_1, m_2)$. Let the belief function $g(\theta_1, \theta_2|m_1, m_2)$ be the DM's posterior beliefs on $\theta_1$ and $\theta_2$ after hearing messages $m_1$ and $m_2$. Because $\theta_1$ and $\theta_2$ are independent and expert $i$ observes only $\theta_i$, the belief function can be decomposed into distinct belief functions $g_1(\theta_1|m_1)$ and $g_2(\theta_2|m_2)$.

Our solution concept is Perfect Bayesian Equilibrium (PBE), which requires:

(i) Given the DM's decision rule $y(m_1, m_2)$ and expert $j$'s communication rule $\mu_j(m_j|\theta_j)$, for each $i$, expert $i$'s communication rule $\mu_i(m_i|\theta_i)$ is optimal.

(ii) The DM's decision rule $y(m_1, m_2)$ is optimal given beliefs $g_1(\theta_1|m_1)$ and $g_2(\theta_2|m_2)$.

(iii) The belief functions $g_i(\theta_i|m_i)$ are derived from the agents' communication rules $\mu_i(m_i|\theta_i)$ according to Bayes rule whenever possible.

We first derive the DM's optimal decision rule $\bar{y}(m_1, m_2)$. Given $m_1$ and $m_2$, $\bar{y}(m_1, m_2)$ maximizes $-E[(y - \theta_1\theta_2)^2|m_1, m_2]$. Because $\theta_1$ and $\theta_2$ are independent and $\theta_1|m_1$ and $\theta_2|m_2$ are independent, it can be readily seen that

$$\bar{y}(m_1, m_2) = E[\theta_1|m_1]E[\theta_2|m_2]. \tag{1}$$

As in CS and Alonso et al. (2008), all PBE are interval equilibria. Specifically, the state space $[0, A_i]$ is partitioned into intervals and expert $i$ only reveals to which interval $\theta_i$ belongs.

**Lemma 1.** *All PBE in the communication game must be interval equilibria.*

Later we will show that the equilibrium number of partitions must be finite (part (ii) of Proposition 1). Define $\overline{m}_i$ as the posterior of state $\theta_i$ given message $m_i$; that is, $E[\theta_i|m_i] \equiv \overline{m}_i$. Before we proceed, we first note a useful result.

**Claim 1.** $E[\theta_i\overline{m}_i] = E[\overline{m}_i^2]$.

Having established that all PBE must be interval equilibria, we now characterize them. Let $N_i$ be the number of partition elements for agent $i$, and $(a_{i,0}, a_{i,1}, \ldots, a_{i,n}, \ldots, a_{i,N_i}) \equiv a_i$ be the partition points with $a_{i,0} = 0$ and $a_{i,N_i} = A_i$. Define $\overline{m}_{i,n}$ as the receiver's posterior of $\theta_i$ after receiving a message $m_{i,n} \in (a_{i,n-1}, a_{i,n})$. It follows that $\overline{m}_{i,n} = (a_{i,n-1} + a_{i,n})/2$. In state $\theta_1 = a_{1,n}$, agent 1 should be indifferent between sending a message that induces a posterior $\overline{m}_{1,n}$ and sending a message inducing a posterior $\overline{m}_{1,n+1}$. That is, $E_{\theta_2}[U^{A_1}|\overline{m}_{1,n}, a_{1,n}] = E_{\theta_2}[U^{A_1}|\overline{m}_{1,n+1}, a_{1,n}]$. More explicitly, the indifference condition can be written as

$$E_{\theta_2}\left[\left\{\overline{m}_2\frac{a_{1,n} + a_{1,n-1}}{2} - (\theta_2 a_{1,n} + b_1)\right\}^2\right] = E_{\theta_2}\left[\left\{\overline{m}_2\frac{a_{1,n} + a_{1,n+1}}{2} - (\theta_2 a_{1,n} + b_1)\right\}^2\right].$$

Using the fact that $E[\theta_i\overline{m}_i] = E[\overline{m}_i^2]$, we can simplify the above indifference condition further as

$$(a_{1,n+1} - a_{1,n}) - (a_{1,n} - a_{1,n-1}) = \frac{E(\theta_2)}{E(\overline{m}_2^2)}4b_1. \tag{2}$$

Similarly, the cutoff points $a_{2,n}$ characterizing agent 2's partition equilibrium satisfy the indifference condition:

$$(a_{2,n+1} - a_{2,n}) - (a_{2,n} - a_{2,n-1}) = \frac{E(\theta_1)}{E(\overline{m}_1^2)}4b_2. \tag{3}$$

Inspecting indifference conditions (2) and (3), we see that there is strategic interaction between the two senders as the term $E(\theta_j)/E(\overline{m}_j^2)$ appears in the condition that determines sender $i$'s cutoff points. In the quadratic-uniform case of CS's one-sender model, the difference between the lengths of any two adjacent intervals, or the incremental step size, is always $4b$. Conditions (2) and (3) show that the strategic interaction between the two senders changes the incremental step sizes: the effective incremental step size is $[E(\theta_j)/E(\overline{m}_j^2)]4b_i$. Note that the amount of information transmitted by agent $j$ is (negatively) measured by the residual variance $E[(\theta_j - \overline{m}_j)^2]$. The residual variance can also be written as

$$E\left[(\theta_j - \overline{m}_j)^2\right] = E\left(\theta_j^2\right) - E\left(\overline{m}_j^2\right) = var(\theta_j) - var(\overline{m}_j).$$

A bigger $E(\overline{m}_j^2)$ or a bigger $var(\overline{m}_j)$ means more information is transmitted by sender $j$. Conditions (2) and (3) imply that, as one agent transmits more information, the effective incremental step size for the other agent decreases.

An equilibrium is characterized by two sequences of partition points, $(a_1, a_2)$, that satisfy the indifference conditions (2) and (3) and the boundary conditions $a_{i,0} = 0$ and $a_{i,N_i} = A_i$. To ease exposition, we define

$$x_i \equiv \frac{E(\theta_i)}{E(\overline{m}_i^2)}.$$

The term $x_i$ can be interpreted as a "bias multiplier." Given $N_i$ and $x_j$ and using the boundary conditions $a_{i,0} = 0$ and $a_{1,N_i} = A_i$, we can solve for the solutions to difference equations (2) and (3):

$$a_{i,n} = A_i \frac{n}{N_i} + 2b_i x_j n(n - N_i). \tag{4}$$

In equilibrium, the following inequality should be satisfied:

$$2|b_i| x_j N_i (N_i - 1) < A_i. \tag{5}$$

That is, the total length of the partition for each agent should be less than the length of the support of $\theta_i$, $A_i$.

**Lemma 2.**

(i) *The expression for $E(\overline{m}_i^2)$ can be written as*

$$E\left(\overline{m}_i^2\right) = \frac{A_i^2}{3} - \frac{A_i^2}{12N_i^2} - \frac{b_i^2 x_j^2 (N_i^2 - 1)}{3}; \tag{6}$$

(ii) $\frac{A_i^2}{4} \leqslant E(\overline{m}_i^2) \leqslant \frac{A_i^2}{3}$.

The following proposition characterizes equilibria of the game.

**Proposition 1.**

(i) *Any equilibrium is characterized by a pair of numbers of partition elements $(N_1, N_2)$ that satisfy*

$$E\left(\overline{m}_1^2\right) = \frac{E(\theta_1)}{x_1} \quad \Leftrightarrow \quad \frac{A_1^2}{3} - \frac{A_1^2}{12N_1^2} - \frac{b_1^2 x_2^2 (N_1^2 - 1)}{3} = \frac{A_1}{2x_1}, \tag{7}$$

$$E\left(\overline{m}_2^2\right) = \frac{E(\theta_2)}{x_2} \quad \Leftrightarrow \quad \frac{A_2^2}{3} - \frac{A_2^2}{12N_2^2} - \frac{b_2^2 x_1^2 (N_2^2 - 1)}{3} = \frac{A_2}{2x_2}, \tag{8}$$

*and the two inequalities*

$$2|b_1| x_2 N_1 (N_1 - 1) < A_1, \qquad 2|b_2| x_1 N_2 (N_2 - 1) < A_2. \tag{9}$$

*The partition elements $(a_1, a_2)$ of an equilibrium with $(N_1, N_2)$ are given by (4).*

(ii) *There exists at least one equilibrium, and in any equilibrium both $N_1$ and $N_2$ are finite.*

As is typical in cheap talk models, there might be multiple equilibria in our model. Among all equilibrium $(N_1, N_2)$, we call the one that maximizes $E(\overline{m}_1^2)E(\overline{m}_2^2)$ the most informative equilibrium, and label the associated number of partition elements as $(N_1^*, N_2^*)$. We follow a common practice in studies of cheap talk: among all equilibria we focus on the most informative equilibrium.[9] The underlying rationale is that the most informative equilibrium is also ex ante Pareto dominant. To see this, we calculate the ex ante equilibrium payoffs for the DM, $U^P$, and for agent $i$, $U^{A_i}$, which are given by:

$$\begin{aligned} U^P &= -E\left[(\overline{m}_1 \overline{m}_2 - \theta_1\theta_2)^2\right] = -E\left(\theta_1^2\right)E\left(\theta_2^2\right) + E\left(\overline{m}_1^2\right)E\left(\overline{m}_2^2\right), \\ U^{A_i} &= -E\left(\theta_1^2\right)E\left(\theta_2^2\right) + E\left(\overline{m}_1^2\right)E\left(\overline{m}_2^2\right) - b_i^2. \end{aligned} \tag{10}$$

Inspecting (10), we can see that the most informative equilibrium, which maximizes $E(\overline{m}_1^2)E(\overline{m}_2^2)$, is also ex ante Pareto dominant.

### 3.2. Partial equilibrium analysis

To examine the strategic interaction between the agents, we conduct the following partial equilibrium analysis. Treating agent $j$'s strategy $\mu_j(m_j|\theta_j)$ (equivalently $E(\overline{m}_j^2)$ and $x_j$) as exogenous, we investigate how agent $i$'s "best response" will change as the amount of information transmitted by agent $j$ varies. Note that given $E(\overline{m}_j^2)$ or $x_j$ the game is essentially reduced to a standard one-sender cheap talk game of CS with agent $i$'s effective bias being $x_j b_i$. In this reduced game, typically there are multiple equilibria.[10] Following CS, we focus on the most informative equilibrium in the reduced game

[9] For equilibrium refinement in cheap talk models, see Matthews et al. (1991) and Chen et al. (2008).

[10] More formally, given agent $j$'s strategy $\mu_j(m_j|\theta_j)$ a (partial) PBE of this reduced game requires the following. First, given the DM's decision rule $y(m_1, m_2)$ and expert $j$'s communication rule $\mu_j(m_j|\theta_j)$, expert $i$'s communication rule $\mu_i(m_i|\theta_i)$ is optimal. Second, the DM's decision rule $y(m_i, m_j)$ is optimal given beliefs $g_i(\theta_i|m_i)$ and $g_j(\theta_j|m_j)$. Third, the belief functions $g_i(\theta_i|m_i)$ and $g_j(\theta_j|m_j)$ are derived from the agents' communication rules $\mu_i(m_i|\theta_i)$ and $\mu_j(m_j|\theta_j)$ according to Bayes rule whenever possible.

and call it agent $i$'s most informative (partial) equilibrium. In particular, in agent $i$'s most informative (partial) equilibrium the number of partition elements is the maximum $N_i$ that satisfies (5), and the partition points $a_i$ follow (4). Note that agent $i$'s most informative (partial) equilibrium depends on $E(\overline{m}_j^2)$ or $x_j$. To proceed, we first provide a useful lemma.

**Lemma 3.** *$E(\overline{m}_i^2)$ exhibits the following comparative statics: it is strictly increasing in $N_i$, strictly decreasing in $x_j$ and $b_i$, and strictly increasing in $E(\overline{m}_j^2)$.*

As agent $j$ transmits more information (i.e., $E(\overline{m}_j^2)$ increases), the bias multiplier for agent $i$, $x_j$, decreases. This leads to a smaller effective incremental step size for agent $i$. By Lemma 3, this further implies that in agent $i$'s most informative (partial) equilibrium $E(\overline{m}_i^2)$ will increase, meaning agent $i$ transmits more information. Therefore, we have the following proposition.

**Proposition 2.** *Treat agent $j$'s strategy as exogenous. As agent $j$ transmits more information, agent $i$ will transmit more information as well in agent $i$'s most informative (partial) equilibrium.*

Proposition 2 shows that the agents' information transmissions exhibit strategic complementarity. To understand this result, consider agent 1's incentives to misrepresent his information. For this purpose, suppose that agent 1 can credibly misrepresent his information by reporting $\hat{\theta}_1$ and that agent 2 sends a message according to $\mu_2(m_2|\theta_2)$. Then the optimal report of agent 1 solves

$$\max_{\hat{\theta}_1} E_{\theta_2}\left[U^{A_1}\middle|\hat{\theta}_1,\theta_1\right] \quad \Leftrightarrow \quad \min_{\hat{\theta}_1} E_{\theta_2}\left[(\hat{\theta}_1\overline{m}_2-\theta_1\theta_2-b_1)^2\middle|\theta_1\right]. \tag{11}$$

The optimal "message distortion" measured by $d_1 \equiv \hat{\theta}_1 - \theta_1$ is given by

$$d_1 = \frac{E(\theta_2)b_1}{E[\overline{m}_2^2]} = \frac{E(\theta_2)b_1}{(E(\theta_2))^2 + var[\overline{m}_2]}. \tag{12}$$

From (12), we see that agent 1's incentive to distort information depends on the informativeness of agent 2's communication. In particular, as agent 2 transmits more information (i.e., $E[\overline{m}_2^2]$ or $var[\overline{m}_2]$ increases), according to (12) agent 1's incentive to distort information, measured by $d_1$, decreases.

To understand the underlying intuition, note that agent 1 ideally wants to distort the DM's decision relative to the DM's ideal action by $b_1$. To achieve this end, however, agent 1 can only distort his own report. Given that the ideal decision is multiplicative, the impact of agent 1's distortion on the DM's action depends on agent 2's report. In particular, if agent 1 expects a higher (lower) $\overline{m}_2$, then any given distortion by agent 1 will be amplified (dampened). More precisely, the distortion in the DM's decision is $d_1\overline{m}_2$. In short, agent 1 tries to make $d_1\overline{m}_2$ as close as possible to $b_1$ by adjusting his own information distortion $d_1$.

Observe that agent 2's information transmission strategy will not affect the ex ante expectation about $\overline{m}_2$. Formally, this is due to the law of iterative expectations: $E[E(\theta_2|m_2)] = E(\theta_2)$. However, agent 2's information transmission strategy will in general affect the variance of $\overline{m}_2$, and this in turn affects agent 1's incentive to distort information. To illustrate this "variance" effect, we decompose agent 1's expected loss in (11).

$$E_{\theta_2}\left[(\hat{\theta}_1\overline{m}_2-\theta_1\theta_2-b_1)^2\middle|\theta_1\right] = (\hat{\theta}_1-\theta_1)^2\,var(\overline{m}_2) - \theta_1^2\,var(\overline{m}_2) + E_{\theta_2}\left[\left(\hat{\theta}_1E(\theta_2)-\theta_1\theta_2-b_1\right)^2\middle|\theta_1\right] \tag{13}$$

$$= d_1^2\,var(\overline{m}_2) + \left[d_1E(\theta_2)-b_1\right]^2 + \text{some terms independent of } d_1. \tag{14}$$

Note that the last term in (13) is agent 1's expected loss if agent 2 does not transmit any information, which is the second term in (14). According to (14), the expected loss can be expressed as the loss due to the variance of the distortion in the DM's action $var(d_1\overline{m}_2)$ (the first term) and the loss due to the difference between $b_1$ and the average distortion in the DM's action $d_1E(\theta_2)$ (the second term). If agent 2 does not transmit any information ($var(\overline{m}_2) = 0$), by (14) agent 1's optimal message distortion is $d_1 = b_1/E(\theta_2)$. If agent 2 transmits some information ($var(\overline{m}_2) > 0$), then the first term in (14) becomes positive while the second term remains the same. Now in order to reduce the expected loss, agent 1 will reduce $d_1$ to a level below $b_1/E(\theta_2)$. By reducing $d_1$, agent 1 can reduce the variance of the distortion in the DM's action, though the average distortion in the DM's action becomes less than the "ideal amount," $b_1$.[11] Generally, as agent 2 transmits more information ($var(\overline{m}_2)$ increases), the loss due to the variance of the distortion in the DM's action becomes more important than the loss resulting from the average distortion being different from the ideal amount. Therefore, agent 1 will further reduce his information distortion $d_1$.

[11] This is because the second effect is zero on the margin when $d_1$ is $b_1/E(\theta_2)$. Note that raising $d_1$ above $b_1/E(\theta_2)$ will hurt agent 1. By increasing $d_1$, agent 1 will lose not only from distorting the DM's action on average by more than his ideal amount $b_1$ but also from increasing the variance of the distortion.

The reason that agent 1 suffers from the variance of the distortion in the DM's action is that he has a quadratic-loss utility function. To illustrate the idea, let us compare the following two cases. In the first case, agent 2 transmits no information (i.e., sends only one message $m_{20}$). In the second case agent 2 has two partition elements with two potential messages: $m_{21}$ and $m_{22}$, $\overline{m}_{21} < \overline{m}_{22}$. Note that in both cases $E(\overline{m}_2) = E(\theta_2)$, and $\overline{m}_{21} < \overline{m}_{20} = E(\theta_2) < \overline{m}_{22}$. In other words, $\overline{m}_2$ in the second case is a mean preserving spread of $\overline{m}_2$ in the first case. Suppose agent 1 distorts his message by the same amount $d_1$ in both cases. Then the distortion of the DM's action, $d_1\overline{m}_2$, in the second case is a mean preserving spread of that in the first case. More precisely, although the average distortion is the same, $|d_1\overline{m}_{21}| < |d_1\overline{m}_{20}| < |d_1\overline{m}_{22}|$. That is, in the second case the resulting distortion in the DM's action is lower (higher) when agent 2's message is $m_{21}$ ($m_{22}$) than that in the first case. The fact that agent 1 has a quadratic-loss utility function implies that the marginal losses are increasing for deviations from agent 1's ideal distortion of the DM's action. Therefore, agent 1 incurs a higher expected loss in the second case, in which $\overline{m}_2$ has a higher variance, than in the first case.

Two assumptions are essential for the strategic complementarity between the agents' information transmissions. The first is that the ideal decision is multiplicative in the states, which leads the impact of one agent's report on the DM's action to depend on the other agent's report. If the ideal decision is additive in two states (e.g., $y^*(\theta_1, \theta_2) = \theta_1 + \theta_2$) as studied by Hori (2006), then the agents' information transmissions are strategically independent. The second is that the agents have quadratic-loss utility functions, which imposes increasing marginal losses for deviations from the ideal action and implies that an agent's expected loss will increase if the receiver's action has a bigger variance. The variance effect and the strategic complementarity disappear if agents only care about the absolute value of deviations. The variance effect and the strategic complementarity, however, will present themselves if the loss function is convex in the absolute value of deviations.

### 3.3. Equilibrium properties

Having conducted partial equilibrium analysis, we now study the properties of the (overall) equilibrium. Recall that $(N_1^*, N_2^*)$ are the number of partition elements in the most informative equilibrium.

**Proposition 3.**

(i) *If there are equilibria with $(N_1', N_2)$ and $(N_1, N_2')$ such that $N_1' > N_1$ and $N_2' > N_2$, then there is an equilibrium with $(N_1', N_2')$.*
(ii) *For any equilibrium $(N_1, N_2)$, $N_1 \leqslant N_1^*$ and $N_2 \leqslant N_2^*$. The lower and upper bounds of $N_i^*$, $i = 1, 2$, are given by*

$$\left\langle -\frac{1}{2} + \frac{1}{2}\left(1 + \frac{A_1 A_2}{|b_i|}\right)^{1/2} \right\rangle \leqslant N_i^* \leqslant \left\langle -\frac{1}{2} + \frac{1}{2}\left(1 + \frac{4A_1 A_2}{3|b_i|}\right)^{1/2} \right\rangle, \tag{15}$$

*where the operator $\langle\ \rangle$ indicates the largest integer that is less than the expression inside it.*
(iii) *If there is an equilibrium with $(N_1, N_2)$ and $1 < N_i \leqslant 8$ for both $i = 1, 2$, then an equilibrium with $(N_1 - 1, N_2 - 1)$ exists as well.*

In standard CS cheap talk models, if the number of partitions in the most informative equilibrium is $N^*$, then for all $N$ such that $1 \leqslant N < N^*$, there is a corresponding equilibrium. Similar results do not hold in our model. In particular, in our model it is possible that there is no equilibrium for some $(N_1, N_2) \leqslant (N_1^*, N_2^*)$ ($(N_1, N_2) \neq (N_1^*, N_2^*)$ and $(N_1, N_2) \neq (1, 1)$).[12] This feature is due to the fact that in our model the agents' information transmissions are strategic complements. If the number of partition elements in one agent's partition is reduced and less information is transmitted, the other agent will have less incentive to transmit information. As a result, the original equilibrium number of partition elements for the second agent might no longer be sustainable.

Part (iii) of Proposition 3 provides a sufficient condition under which the existence of a given equilibrium implies the existence of equilibria with partitions of smaller size. This sufficient (but not necessary) condition is that the partition sizes of both agents in the given equilibrium are no greater than 8. The bound being 8 is related to the upper bound of the amount of strategic complementarity: the ratio of the lower bound of the bias multiplier, $x_i$, to its upper bound is 3/4 (implied by part (ii) of Lemma 2). If an equilibrium with $(N_1, N_2) < (8, 8)$ does not exist, then even the maximum amount of strategic complementarity is not strong enough to make $(N_1 + 1, N_2 + 1)$ an equilibrium. Therefore, the existence of an equilibrium with $(N_1, N_2) \leqslant (8, 8)$ implies the existence of an equilibrium with $(N_1 - 1, N_2 - 1)$. This property further implies that, if in the most informative equilibrium $(N_1^*, N_2^*)$ satisfies $1 \leqslant N_i^* \leqslant 8$ for both $i = 1, 2$, then, for $i = 1, 2$, for every $N_i$ such that $1 \leqslant N_i \leqslant N_i^*$, there is an equilibrium with $(N_i, N_j)$ for some $N_j$.

Proposition 3 also implies that in the most informative equilibrium a decrease in one agent's bias will not only increase the amount of information transmitted by this agent but also lead to more information being transmitted by the other agent. More precisely, fix all other parameter values and change $b_i$ to $b_i'$, with $|b_i'| < |b_i|$. Then in the most informative

[12] Here we provide an example in which $(N_1^*, N_2^*) > (1, 1)$, but an equilibrium with $(N_1^*, 1)$ does not exist. Specifically, let $A_1 = 1$, $A_2 = 1$, $b_1 = 1/1360$, and $b_2 = 0.06$. In the most informative equilibrium, $N_1^* = 20$ and $N_2^* = 2$. However, there is no equilibrium with $(N_1, N_2) = (20, 1)$. Moreover, there is no equilibrium with $(N_1, N_2) = (N_1^* - 1, N_2^* - 1) = (19, 1)$.

equilibrium, the numbers of partition elements for both agents will weakly increase ($(N_1^*, N_2^*) \leqslant (N_1^{*\prime}, N_2^{*\prime})$), and both agents will transmit more information ($E[(\overline{m}_i')^2] > E(\overline{m}_i^2)$ and $E[(\overline{m}_j')^2] > E(\overline{m}_j^2)$). This is again due to the fact that the agents' information transmissions are strategic complements.

Another property worth mentioning is that in our model the equilibrium information transmissions depend only on the absolute values of the biases; the signs of the biases do not matter. This implies that the equilibrium information transmissions do not depend on whether the agents have opposing or like biases.[13] This property holds in our model because the interaction between the agents' communication occurs only through the terms $E(\overline{m}_1^2)$ and $E(\overline{m}_2^2)$. When $b_i$ changes sign, only the direction of the partition of $m_i$ reverses; the same amount of information is transmitted in equilibrium.

Suppose the ideal action, instead of being $\theta_1\theta_2$, is $y^*(\theta_1, \theta_2) = \gamma\theta_1\theta_2$, where $\gamma \neq 0$ is some known constant. This modification will not qualitatively change the equilibrium characterization and the results of the basic model. In this setting, agent 1's optimal message distortion is $\hat{\theta}_1 - \theta_1 = \frac{E(\theta_2)b_1}{\gamma E[\overline{m}_2^2]}$. Note that this expression is very similar to that in the basic model, $\frac{E(\theta_2)b_1}{E[\overline{m}_2^2]}$. From this expression we can see that the presence of $\gamma$ does not change the variance effect as it operates through $E[\overline{m}_2^2]$. The presence of $\gamma$ does change agent 1's average incentive to distort information. The smaller the $|\gamma|$, the stronger the agents' incentives to distort information. If $\gamma$ is negative, the agents will reverse the direction in which they distort information compared to the case in which $\gamma$ is positive.

So far we have assumed that the support of $\theta_i$ is $[0, A_i]$. Here we briefly discuss what happens if the support of $\theta_i$ is extended into the negative domain. Suppose for $i = 1, 2$, $\theta_i$ is uniformly distributed on $[-B_i, A_i]$, with $A_i > 0$ and $B_i > 0$. The fact that $\theta_j$ can be positive or negative leads to the following complication. Suppose $b_i > 0$ so that agent $i$ wants to pull the DM's decision to the right. Given that the DM's ideal action is multiplicative in states, to achieve this goal agent $i$ needs to overstate his information when the realized $\theta_j$ is positive but understate his information when the realized $\theta_j$ is negative. The analysis of the basic model, however, applies with slight modification.[14] Now the direction in which agent $i$ wants to distort information is determined by the sign of $b_i E(\theta_j)$.[15] Intuitively, $E(\theta_j)$ is the average of $\theta_j$, which determines on average how agent $i$'s information distortion will translate into distortion of the DM's action. One important difference from the basic model is that when $B_i = A_i$ so that $E(\theta_i) = 0$ and agent $i$'s communication is informative, agent $j$ reveals his information fully in the most informative equilibrium.[16] For generic cases in which $B_i \neq A_i$ so that $E(\theta_i) \neq 0$, any equilibrium has a finite number of partitions and the results of the basic model hold qualitatively. In particular, the agents' information transmissions are still strategic complements.[17]

## 4. Conclusion and discussion

We study a two-sender cheap talk model in which two experts have partial and non-overlapping private information and communicate to the receiver simultaneously. The receiver's ideal action is multiplicative in the experts' private information, and each expert's ideal action differs from that of the receiver's by some constant. In this setting, we show that information transmission displays strategic complementarities in that more informative communication from one expert induces more informative communication from the other. Moreover, the informativeness of communication in equilibrium does not depend on whether the two senders have like or opposing biases, but only depends on the magnitudes of these biases.

Delegation for informational reasons has been widely studied in the literature (Melumad and Shibano, 1991; Dessein, 2002; Alonso and Matouschek, 2007, 2008). In an early working paper (McGee and Yang, 2011), we study when decision rights should be delegated to one of the two experts. We show that the DM, if he ever delegates, always prefers to delegate decision rights to the expert with the smaller bias in absolute value. Comparing delegation to simultaneous communication, we demonstrate that when the experts have like biases, delegation is always superior for the DM whenever informative communication is possible. On the other hand, simultaneous communication dominates delegation when the experts have opposing biases and the smaller bias is big enough.

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13 In Krishna and Morgan (2001b) who study sequential communication, equilibrium information transmission depends on whether the agents have opposing or like biases.

14 Inspecting the proof, we can see that Lemma 1 still applies. Specifically, the following conditions still hold: $\frac{\partial^2}{\partial\theta_1\partial v_1}[U^{A_1}|v_1, \theta_1] = 2E[\theta_2^2|\mu_2(\cdot)] > 0$ and $\frac{\partial^2}{\partial\theta_1^2}[U^{A_1}|v_1, \theta_1] = -2E[\theta_2^2] < 0$. Hence all PBE must be interval equilibria. Moreover, the difference equations that characterize the partition points, (2) and (3), remain the same. Only equations (7), (8), and (9) need to be modified slightly.

15 Suppose $b_1 > 0$. If $E(\theta_2) > 0$, then agent 1 wants to overstate his information. If $E(\theta_2) < 0$, then agent 1 wants to understate his information.

16 $E(\theta_i) = 0$ implies that the right-hand side of (2), the effective incremental step size, is 0. To understand the intuition, suppose $b_1 > 0$. When $E(\theta_2) = 0$, the posterior regarding $\theta_2$ is equally likely to be positive or negative, implying that agent 1 can gain nothing by overstating or understating his information. As a result, he has no incentive to distort his information and reveals his information fully in equilibrium.

17 When $\theta_j$ can be negative as well as positive, the magnitude of the distortion in the DM's action depends on the report of $\theta_j$, and the direction of the distortion in the DM's action can be different from that of agent $i$'s information distortion. However, the following result still holds: as the reports of $\theta_j$ become more uncertain, how agent $i$'s information distortion translates into distortion of the DM's action also becomes more uncertain. Essentially, changes in the support of $\theta_j$ are qualitatively equivalent to changes in $b_i$, as agent $i$'s incentive to distort information is given by $\hat{\theta}_i - \theta_i = \frac{E(\theta_j)b_i}{E[\overline{m}_j^2]}$.

Our model is highly stylized. There are several ways to extend our analysis.[18] The first is to consider more general distributions and more general functions for the ideal decision (some results along these lines can be found in McGee and Yang, 2011). The second possible extension is to model agents' biases differently.[19] The third possible extension is to consider sequential communication and ask the following questions.[20] First, which agent should communicate first in order to maximize the receiver's payoff? Second, how does sequential communication compare to simultaneous communication and delegation in terms of the receiver's payoff? Finally, it would be interesting to consider the case in which the experts' pieces of private information are correlated. We leave extensions along the above lines for future research.

## Appendix A. Examples and proofs

### A.1. Examples in which the ideal action is multiplicative in two states

Consider the CEO example in which $\theta_1$, observed only by the marketing manager, is the demand size and $\theta_2$, observed only by the production manager, is the efficiency of production. Denote the output (size) of the new factory as $Q$, and suppose the firm is a monopoly. The demand faced by the firm is $Q = \theta_1(K - P)$, where $P$ is the price and $K$ some known constant. $K - P$ can be interpreted as the demand curve for individual consumers, while $\theta_1$ is the number of consumers or market size. The marginal cost of production is $c$. We define the efficiency of production as $\theta_2 \equiv (K - c)/2$. In the current setup, the marginal revenue is given by $MR(Q) = K - 2Q/\theta_1$. Setting the marginal revenue equal to the marginal cost, we derive the optimal (profit maximizing) output $Q^*$ as follows: $Q^* = \theta_1(K - c)/2 = \theta_1\theta_2$. That is, the CEO's ideal action is multiplicative in the two states.

In the previous example, the individual demand curve is linear. Now consider demand curves with constant elasticities. Suppose $Q = \theta_1 P^{-\epsilon}$, where the elasticity parameter $\epsilon$ ($> 0$) is common knowledge. We define the efficiency of production as $\theta_2 \equiv (\frac{\epsilon}{\epsilon-1})^{-\epsilon} c^{-\epsilon}$. In this situation, the marginal revenue is given by $MR(Q) = \frac{\epsilon-1}{\epsilon}\theta_1^{1/\epsilon} Q^{-1/\epsilon}$. The profit maximizing output $Q^*$ satisfies $Q^* = (\frac{\epsilon}{\epsilon-1})^{-\epsilon} c^{-\epsilon}\theta_1 = \theta_1\theta_2$. Again, the CEO's ideal action is multiplicative in the two states.

**Proof of Lemma 1.** First note that in any PBE of the communication game, the optimal decision given beliefs satisfies (1). Given that the agents are symmetric, it is sufficient to show that given any communication rule for agent 2, $\mu_2(\cdot)$, agent 1's optimal communication rule is of the interval form. Suppose the DM holds a posterior belief $v_1$ regarding $\theta_1$. Then agent 1's expected utility is

$$E_{\theta_2}\big[U^{A_1}\big|v_1, \theta_1\big] = -E_{\theta_2}\big[\big\{v_1 E\big[\theta_2\big|\mu_2(\cdot)\big] - \theta_1\theta_2 - b_1\big\}^2\big]. \tag{16}$$

It can be readily seen that $\frac{\partial^2}{\partial\theta_1\partial v_1}[U^{A_1}|v_1, \theta_1] = 2E[\theta_2^2|\mu_2(\cdot)] > 0$ and $\frac{\partial^2}{\partial\theta_1^2}[U^{A_1}|v_1, \theta_1] = -2E[\theta_2^2] < 0$. This implies that for any two different posterior beliefs of the DM, say $\underline{v}_1 < \overline{v}_1$, there is at most one type of agent 1 that is indifferent between both. Now suppose that contrary to interval equilibria, there are two states $\underline{\theta}_1 < \overline{\theta}_1$ such that $E_{\theta_2}[U^{A_1}|\overline{v}_1, \underline{\theta}_1] \geqslant E_{\theta_2}[U^{A_1}|\underline{v}_1, \underline{\theta}_1]$ and $E_{\theta_2}[U^{A_1}|\underline{v}_1, \overline{\theta}_1] > E_{\theta_2}[U^{A_1}|\overline{v}_1, \overline{\theta}_1]$. Then $E_{\theta_2}[U^{A_1}|\overline{v}_1, \overline{\theta}_1] - E_{\theta_2}[U^{A_1}|\underline{v}_1, \overline{\theta}_1] < E_{\theta_2}[U^{A_1}|\overline{v}_1, \underline{\theta}_1] - E_{\theta_2}[U^{A_1}|\underline{v}_1, \underline{\theta}_1]$, which contradicts $\frac{\partial^2}{\partial\theta_1\partial v_1}[U^{A_1}|v_1, \theta_1] > 0$. □

**Proof of Claim 1.** Note that $\overline{m}_i = E[\theta_i|m_i]$ and $m_i$ is coarser than $\theta_i$. Therefore,

$$E[\theta_i\overline{m}_i] = E\big[\theta_i E[\theta_i|m_i]\big] = E\big\{E\big[\theta_i E[\theta_i|m_i]\big]|m_i\big\} = E\big\{E[\theta_i|m_i]E[\theta_i|m_i]\big\} = E\big[\overline{m}_i^2\big]. \qquad \square$$

**Proof of Lemma 2.** We prove the results for $E(\overline{m}_1^2)$. The results for $E(\overline{m}_2^2)$ can be proved similarly. By definition and (4), we have

$$E\big(\overline{m}_1^2\big) = \sum_{n=1}^{N_1}\int_{a_{1,n-1}}^{a_{1,n}} \frac{1}{A_1}\frac{(a_{1,n} + a_{1,n-1})^2}{4} = \frac{1}{4A_1}\sum_{n=1}^{N_1}(a_{1,n} - a_{1,n-1})(a_{1,n} + a_{1,n-1})^2 = \frac{A_1^2}{3} - \frac{A_1^2}{12N_1^2} - \frac{b_1^2 x_2^2(N_1^2 - 1)}{3}.$$

This proves part (i). To prove part (ii), note that $E(\overline{m}_1^2) = (E(\theta_1))^2 + var[E(\theta_1|m_1)]$. Because the conditional variance $var[E(\theta_1|m_1)] \in [0, var(\theta_1)]$, $(E(\theta_1))^2 \leqslant E(\overline{m}_1^2) \leqslant E(\theta_1^2)$, and part (ii) immediately follows. □

[18] Our basic model with two agents can be extended to $n$ agents in a straightforward way, and the main results do not change qualitatively. With more senders, the strategic complementarity of information transmissions among senders is potentially stronger.

[19] In McGee and Yang (2011), we consider an alternative setting in which agent $i$'s utility function is $U^{A_i} = -(y - \theta_i)^2$. That is, each agent wants the DM's action to match his own private information. Similar biases appear in Martimort and Semenov (2008). In this setting, we show that the agents' information transmissions could be strategic substitutes in some situations and strategic complements in others. The driving force behind the strategic interaction between the experts' information transmissions is again a variance effect. For another way of modeling state-dependent biases, see Gordon (2010).

[20] Ottaviani and Sorensen (2001) show that in committee debates the order of speech affects information transmission and thus matters. In our model, under sequential communication characterizing the first sender's equilibrium strategy is difficult. By changing his report, the first sender can potentially induce different partitions from the second sender.

**Proof of Proposition 1.** Part (i) follows immediately from previous analysis. In particular, given $N_1$ and $N_2$, $x_1$ and $x_2$ are determined from Eqs. (7) and (8). If $N_1$, $N_2$, $x_1$ and $x_2$ satisfy the inequalities (9), then there is an equilibrium associated with $N_1$ and $N_2$ with the partition elements being characterized by (4).

Part (ii). The existence of equilibrium is guaranteed since a babbling equilibrium in which $N_1 = N_2 = 1$ and the DM ignores the messages always exists. To show that the equilibrium partition elements are finite, note that by Lemma 2 $x_i$ has a lower bound $\frac{3}{2A_i} > 0$. Now inspecting the inequalities (9), we can see that as $N_i$ goes to infinity one of the inequalities must be violated. Therefore, both $N_1$ and $N_2$ must be finite. □

**Proof of Lemma 3.** Consider the change in $E(\overline{m}_1^2)$ when $N_1$ decreases to $N_1 - 1$:

$$E(\overline{m}_1^2)(N_1) - E(\overline{m}_1^2)(N_1 - 1) = \frac{A_1^2}{12}\left[\frac{1}{(N_1-1)^2} - \frac{1}{N_1^2}\right] - \frac{b_1^2 x_2^2}{3}\left[N_1^2 - (N_1-1)^2\right] \propto A_1^2 - 4b_1^2 x_2^2 N_1^2 (N_1-1)^2 > 0.$$

The last inequality follows from $a_{1,1} > 0$, which implies that $A_1 > 2b_1 x_2 N_1(N_1 - 1)$. Thus $E(\overline{m}_1^2)$ is strictly increasing in $N_1$. The term $x_2$ affects $E(\overline{m}_1^2)$ in two ways. First, a decrease in $x_2$ directly increases $E(\overline{m}_1^2)$. Second, by (5) a decrease in $x_2$ leads to a weakly larger $N_1$, which increases $E(\overline{m}_1^2)$ as well. Therefore, $E(\overline{m}_1^2)$ is strictly decreasing in $x_2$. By similar logic, $E(\overline{m}_1^2)$ is strictly decreasing in $b_1$. Since $x_2$ is decreasing in $E(\overline{m}_2^2)$, it follows that $E(\overline{m}_1^2)$ is strictly increasing in $E(\overline{m}_2^2)$. □

**Proof of Proposition 3.** Part (i). Note that Eqs. (7) and (8) define the "bias multipliers" as functions of $N_1$ and $N_2$: $x_1(N_1, N_2)$ and $x_2(N_1, N_2)$. By Lemma 3, both $x_1(N_1, N_2)$ and $x_2(N_1, N_2)$ are strictly decreasing in $N_1$ and $N_2$. Now the fact that $N_1' > N_1$ and $N_2' > N_2$ implies that $x_2(N_1', N_2') < x_2(N_1', N_2)$ and $x_1(N_1', N_2') < x_1(N_1, N_2')$. Because equilibria with $(N_1', N_2)$ and $(N_1, N_2')$ exist, we have $2|b_1|x_2(N_1', N_2)N_1'(N_1' - 1) < A_1$ and $2|b_2|x_1(N_1, N_2')N_2'(N_2' - 1) < A_2$. Combining the above inequalities, we have $2|b_1|x_2(N_1', N_2')N_1'(N_1' - 1) < A_1$ and $2|b_2|x_1(N_1', N_2')N_2'(N_2' - 1) < A_2$, which implies that an equilibrium with $(N_1', N_2')$ exists.

Part (ii). We see that by (10) $E(\overline{m}_1^2)$ and $E(\overline{m}_2^2)$ should be maximized in the most informative equilibrium. By Lemma 3, $E(\overline{m}_i^2)$ is increasing in $N_i$ and $E(\overline{m}_j^2)$. Thus $E(\overline{m}_i^2)$ is increasing in $N_j$ as well. Therefore, $N_1$ and $N_2$ are maximized in the most informative equilibrium subject to condition (9). The bounds of $N_i^*$ come from part (ii) of Lemma 3. In particular, (15) follows from the fact that $x_i \in [\frac{3}{2A_i}, \frac{2}{A_i}]$. Now we show that there is no equilibrium with $N_i > N_i^*$. Because the agents are symmetric, we show that there is no equilibrium with $N_1 > N_1^*$. Suppose there is an equilibrium with $(N_1, N_2)$ such that $N_1 > N_1^*$ and $N_2 \geqslant N_2^*$. Then the equilibrium with $(N_1, N_2)$ is more informative than the equilibrium with $(N_1^*, N_2^*)$, which contradicts the fact that the equilibrium with $(N_1^*, N_2^*)$ is the most informative equilibrium. Now suppose there is an equilibrium with $(N_1, N_2)$ such that $N_1 > N_1^*$ and $N_2 < N_2^*$. By the result in part (iii), this implies that an equilibrium with $(N_1, N_2^*)$ exists as well, but this contradicts the fact that the equilibrium with $(N_1^*, N_2^*)$ is the most informative equilibrium as it is less informative than the equilibrium with $(N_1, N_2^*)$.

Part (iii). By part (ii) of Lemma 2 we see that the ratio of the upper bound of $x_i$ to its lower bound is 4/3. On the other hand, the ratio of $\frac{(N_i-1)(N_i-2)}{N_i(N_i-1)} = (N_i - 2)/N_i$, and this ratio is less than 3/4 if $N_i$ is less than 8. This implies that if $1 < N_i \leqslant 8$ for both $i = 1, 2$, then the left-hand side of conditions (9) under $(N_1 - 1, N_2 - 1)$ are smaller than those under $(N_1, N_2)$. Therefore, if an equilibrium with $(N_1, N_2)$ exists and $1 < N_i \leqslant 8$ for both $i = 1, 2$, conditions (9) must be satisfied with $(N_1 - 1, N_2 - 1)$, which implies an equilibrium with $(N_1 - 1, N_2 - 1)$ exists. □

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