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Survival Auctions

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Abstract Dynamic clock auctions with drop-out information typically yield outcomes closer to equilibrium predictions than do comparable sealed-bid auctions. However, clock auctions require congregating bidders for a fixed time interval, which has limited field applicability and introduces inefficiencies of its own given the time cost of congregating bidders. In this experiment we explore the effects of removing these inefficiencies through survival auctions – a multi-round sealed-bid auction which is theoretically isomorphic to the dynamic clock auction with drop-out information.

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1 Introduction

Dynamic (clock) auctions with rivals drop-out information provided have consistently yielded closer conformity to equilibrium bidding strategies than comparable sealed-bid auctions for a variety of auction institutions and demand structures: uniform-price multi-unit demand auctions with and without synergies (Kagel and Levin [6,7]), single-unit, private-value auctions (Kagel, Harstad, and Levin [3]), and single-unit common value auctions (Levin, Kagel, and Richard [8]). In most, but not all cases, the superior performance of the dynamic auction with drop-out information has been attributed to a transparency and simplicity that is simply lacking in most sealed-bid auctions (see, for example, Kagel and Levin [6]). However, dynamic clock auctions suffer from a number of practical disadvantages that are likely to limit their applications to field settings as they require congregating all bidders for a fixed time interval. This introduces inefficiencies of its own due to the time cost of congregating bidders. Alternatively, conducting quasi-dynamic clock auctions similar to those underlying the spectrum (air-wave) rights auctions (Cramton [2]) can lead to exceedingly long auctions with an uncertain end point.

One promising practical alternative to the ascending-price clock auctions is "survival" auctions: multi-round sealed-bid auctions in which low bids are successively eliminated in every round with the low-bid price announced and needing to be met or exceeded in subsequent rounds. These survival auctions have been shown to be strategically equivalent to ascending-price clock auctions (McAdams, Fujishima and Shoham [9]). Survival auctions do not require congregating bidders for a fixed time interval and have a certain and swift end period. They also provide the drop-out information that is valuable for raising revenue in a number of auction settings, and has been shown to be the key factor in obtaining closer to equilibrium outcomes in ascending-price auctions. The present paper compares two different versions of the survival auction to a dynamic ascending-price auction with drop-out information provided. We do so for the case of private value, Vickrey style auctions in which bidders demand multiple (up to two) units each. The ascending-price auction employs the Ausubel [1] format, with drop-out information provided.

More specifically, we compare two different survival auction mechanisms to the Ausubel auction: (1) An s-stage survival auction (where s is equal to the number of units initially demanded, or initially bid on minus the number of units supplied) so that one active bid is dropped in each auction round until all units are allocated ([9]), and (2) A two-stage survival format where everyone bids in the first stage, and only a limited number of high (stage-one) bids are permitted to bid in the second-stage (Perry, Wolfstetter and Zamir [12]). We show that the strategic equivalence between the s-stage survival auction and the Ausubel auction still holds when bidders have multi-unit demands and Vickrey pricing rules are employed, and that all three auctions have the same equilibrium outcomes via sincere bidding (i.e., bidders bid their private values).

We find that the Ausubel auction achieves the highest level of sincere bidding and efficiency, with the s-stage survival auction showing the most improvement over time so that by the last several auctions the level of sincere bidding and efficiency approaches that of the Ausubel auctions. Deviations from sincere bidding in both the survival and two-stage auctions result primarily from bidding too low (below bidders' private values). This stands in marked contrast to the pervasive bidding above value found in comparable one-shot Vickrey auctions. The efficiency measures are compared to those resulting from purely random bidding and two modified random bidding rules, as additional reference points against which to evaluate the different auction formats. We also compare seller revenue and bidder profits between auction institutions, and the random bidding rules.

2 Theoretical Considerations

We consider an auction in which K indivisible identical objects are sold to n bidders, where $n > K$. Each bidder i ($i = 1, 2, \cdots, n$) demands up to two units of the good. Bidder i's valuation of object j is v_{ij}, $j = 1, 2$. v_{ij} is observable to bidder i but not observable to the other bidders. Ex ante, v_{ij}'s are independently drawn from a uniform distribution with support $[0, \bar{v}]$.

Below we will first describe the three auction formats to be examined in our experiment. We will then present theoretical results regarding these auction mechanisms.

2.1 Auction Formats

Ausubel auction with drop-out information provided: Bidders start out actively bidding on all units. A price "clock" starts at zero and increases continuously thereafter, with bidders deciding at what price to drop out of the bidding. Dropping out is irrevocable so a bidder can no longer bid on a unit he has dropped out on. Winning bidders pay the price at which they have "clinched" an item. Clinching works as follows: With K objects for sale, suppose at a given price, p_0, bidder i still demands two units, but the aggregate demand of all other bidders just dropped from K to $K-1$. Then in the language of team sports, bidder i has clinched winning an item no matter how the auction proceeds. As such bidder i is awarded one item at the clinching price, p_0. This process repeats itself with the supply reduced from K to $K-1$ and with i's demand reduced by one unit. In this way the auction sequentially implements the Vickrey rule that each bidder pays the amount of the kth highest rejected bid, other than her own, for the kth object won. During the auction process, all drop-out prices are publicly reported when they occur, along with units clinched and the prices at which they were clinched. Ausubel [1] shows that sincere bidding is the unique equilibrium surviving iterated elimination of (weakly) dominated strategies.

S-stage survival auction: The auction begins with each bidder submitting a 2×1 vector of bids for both units. Bids from all subjects are ranked from highest to lowest with the low bid announced and the bid on that unit excluded in subsequent rounds. The lowest bid also becomes the minimal bid required for the following round. The auction proceeds in this way dropping the low bid in each round and determining who wins items, and the price paid for those items, using the clinching rules described for the Ausubel auction. (Clinched units and the prices paid for these units are announced as well.) This process repeats itself until all the items have been clinched, which takes exactly s stages, where s is equal to the number of units initially bid on minus the number of units supplied. For example, with 4 bidders bidding for 2 objects, and initially demanding two units each, it takes 6 stages ($s = 2 \times 4 - 2 = 6$) to complete the auction. In what follows we refer to the s-stage survival auction as *the survival auction*.

Two-stage survival auction: This is the same as the multi-stage survival auction described above except that it proceeds in two rather than multiple stages. Each auction consists of two rounds with only m high bids being active in the second round, where m is the minimal integer number such that clinching is not possible in the first round. For example, with 4 bidders bidding for 2 objects, setting $m = 4$ assures that there will be no clinching in round one (stage one of the bidding process); similarly, with 4 bidders bidding for 3 objects, it can be verified that $m = 5$. The $2n - m$ low bids dropped in the first stage are revealed to the surviving bidders prior to bidding in the second stage, with all bids in the second stage required to be greater than or equal to the highest of the dropped bids from stage-one. In what follows we will refer to the two-stage survival auction as *the two-stage auction*.

2.2 Theoretical Predictions

The survival auction was first analyzed in McAdams, Fujishima and Shoham [9], who show that an ascending-price auction using English clock rules (aka Japanese clock auction; see Milgrom and Weber [10]) and a survival auction are strategically equivalent for both single-unit and multi-unit object auctions; i.e., all high bidders pay the *same* price equal to the highest rejected bid price. In our case, the auction format under consideration involves bidders demanding multiple units and adopts Vickrey allocation and pricing rules designed to induce sincere bidding. Following the basic arguments in [9], the strategic equivalence between a survival auction and an Ausubel auction can also be established.

Proposition 1 *The survival auction and the Ausubel auction are strategically equivalent.*

Proof: See Appendix A.

Two auctions are strategically equivalent if there exists an isomorphism between their strategy spaces which preserves payoffs. The proof is completed by showing that there exists an identity mapping between the information sets and their precedence relations in the survival auction and the Ausubel auction, leading to an identity mapping between the strategy spaces which preserves payoffs.

Strategic equivalence is the strongest possible formal relationship that can be established between two mechanisms. Proposition 1 thus implies that survival auction and Ausubel auction are outcome equivalent.[1] Ausubel [1] shows that sincere bidding by all bidders is the unique outcome of iterated elimination of weakly dominated strategies in the Ausubel auction with drop-out information provided. In view of the strategic equivalence between the survival auction and the Ausubel auction, we have the following corollary:

Corollary: *Sincere bidding in each round is the unique outcome of iterated elimination of weakly dominated strategies in both the survival auction and the Ausubel auction.*

We can show that sincere bidding is also the unique outcome of iterated elimination of weakly dominated strategies in the two-stage survival auction. Thus a two-stage survival auction is outcome equivalent to an Ausubel auction.

Proposition 2 *A two-stage survival auction is (Nash) outcome equivalent to an Ausubel auction.*

Proof: See Appendix A.

The proof is straightforward. Suppose the highest rejected bid in the first stage is b^*, then bidding $\max\{b^*, v_{ik}\}$ for $k \in \{1, 2\}$ is the weakly dominant strategy for each remaining bidder i who is still active on object k in the second stage. Given that bidders submit $\max\{b^*, v_{ik}\}$ in the second stage (the outcome of single-round elimination of weakly dominated strategies), the proof is completed by showing that sincere bidding is the weakly dominant strategy in the first stage.

[1]Two auctions are outcome equivalent if they possess (Nash) equilibria in which the items are allocated to the same set of bidders at the same set of prices.

3 Experimental Design and Procedures

Experimental sessions were conducted at Ohio State University using subjects from a wide cross-section of undergraduate students. Each session lasted approximately $1.5 - 2$ hours, and the earnings per subject were between \$35 and \$40 on average. Each experimental session had two or more markets operating simultaneously, with subjects randomly reassigned to new markets in each auction period. There were four bidders in each market, with each bidder demanding two units. Bidders' demands were weakly decreasing, with two independent draws from a uniform distribution with support $[0, \$7.50]$ (with new draws in each auction period). Each auction employed an ABA design with supply (K) equal to 2 units in the first 12 auctions, $K = 3$ in the next 12, and $K = 2$ in the last 12 auctions.[2] Each auction began with two dry runs with $K = 2$.

Table 1 shows the number of sessions under each auction format along with the total number of experimental subjects.[3]

Table 1: Experimental Treatments

Institution	Number of sessions	Number of subjects
Survival	2	20
Two-stage	2	20
Ausubel	1	16

All of the *Ausubel auctions* employed a "digital" price clock with a price increment of \$0.25 every 3 seconds.[4] During the auction, the current price of the item, the number of items for sale, and the number of units actively bid on were posted on each bidder's screen at all times. Drop-outs and clinching prices were also reported on all bidders' screens as they occurred. When a bidder clinched an item the clinching price was automatically recorded on her computer screen just below the value of the item, with the profits earned for that item reported just below this. We employ a single Ausubel auction session as Kagel and Levin [6] have conducted a number of such auctions with a single human competing against computerized rivals with results quite similar to those reported here for all human bidders. These

[2] In one of the two-stage auctions there were 13 $K = 3$ rounds. Data for the extra $K = 3$ auction have been dropped from the analysis.

[3] In the Ausubel auction, there were 8 subjects in the last $K = 2$ set of auctions as the time period we had recruited subjects for required that we permit those who needed to leave to do so for the last K= 2 treatment.

[4] Drop-outs occurring within a given tick of the price clock were counted as having dropped out at the same price but with the drop-out order determined by when the file server recorded the drop-out.

results are reported in a format compatible with the present results, along with the structure of these auctions, in Appendix B.

In the *survival auctions* bids were submitted in each round with all but the unit with the lowest bid continuing to be actively bid on in the next round. After every round the bid on the unit that was dropped that round was reported along with drop-out bids on all units from previous rounds. Active bidders resubmitted their bids in every round with the restriction that bids in later rounds must be greater than or equal to the drop-out bid of the previous round.[5]

In *two-stage auctions,* m active bids survive from the first-stage bidding. The $2n - m$ low bids are reported to subjects after stage 1.[6] The number of second-stage bids, m, was set so that clinching was not possible in the first round ($m = 4$ for $K = 2$ and $m = 5$ for $K = 3$).

Following completion of all auctions all dropout prices and valuations were reported back to subjects, with dropout prices ranked from highest to lowest, and with own bids clearly distinguished from rivals'.[7] Furthermore, the same clinching metaphor employed in the Ausubel auctions was used to describe who earned items and the prices paid in both the survival and two-stage auctions.

Instructions were read out loud to subjects, with copies for them to follow along with as well. The instructions also included examples of how the pricing rules worked.[8]

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[5] In case of ties for the low bid all tied bids were dropped. In case of ties for clinching, the computer randomly determined who earned a unit.

[6] In case of ties, the rank of the bids was determined randomly. Then to ensure m active bids in the second stage, $2n - m$ low bids were dropped.

[7] There are no drop-out prices for clinched items in the Ausubel auction. In this treatment "xxx" was used in place of the drop-out price.

[8] See http://www.econ.ohio-state.edu/lixinye/Experiment/Survival/Instruction/

4 Results

In this section we present our results by focusing on bid patterns, efficiency, profits, and revenue.

Table 2: Frequency of Sincere Bidding
(Standard errors of the mean in parenthesis. Differences from survival auctions in bold.)

	Higher Value Unit			Lower Valued Unit		
	Survival	Two-stage	Ausubel	Survival	Two-stage	Ausubel
K = 2	0.246 (0.063)	0.296 (0.073) **-0.050**	0.828 (0.052) **-0.582****	0.375 (0.067)	0.292 (0.056) **0.083**	0.761 (0.077) **-0.386****
K = 3	0.425 (0.078)	0.342 (0.085) **0.083**	0.833 (0.042) **-0.408****	0.567 (0.074)	0.417 (0.076) **0.150**	0.676 (0.083) **-0.109**
K = 2	0.600 (0.084)	0.396 (0.085) **0.204+**	0.783 (0.079) **-0.183**	0.658 (0.069)	0.429 (0.065) **0.229****	0.556 (0.135) **0.102**

\+ Significantly different from 0 at the 10% level
** Significantly different from 0 at the 1% level

Bid Patterns: Table 2 compares the frequency of sincere bidding between the three auction procedures. The unit of observation employed is the subject average mean frequency with which individual bidders were bidding sincerely computed over all 12 auctions for each value of K. Specifically the first row with $K = 2$ corresponds to rounds $1 - 12$, $K = 3$ to rounds $13 - 24$ and last row with $K = 2$ to rounds $25 - 36$. Mann-Whitney tests are used to check for significant differences between auction institutions in all tables unless noted otherwise. Furthermore, since 25¢ bid increments were employed in the clock auctions, bids are counted as sincere in the clock auctions if the dropout occurred at the clock tick just below the actual value or just above the actual value.[9] To give the sealed-bid auctions the same flexibility, a bid is counted as sincere if it occurred within plus or minus 12.5¢ of the actual value.[10]

For the first $K = 2$ auctions, the frequency of sincere bidding is substantially (and significantly) lower

[9] For example, suppose the value was \$4.12, the bid would be counted as sincere if the drop occurred at 4.00 or 4.25.

[10] Effectively, 13¢ of the actual value. Winning bids are censored in the Ausubel auction but not in the survival auctions. Accounting for this by dropping winning bids from the calculations for the survival auctions has no material effect on the results reported. Furthermore, for the survival and two-stage auctions we only employ bids that resulted in being dropped from the auction; e.g., for a bidder dropping out in round three of the survival auction we ignore all earlier bids by this subject on this unit for this auction.

in the survival auctions than in the Ausubel auctions for both high and low valued units. Furthermore, there is little to distinguish between the survival and two-stage auctions on this account. However, there is substantial growth in the frequency of sincere bidding in the survival auctions for both units, and some deterioration, particularly for the lower valued unit, in the Ausubel auctions so that by the last $K = 2$ treatment there are no longer any significant differences between the two treatments.[11] There is much more modest improvement in sincere bidding in the two-stage auctions than the survival auctions, so that by the last $K = 2$ auctions, there are statistically and economically meaningful differences in the frequency of sincere bidding between the survival auctions and the two-stage auctions. Finally, there is significantly less sincere bidding in the two-stage auctions than the Ausubel auctions for all but the lower valued unit in the last $K = 2$ treatment.

Observation 1 *There is substantially more sincere bidding to begin with in the Ausubel auctions than in either the survival or the two-stage auctions. These differences persist for the two-stage auctions, but are gradually eliminated for the survival auctions.*

[11] Part of the reason for the reduced frequency of sincere bidding in the last $k = 2$ treatment is due to sample selection. Average frequency of sincere bidding for the eight subjects present in the first and last $k = 2$ treatments was .680 in the second $k = 2$ treatment for the lower valued unit (the one with the most change under Ausubel) which makes for a somewhat smaller spread than reported in Table 2. Of these eight subjects 4 out of 7 had more sincere bidding for their higher valued unit in the second $k = 2$ treatment versus the first $k = 2$ (1 no change) and 2 out of 6 had more sincere bidding for their lower valued unit (2 no change). Bidders rarely win a second (lower valued) unit which may also be partly responsible for the slippage in sincere bidding on that unit.

Table 3: Comparison of Bid Patterns Between Auction Mechanisms

(Frequencies with standard errors of the mean in parenthesis.
Differences from survival auctions in bold.)

	Higher Valued Unit			Lower Valued Unit		
K=2	Survival	Two-stage	Ausubel	Survival	Two-stage	Ausubel
Won and earned negative profits	0.021 (0.015)	0.046 (0.027) **-0.025**	0.009 (0.047) **0.012**	0.194 (0.125)	0.286 (0.184) **-0.092**	0.125 (0.210) **0.069**
Bid > v and not win	0.047 (0.025)	0.127 (0.052) **-0.080**	0.050 (0.112) **-.003**	0.119 (0.047)	0.187 (0.063) **-0.068**	0.180 (0.134) **-0.061**
Bid < v	0.648 (0.078)	0.570 (0.084) **0.078**	0.122 (0.089) **0.526****	0.498 (0.079)	0.510 (0.073) **-0.012**	0.060 (0.079) **0.438****
K=3						
Won and earned negative profits	0.011 (0.008)	0.027 (0.013) **-0.016**	0.000 (0.000) **0.011**	0.159 (0.077)	0.143 (0.082) **0.016**	0.103 (0.148) **0.056**
Bid > v and not win	0.158 (0.058)	0.033 (0.019) **0.125+**	0.069 (0.101) **0.089**	0.190 (0.068)	0.123 (0.053) **0.067**	0.294 (0.142) **-0.104**
Bid < v	0.325 (0.072)	0.616 (0.086) **-0.291***	0.098 (0.094) **0.227***	0.227 (0.063)	0.453 (0.074) **-0.226***	0.031 (0.055) **0.196****
K=2						
Won and earned negative profits	0.029 (0.020)	0.045 (0.022) **-0.016**	0.000 (0.000) **0.029**	0.000 (0.000)	0.167 (0.105) **-0.167**	0.250 (0.421) **-0.250+**
Bid > v and not win	0.053 (0.027)	0.127 (0.059) **-0.074**	0.139 (0.174) **-0.086**	0.134 (0.053)	0.164 (0.063) **-0.030**	0.400 (0.215) **-0.266***
Bid < v	0.379 (0.082)	0.479 (0.094) **-0.100**	0.078 (0.124) **0.310***	0.200 (0.052)	0.389 (0.074) **-0.189+**	0.044 (0.106) **0.156***

\+ Significantly different from 0 at the 10% level
* Significantly different from 0 at the 5% level
** Significantly different from 0 at the 1% level

Table 3 compares the pattern of deviations from sincere bidding between auction institutions.[12] For both the survival and two-stage auctions the frequency of winning items and earning negative profits is quite low, comparable to the results reported for the Ausubel auctions. (Note, this measure for lower valued units is deceptively high as typically one would have to bid above value to win two units given

[12]The base for the category "won and earned negative profits" is all units actually won. The base for all bids greater than v and not win (as well as all bids less than v) is all non-winning bids. The excluded category – bids essentially equal to value for non-winning bids – is reported in Table 2.

how others were bidding, with relatively few subjects winning two units.[13]) What Table 3 shows is that the primary source of deviations from sincere bidding for both the survival and two-stage auctions is bidding *below* value. This holds to the point that there is significantly more bidding below value in both the survival and two-stage auctions throughout compared to the Ausubel auctions. What differentiates the survival and two-stage auctions on this score is that after the first $K = 2$ treatment, there is a substantial reduction in the frequency of bidding below value in the survival auctions, with much more modest improvement in the two-stage auctions. This is consistent with the changes in sincere bidding reported in Table 2.

Observation 2 *Deviations from sincere bidding for both the two-stage and survival auctions result primarily from bidding below value, involving opportunity costs rather than out-of-pocket losses.*

What is perhaps most striking about the underbidding in the two-stage and survival auctions is the contrast to the pattern found in the standard (one-stage) sealed-bid Vickrey auctions, which typically involves bidding *above* value (Kagel and Levin [5]; Kagel, Kinross, and Levin [4]). For example, in a standard one-round multi-unit demand Vickrey auction with the same parameters and procedures as those employed here (Kagel, Kinross, and Levin [4]), the frequency of bidding above value on units not won averaged 64.6% and 65.9% for higher and lower valued units respectively (and not below 57.5% for *any* value of K). This compares to a maximum frequency of under 20% reported here for the survival and two-stage auctions.[14]

These results pose two important questions. First, why are there differences in the bid patterns between the standard, single-stage Vickrey auction and the two-stage and survival auctions? Second, why are there improvements in performance under the survival auctions as opposed to the much slower adjustments in the two-stage auctions?

A number of contributing factors could be at work. First, in a multi-unit demand context, characterizing the auction mechanism in terms of the clinching rules appears to play a significant role in getting subjects not to bid above their value. The evidence for this comes from a series of dynamic Vickrey auctions (without any dropout information) using the same terminology as in the sealed-bid

[13]For example, in the last $K = 2$ treatment for the Ausubel auctions we have two subjects who actually won a low valued unit.

[14]In addition, in the standard sealed-bid Vickrey auction, the average frequency of bidding above value on their higher valued unit and losing money was just under 25%, with the frequency of bidding below value on units not won being 17.4% and 14.3% for the higher and lower valued units respectively. These are strikingly at odds with the data reported in Table 3.

auctions compared to using the clinching terminology employed here.[15] Using the sealed-bid terminology subjects deviate from sincere bidding by bidding above value. In contrast, using the clinching terminology subjects typically deviate from sincere bidding by bidding below their values. That is, there appears to be a clear framing effect as a consequence of the language used to describe the auction mechanism. Both the two-stage and survival auctions reported here used the clinching terminology, so we might presume similar effects.

Second, with overbidding largely eliminated, subjects have only one type of error they can make – underbidding. With this in mind there are a number of differences between the auction institutions studied here. In the Ausubel auctions they must repeatedly decide (with each tick of the clock) to stay in or to drop out. In the survival auctions they must decide in each round how much to bid relative to their value. In the two-stage auction they make this decision twice. As such, subjects have, in effect, much more experience with the auction in the continuous clock case, an intermediate level of experience in the case of the survival auctions, and minimal experience in the two-stage auction. Thus, to the extent that subjects are learning from experience, we should see the quickest convergence to equilibrium for the Ausubel auctions, followed by the survival auctions, with the two-stage auctions showing the least amount of learning. This is exactly what we see in the data.[16]

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[15] These results, along with the instructions employed, can be found at:
http://www.econ.ohio-state.edu/kagel/survivor_appendix

[16] We find some evidence for subjects learning not to bid below value within the Ausubel auctions. Looking at the first $K = 2$ treatment, and breaking the data up into the first 6 auctions versus the last 6 auctions, we find underbidding frequencies of 14.3% (0.063) versus 6.9% (0.045) for the high valued unit (with standard errors of the mean in parentheses) and 5.2% (0.025) versus 6.9% (0.030) for the low valued unit. Although a within subject Mann-Whitney (sign) test does not reject a null hypothesis of no difference at conventional levels, the difference with respect to the high valued unit is suggestive ($p < .15$ using a one-tailed test). For data on learning within dynamic clock auctions, but in a different context, see Kagel and Levin [6].

Table 4: Comparisons of Efficiency, Profits and Revenue
(Standard errors of the mean in parentheses. Differences from Survival auctions in bold.)

	Efficiency			Revenue[b] (difference from sincere bidding)			Profits[b] (difference from sincere bidding)		
	Survival	Two-stage	Ausubel	Survival	Two-stage	Ausubel	Survival	Two-stage	Ausubel
K = 2	93.8% (1.33)	93.1% (1.72) **0.70%**	98.3% (0.77) **-4.50%****	-0.951 (0.227)	-1.085 (0.249) **0.134**	-0.211 (0.101) **-0.740****	0.110 (0.314)	0.209 (0.373) **-0.099**	0.018 (0.138) **0.092***
K = 3	97.4% (0.61)	94.9% (0.96) **2.50%***	98.8% (0.59) **-1.40%****	-0.391 (0.208)	-2.071 (0.257) **1.680****	-0.134 (0.089) **-0.257**	-0.066 (0.250)	1.217 (0.313) **-1.283****	-0.061 (0.114) **-0.005**
K = 2	98.7% (0.41)	95.2% (1.05) **3.50%**+	98.8% (1.24) **-0.01%****	-0.325 (0.090)	-0.988 (0.188) **0.663****	0.154 (0.098) **-0.479****	0.164 (0.108)	0.382 (0.234) **-0.220**	-0.316 (0.228) **0.480***
Random[a]	60.9% [62.5%]			-0.32 [0.36]			-4.63 [-6.71]		
Random Budget Constrained[a]	92.3% [93.0%]			-4.35 [-5.65]			3.36 [4.45]		
Random Survival[a]	91.9% [93.2%]			-4.11 [-5.82]			3.08 [4.65]		

[a] Simulations for K = 3 are in brackets; K = 2 not in brackets.
[b] Actual revenue (profit) less predicted revenue (profit).
[+] Significantly different from 0 at the 10% level.
[*] Significantly different from 0 at the 5% level.
[**] Significantly different from 0 at the 1% level.

Efficiency, profits and revenue: Table 4 compares data on efficiency, profits and revenue between the three auctions mechanisms. Efficiency is measured in the usual way – the sum of the K winning valuations divided by the sum of K highest valuations. Since in each auction valuations are drawn randomly, we report revenue and profits in terms of deviations from the equilibrium prediction. In all cases the unit of observation is the individual auction market.

The Ausubel auction generates the highest efficiency of all three mechanisms for all three auction sets. However, after the first $K = 2$ treatment the survival auction comes quite close, with essentially no difference in average efficiency between the two mechanisms in the last $K = 2$ treatment.[17] In contrast, the two-stage auctions show minimal improvement in efficiency, with the level significantly lower than under survival bidding for $K = 3$ and the final $K = 2$ treatment.

[17] The Mann-Whitney test yields a significant difference in the last $K = 2$ treatment as all but one of the Ausubel auctions had 100% efficiency, whereas a number of the survival auctions had less than 100% efficiency. A t-test shows no significant difference between treatments here.

For comparative purposes we also report efficiency, revenue and profits for three simulated "naive" bidding models in Table 4. The first benchmark involves totally random bidding – bids based on random draws from the uniform distribution over the entire interval of possible values so that bidders take no account of their valuations in determining what to bid.[18] The auction mechanism assumed is a single stage sealed bid auction. The second benchmark ("random budget constrained") again assumes a single stage sealed bid auction mechanism, but now bidders are budget constrained (i.e., they bid at or below their private valuations). Furthermore, bids for the higher valued unit are constrained to be no lower than the random bid for the lower valued unit. The third measure ("random survival") employs the random budget constrained rules, but does so within the context of the survival auction mechanism (i.e., bids in each round must be greater than or equal to the bid in the previous round). We ran 100 simulations for each of the three bidding rules, in each case using the entire set of valuations drawn for the survival auctions. Efficiency and profits improve substantially in going from unconstrained random bidding to budget constrained random bidding. However, adding in the survival rules has, on average, essentially no impact on outcomes.

These naive bidding models provide a benchmark against which to evaluate the efficiency measures. First, all mechanisms do substantially better than unconstrained random bidding. Second, the two-stage auctions do not do much better than the budget constrained random bidding, but both the Ausubel and survival auctions (the latter, after a bit of experience) do substantially better.

Observation 3 *The Ausubel auctions start out with high levels of efficiency and stay that way throughout. The survival and two-stage auctions start out with much lower efficiency levels, comparable to what the budget constrained random bidding simulations suggest. Efficiency improves rather dramatically for the survival auctions, rivaling the levels found in the Ausubel auction in the last $K = 2$ treatment, but there is minimum improvement in efficiency for the two-stage auctions.*

Average revenue under the different auction formats is reported in the middle columns of Table 4. The first thing to note is that revenue is lower than predicted in both the survival and two-stage auctions. This results from the high frequency of bidding below value reported earlier. Revenue is closest to the equilibrium prediction across all three auction sets for the Ausubel auction. As a result

[18] There are two ways to do this. In what is reported the higher of the two draws is the bid for the higher valued unit and the lower of the two draws is the bid for the lower valued unit. We have also run simulations for "restricted naive bidding" when the bid for the higher valued unit, b_h, is a draw from uniform distribution on $[\underline{v}, \bar{v}]$ and the bid for lower valued unit, b_l, is a draw from uniform distribution on $[\underline{v}, b_h]$. The effect of this is to raise efficiency a bit relative to the totally random bidding reported.

average revenue is lower in the survival auctions than in the Ausubel auctions for all three auction sets, and is significantly less than the Ausubel auctions for both $K = 2$ treatments. The two-stage auctions have even lower revenue than the survival auctions with these differences statistically significant in the $K = 3$ treatment and the last $K = 2$ treatment, as the survival auctions are converging to the equilibrium prediction, while the two-stage auctions show little improvement.

Average bidder profits are reported in the far most columns of Table 4. With the exception of the two-stage auction profits with $K = 3$, there is little in the way of economically meaningful differences in profits between the auction institutions, as the average difference from predicted profits is less than fifty cents per auction. For the $K = 3$ treatment, the high frequency of underbidding found in the two-stage auctions results in higher average profits than predicted of \$1.28 per auction, well above average profits earned under either of the alternative auction institutions.[19]

Observation 4 *Average revenue in the survival and two-stage auctions tends to be below the level in the Ausubel auctions as a result of the higher frequency of bidding below value reported in the first two cases. This translates into higher bidder profits in the two-stage and survival auction compared to the Ausubel auctions.*

5 Summary and Conclusions

This paper looks at the applicability of survival auctions as an alternative to ascending-price, clock auctions which have been shown in a large variety of laboratory experiments to yield outcomes much closer to equilibrium predictions than sealed-bid auctions. We extend the survival auction mechanism originally suggested by McAdams, Fujishima, and Shoham [9] to multi-unit demand auctions that employ Vickrey allocation and pricing rules, and show that strategic equivalence holds between the survival auctions and ascending-price (Ausubel) auctions with drop-out information. This implies efficient unit allocations via sincere bidding. Realization of this theoretical prediction would favor using survival auctions over ascending-price auctions as the former have quick and predictable termination times, do not require bidder congregation, and have an information-revelation component.

We find that the survival auctions have excellent results, in terms of allocative efficiency, compared to the benchmark Ausubel auction following an initial learning phase in which they do substantially worse than the Ausubel auctions. The survival auctions do significantly better than the static Vickrey

[19] Profits (revenue) under the budget constrained random bidding rules are great (lower) than predicted profits as subjects consistently bid less than with sincere bidding.

auction after an initial learning phase as well (see Kagel, Kinross, and Levin [4], for static Vickrey auction results using a design comparable to the one employed here). In contrast, a two-stage version of the survival auction suggested by Perry, Wolfstetter, and Zamir [12] does not do nearly as well as the survival or Ausubel auctions even after bidders have gained substantial experience with the mechanism.

One surprising behavioral result from the present experiment is that subjects tend to deviate from sincere bidding by bidding below their value as opposed to bidding above their value as in the single-round Vickrey auction. We attribute this to the use of the clinching terminology, which is quite natural for the quasi-dynamic nature of the survival and two-stage auctions, as opposed to the static explanation of the price and allocation rules that is natural to employ in the single-round Vickrey auction. The data also indicate that the repeated nature of decisions made in the survival auctions helps to reduce the bidding below value, so that bids are converging on the sincere bidding predicted as bidders gain experience with the mechanism.

One skeptical response to the differences between mechanisms reported here is that all of this does not matter for "real world" bidders who are sophisticated and need only be told the logic underlying sincere bidding to achieve this outcome for any of the three mechanisms. There are several possible responses to this criticism. First, this view is far from universal as the debates leading up to the design of the FCC spectrum auctions show.[20] Second, the results reported here shift the burden of proof from those who believe that the details of the mechanism do not matter to "sophisticated" bidders to demonstrate that their view is correct. Third, Rutstrom [13] has conducted an experiment comparing an English clock mechanism to a second-price sealed-bid mechanism for a single-unit private-value auction in which she went to great pains to explain the dominant bidding strategy.[21] This resulted in *no* difference from the typical pattern reported in the lab in cases where subjects are not offered any explanation for the dominant bidding strategy – prices were significantly higher in the second-price auctions. At a minimum this indicates that a more dynamic mechanism is more likely to continue to produce closer to equilibrium outcomes even when subjects are tutored on the correct bidding strategy.

The results reported here show promise for survival auctions as a viable alternative to ascending-price auctions in terms of generating desirable equilibrium outcomes, while having a number of prefer-

[20]For example, in their comments to the Federal Communications Commission describing the multi-unit Vickrey auction Nalebuff and Bulow [11] write (p. 29): "However, experience has shown that even Ph. D. students have trouble understanding the above description [of the dominant bidding strategy] ... The problem is that if people do not understand the payment rules of the auction then we do not have confidence that the end result will be efficient."

[21]The auctioned item was a box of chocolates which is, presumably, strictly private value or very close to it.

able institutional characteristics compared to ascending-price auctions. Future research should more thoroughly explore the properties of these survival auctions, particularly in the case of common value auctions, or auctions with affiliated private values, where the information aggregation inherent in revealing drop-out prices is predicted to raise revenue for sophisticated/experienced bidders and to reduce the incidence of the winner's curse for naive/inexperienced bidders (Levin, Kagel and Richard [8]).

References

[1] Ausubel, L. M.: An efficient ascending-bid auction for multiple objects, American Economic Review 94, 1452-1475 (2004).

[2] Cramton, P.: The FCC spectrum auctions: an early assessment, Journal of Economics and Management Strategy 6, 431-495 (1997).

[3] Kagel, J. H., Harstad, R. M., Levin, D.: Information impact and allocation rules in auctions with affiliated private values: a laboratory study, Econometrica 55, 1275-1304 (1987).

[4] Kagel, J. H., Kinross, S., Levin, D.: Implementing efficient multi-object auction institutions: an experimental study of the performance of boundedly rational agents, OSU Working Paper (2003).

[5] Kagel, J. H., Levin, D.: Independent-private values auctions: bidder behavior in first-, second- and third-price auctions with varying numbers of bidders, Economic Journal 103, 868-879 (1993).

[6] Kagel, J. H., Levin, D.: Behavior in multi-unit demand auctions: experiments with uniform price and dynamic Vickrey auctions, Econometrica 69, 413-454 (2001).

[7] Kagel, J. H. Levin, D.: Multi-unit demand auctions with synergies: some experimental results, Game and Economics Behavior 53, 170-207 (2005).

[8] Levin, D., Kagel, J. H., Richard, J.-F.: Revenue effects and information processing in English common value auctions, American Economic Review 86, 442-460 (1996).

[9] McAdams, D., Fujishima, Y., Shoham, Y.: Speeding up ascending-bid auctions, Proceedings of the International Journal Conference in Artificial Intelligence, 554-559 (1999).

[10] Milgrom, P., Weber, R.: A theory of auctions and competitive bidding, Econometrica 50, 1082-1122 (1982).

[11] Nalebuff, B. J., Bulow, J. I.: Designing the PCS Auction, Comment to the FCC on Behalf of Bell Atlantic (1993).

[12] Perry, M., Wolfstetter, E., Zamir, S.: A sealed-bid auction that matches the English auction, Games and Economic Behavior 33, 265-273 (2000).

[13] Rutstrom, E. E.: Homegrown values and incentive compatible auction design, International Journal of Game Theory 27, 427-441 (1998).

Appendix A

Proof of Proposition 1: Basically we can show that there exists an identity mapping between the information sets and their precedence relations in the two auctions, which leads to an identity mapping between the strategy spaces. We then verify that the (one-to-one) corresponding strategies induce the same payoffs. Following arguments paralleling those in McAdams, Fujishima, and Shoham [9], we can proceed in three steps:

(1) In the survival auction, the new information that bidder i possesses in between round t and $t+1$, if he survives round t (i.e., if he is still active on at least one item), is the losing bid information and clinching information in round t. Losing bid information consists of the identity of the bidder who lost a bid, the unit on which the bid lost, and the bid amount. Thus the losing bid information in between round t and round $t+1$ can be described by a $3t$-dimensional vector. The clinching information can also be described by a $3t$-dimensional vector, consisting of the identity of the bidder who clinched a unit, the unit being clinched, and the price at which the unit was clinched (if no clinching occurs at round t, then all entries are filled by, say, letter "N "). Therefore, each surviving bidder's decision points in the $t+1$st round can be represented by a $6t$-dimensional vector. In the Ausubel auction, the new information that bidder i possesses in between round t and $t+1$, if he still stays in the auction (not dropping from all items), is the drop-out information and clinching information in round t. The drop-out information consists of the identity of the bidder who dropped out, the item on which this bidder dropped out, and the drop-out price. Thus the losing bid information in between round t and $t+1$ can be described by a $3t$-dimensional vector. Similarly, the clinching information can also be described by $3t$-dimensional vector. Therefore the isomorphism of decision point sets between two auctions is the identity mapping.

(2) In the survival auction, each bidder in the $(t+1)$st round, if still active, can make any new bid which is higher than the minimal bid in that round, i.e., the eliminated bid in the tth round. In the Ausubel auction, each bidder after the tth drop-out can decide to wait until any price higher than the last drop-out before being the next to drop out. Thus the feasible action sets are identical and the decision point precedence relation is preserved by the identity mapping.

(3) In both auctions, under the identity mapping, if the "same" terminal point is reached, then the actions must be the "same" at the "same" decision points – "same" in the sense that they are equivalent under the identity mapping. This implies the following: (a) the objects will be allocated to the same set of bidders, (b) the winners of the items will pay the same amounts, and (c) the information available to all bidders at the end of the auction will be the same.

(1) and (2) imply that there exists an isomorphism (the identity mapping in this case) between the strategic sets in Ausubel and Survival auctions. (3) implies that the payoffs are preserved under this identity mapping. *Q.E.D.*

Proof of Proposition 2: We start with the second stage in a two-stage survival auction. Suppose the highest rejected bid in the first stage is b^*, then bidding $\max\{b^*, v_{ik}\}$ for $k \in \{1, 2\}$ is the weakly dominant strategy for each remaining bidder i who is still active on object k. All other strategies are weakly dominated. Now consider the first stage bidding. Given that bidders submit $\max\{b^*, v_{ik}\}$ in the second stage bidding (the outcome of one-round elimination of weakly dominated strategies), we claim that sincere bidding is the weakly dominant strategy in the first stage. (1) Bidding more than v_{ik} for bidder i on item k, say, bidding $v_{ik}^+ > v_{ik}$ is weakly dominated by bidding v_{ik}, as there is some positive probability that v_{ik}^+ will become binding for bidder i who ends up winning the item, in which event bidder i incurs a loss. (2) Bidding less than v_{ik} for bidder i on item k, say, bidding $v_{ik}^- < v_{ik}$ is weakly dominated by bidding v_{ik}, as there is some positive probability that bidder i will be excluded from the second stage bidding, while she would make it to the second stage with positive profit if she bid sincerely. Thus all the other strategies in the first stage are weakly dominated by sincere bidding, given that in the second stage each bidder bids $\max\{b^*, v_{ik}\}$. This shows that sincere bidding is the unique outcome of iterated elimination of weakly dominated strategies in the two-stage survival auction. *Q.E.D.*

Appendix B

This appendix reports results for Ausubel auctions with drop-out information from Kagel and Levin [6] in a format that is compatible with the results reported in the text. The results are summarized in Table A1 below, which is based on sessions 9 and 10 containing a total of 27 subjects.

Table A1: Ausubel Auction with Drop-Out Information and Computer Rivals

n=3	Unit 1	Unit 2		
Won and earned negative profit[a]	0.068 (0.033)	0.047 (0.039)	Bidder Earnings[b]	-0.094 (0.043)
Bid $> v_h$ with possible negative profit[a]	0.214 (0.060)	0.042 (0.015)	Efficiency[c]	99.1% (0.38)
Bid $< v_h$ [a]	0.160 (0.052)	0.255 (0.054)	Revenue[b]	-0.027 (0.134)
n=5			n=5	
Won and earned negative profit[a]	0.061 (0.031)	0.073 (0.046)	Bidder Earnings[b]	-0.091 (0.043)
Bid $> v_h$ with possible negative profit[a]	0.085 (0.027)	0.023 (0.012)	Efficiency[c]	99.3% (0.35)
Bid $< v_h$ [a]	0.131 (0.037)	0.219 (0.048)	Revenue[b]	-0.025 (0.040)

[a] Frequencies
[b] Difference from sincere bidding: sincere bidding less actual bids
[c] As a percentage of sincere bidding
S_m = standard error of the mean

Valuations were drawn iid from a uniform distribution with support $[0, \$7.50]$. Bidders with single unit demands were represented by computers programmed to submit bids equal to their valuation. Bidder h was played by human subjects drawn from a wide cross-section of undergraduate and graduate students at University of Pittsburgh and Carnegie-Mellon University.

Each human (h) operated in her own market with her own set of computer rivals. h's knew they were bidding against computers, the number of computers, and the computers' bidding strategy (but not the logic underlying this strategy). Supply, m, was set at two in all auctions. Each h had flat demand for two (2) units based on their random draw form the interval $[0, \$7.50]$.

All of the auctions employed a "digital" price clock with price increments of \$0.01 per second; Otherwise procedures were essentially the same as those reported in the text.

Session 9 began with 13 auctions with 3 computerized rivals, followed by 14 auctions with 5 computerized rivals (a total of 19 subjects in this session). Session 10 began with 13 auctions with 5 computerized rivals, followed by 14 auctions with 3 computerized rivals (a total of 18 subjects in this session).