# Auctions with Entry and Resale*

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**Abstract**

We study how resale affects auctions with costly entry in a model where bidders possess two-dimensional private information signals: entry costs and valuations. We establish the existence of symmetric entry equilibrium and identify sufficient conditions under which the equilibrium is unique. Our analysis suggests that the opportunity of resale affects both entry and bidding, and, in particular, it induces motivation for speculative entry and resale hunting abstentions. Our numerical results suggest that while expected entry is higher when resale is allowed, the effects of resale on expected revenue and efficiency are both ambiguous.

Keywords: Auctions, auctions with resale, auctions with entry

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## 1 Introduction

Starting with the seminal work of Vickrey (1961) and continuing until recently, the auction literature has mostly adopted the paradigm of a fixed number of bidders. This simplifying assumption has

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allowed for enormous progress in the characterization of optimal auctions and analysis of revenue equivalence and revenue rankings of different auction formats (see, for example, Riley and Samuelson, 1981, Myerson, 1981, Milgrom and Weber, 1982). However, in many auctions the number of rivals is not known when bids are placed.[1] This observation motivated several papers (e.g., McAfee and McMillan, 1987, Matthews, 1987, and Harstad et al., 1990) to treat the number of rival bidders as coming from exogenously determined distributions. However, this approach with a stochastic number of bidders still ignores an important issue. In many auctions the cost of bid preparation or information acquisition is far from trivial. We cannot simply rank auctions by assuming a fixed or stochastic number of bidders without accounting for the fact that different auctions are likely to induce different entry incentives. Endogenous entry must be taken into account in order to compare expected revenue or to design optimal auctions in those situations.

These considerations have motivated a growing literature on auctions with costly and endogenous entry.[2] Independent-private-value (IPV) auctions with costly entry often result in inefficient allocation, since the auction is typically conducted among the actual bidders, which is a subset of all potential bidders. If the bidder with the highest value is excluded from the auction, the auction outcome is necessarily inefficient *ex post*.

This sort of inefficiency may create a motive for post-auction resale and such resale opportunity affects significantly, as we shall show, bidders' bidding behavior and entry strategies. In this paper, we study the effect of allowing resale opportunity in an auction model where each potential bidder possesses two-dimensional private information signals: entry cost ($c$) and valuation ($v$). When the opportunity of resale is absent, this framework is first analyzed by Green and Laffont (1984), who demonstrate that under a Vickrey auction the entry equilibrium is characterized by a unique entry cutoff curve (or entry indifference curve) $C(\cdot)$ such that a bidder with type $(c, v)$ enters the auction if and only if $c \leq C(v)$. In such an equilibrium, it is possible that a bidder with a high value chooses not to enter the auction, simply because her entry cost is above her entry cutoff.[3] In a general

[1] This is the case, for example, in most of the sealed-bid procurement auctions.

[2] See, among others, French and McCormick (1984), Green and Laffont (1984), Samuelson (1985), McAfee and McMillan (1987), Tan (1992), Engelbrecht-Wiggans (1993), Levin and Smith (1994), Stegeman (1996), Tan and Yilankaya (2006), Ye (2004, 2007), Lu (2010), Lu and Sun (2010), and Moreno and Wooders (2011). Also see Bergemann and Välimäki (2006) for an extensive survey of the literature.

[3] Following Green and Laffont's analysis, Lu and Sun (2010) study entry implementability and derive *ex ante* efficient auctions when bidders own two-dimensional private information on values and entry costs.

procurement setting allowing for correlations between participation costs and production costs, Gal et al. (2007) establish the existence and uniqueness of the equilibrium entry cutoff curve.

Introducing post-auction resale into models with costly entry enriches the analysis, yet complicates matters so that the existence and uniqueness of entry equilibrium are no longer obvious. The additional difficulty emerges since, with resale, the option value for staying out is also positive and varies with types. To make our analysis tractable, we assume that a bidder's entry cost can only be either high ($c_H$) or low ($c_L$) in this paper. With such a modeling simplification, we are able to show that with resale, a symmetric entry equilibrium is characterized by entry thresholds ($v_L$, $v_H$), where $v_H > v_L$, so that bidders with entry cost $c_L$ (henceforth often referred to as type-$L$ bidders) will enter the auction if and only if their values $v \geq v_L$, and bidders with entry cost $c_H$ (henceforth often referred to as type-$H$ bidders) will enter the auction if and only if their values $v \geq v_H$. We show that while the opportunity of resale only affects the entry threshold for type-$H$ bidders, it affects both the entry threshold and bidding for type-$L$ bidders. More specifically, type-$L$ bidders tend to bid more aggressively with the resale opportunity, and the lower the value, the higher the magnitude of overbidding above value (the *competition effect* induced by the resale opportunity).

While the symmetric entry equilibrium is unique in the no-resale benchmark, the uniqueness cannot be established when resale is allowed. In the case with multiple equilibria, we show that the equilibria can be nicely ranked in the increasing order of $v_L$, or equivalently, in the decreasing order of $v_H$. We also identify sufficient conditions that assure the uniqueness of the symmetric entry equilibrium.

We compare the symmetric entry equilibrium when resale is allowed to the equilibrium when resale is banned. Our first finding is that with resale, the entry cutoff is higher for bidders with higher entry costs (type-$H$ bidders), and lower for bidders with lower entry costs (type-$L$ bidders). This suggests that when resale is allowed, bidders with low entry costs are more likely to enter, while bidders with high entry costs are less likely to enter. In other words, resale naturally induces a *speculative* motivation for entry and a *resale-hunting* motivation for staying out. Speculators are those with low entry costs and low valuations who would not enter without resale, but enter when resale opportunity is available. Resale hunters are those with high entry costs and high valuations who would refrain from entering when a resale market is available, but enter when resale is unavailable. Thus, although we may be tempted to think that the additional opportunity to trade in the resale market provides an additional motive to enter, the opposite incentive to avoid entry cost by staying out and buying in the resale market may more than offset it. That high-valued type-$H$ bidders are replaced by low-valued type-$L$ bidders can be referred to as the *displacement effect* induced by the resale opportunity.

In our model, it is difficult to work out a closed-form solution for the entry equilibrium with a general value distribution. In order to understand how resale opportunities affect entry, expected revenue, and efficiency, we turn to numerical computations. Our findings suggest that, while resale increases expected entry, its effects on expected revenue and efficiency are both ambiguous. That resale has an ambiguous effect on expected revenue has been well-documented in other auction settings with resale (e.g., Haile, 2003 and Pagnozzi, 2007), and it can be explained in our setting based on the competitive effect and the displacement effect induced by resale opportunities. The ambiguous effect of resale on efficiency, however, is less trivial and somewhat counter-intuitive given that bidders are symmetric in our setting.[4]

The literature on auctions with resale is relatively new. Gupta and Lebrun (1999) consider first-price auctions with resale. Haile (2003) considers an IPV setup in which bidders only have noisy signals at the auction stage, where the motive for resale arises when the true value of the auction winner turns out to be low. Zheng (2002) identifies conditions under which the outcome of optimal auctions can be achieved with resale. Garret and Tröger (2006) consider a model with a speculator, who can only benefit from participating in the auction when she can resell the item to the other bidder. Pagnozzi (2007) demonstrates why a strong bidder may prefer to drop out of an auction before the price reaches her valuation, simply because she anticipates that she will be in an advantageous position in the post-auction resale stage. Hafalir and Krishna (2008) analyze auctions with resale in an asymmetric IPV auction environment with two bidders and show that the expected revenue is higher under a first-price auction than under a second-price auction. In a follow-up study, Hafalir and Krishna (2009) focus on some distribution families where the equilibrium bidding functions can be solved in closed forms. They show that resale increases the original seller's revenue, but the effect of resale on social surplus is ambiguous.[5] Finally, Garratt et al. (2009) show that when resale is allowed, the English ascending auction is susceptible to tacit collusion. Our paper contributes to the

[4] The ambiguous effect of resale on efficiency is much easier to understand in asymmetric auctions with resale, as demonstrated by Krishna and Hafalir (2009).

[5] The intuition is as follows: the weak bidder bids more aggressively with resale, which hurts efficiency; on the other hand, resale helps restore efficiency after the auction. The net effect depends on the specific distribution or parameter values.

literature by being the first to integrate and analyze both entry and resale in an auction model with two-dimensional private information, an arbitrary number of bidders, and an incomplete information setting for the resale stage.[6]

The paper is organized as follows. Section 2 lays out the model. Section 3 characterizes the equilibrium. Section 4 is devoted to comparisons of the equilibria in the two cases. Section 5 discusses some assumptions and extensions in our analysis, and Second 6 concludes.

# 2 The Model

There is a single, indivisible object for sale to $n \geq 2$ potentially interested buyers (firms) through an auction. The seller's valuation is normalized to 0. It is costly for a bidder to participate in the auction. So, unlike most auction models, in our model each bidder possesses two-dimensional private information about her participation cost ($c$) and value ($v$). We assume that $c = c_L$ with probability $q$ and $c = c_H$ with probability $1 - q$, and $0 < c_L < c_H < 1$;[7] $v$ is distributed according to the cumulative distribution function $F(\cdot)$ on $[0, 1]$. We assume that for any $a \in (0, 1]$, the truncated distribution of $F(\cdot)$ on $[0, a]$ satisfies the monotonic hazard rate property, that is, $[F(a) - F(x)] / f(x)$ strictly decreases in $x$ for $x \in [0, a]$. We assume that a second-price sealed-bid auction is conducted, which is without loss of generality as we will show that revenue equivalence holds among all the standard auctions in our setting. For simplicity of analysis, we assume that the seller does not set a reserve price other than his own reservation value 0. After the auction, the winner can conduct a post-auction resale.[8] In the resale, we assume that the winner of the item from the initial auction (the reseller) will conduct an optimal auction (with an optimal reserve price) to resell the item.

The seller makes the selling mechanism publicly known before participation (or entry) occurs. After learning the realizations of their private entry costs and values, the potential bidders make entry decisions simultaneously and independently. After entry, the bidders observe the number of other entering bidders and then bid for the item. We will consider the two scenarios when resale is

[6] In Gupta and Lebrun (1999) and Pagnozzi (2007), complete information is assumed in the resale stage.

[7] When $c_L = c_H = c$, resale never occurs in the symmetric monotone entry equilibrium we will focus on, hence is omitted in our analysis.

[8] There is no obvious reason to believe that the heterogeneous entry costs that we envision also apply to the resale. We thus assume that the entry costs for the resale market are the same. We further assume that this cost is small, and is hence omitted. We discuss the implications of this assumption in Section 5.

allowed and not allowed.

# 3 Equilibrium Characterizations

In this section we characterize the equilibrium in the no-resale benchmark case and in the case when resale is allowed. We will focus on the symmetric entry equilibrium characterized by a pair of entry thresholds, ($\tilde{v}_L$, $\tilde{v}_H$) in the case when resale is banned and ($v_L$, $v_H$) in the case when resale is allowed, so that a bidder with a type $(c_k, v)$ enters the auction if and only if $v \geq \tilde{v}_k$ in the no resale benchmark and $v \geq v_k$ in the case with resale, $k \in \{L, H\}$; in other words, bidders with the same entry cost type will follow the same entry threshold in a symmetric entry equilibrium.

## 3.1 Entry without Resale

In this section we analyze the benchmark case when resale is banned. Since bidders are *ex ante* symmetric, without loss of generality we can focus on bidder $i$'s entry strategy. To show that ($\tilde{v}_L$, $\tilde{v}_H$) characterizes a symmetric entry equilibrium, we need to verify that there is no incentive for bidder $i$ to deviate from the prescribed entry strategy given that all the other bidders follow the entry thresholds ($\tilde{v}_L$, $\tilde{v}_H$).

First, given that $0 < c_L < c_H < 1$, we can rule out both $\tilde{v}_L = 0$ and $\tilde{v}_H = 0$ (otherwise bidders will incur net losses with realized values lower than their entry costs). We can also rule out the case $\tilde{v}_L = \tilde{v}_H = 1$ (otherwise a single bidder with a value higher than her entry cost would find it profitable to deviate to enter the auction). We next show that in any symmetric entry equilibrium, $0 < \tilde{v}_L < \tilde{v}_H \leq 1$. Suppose in negation we have $1 \geq \tilde{v}_L > \tilde{v}_H > 0$ or $1 > \tilde{v}_L = \tilde{v}_H > 0$. But then a bidder with type $(c_L, \tilde{v}_H)$ who is supposed to stay out (or be indifferent between entry and staying out) would be strictly better off by entering, as a bidder with $(c_H, \tilde{v}_H)$ is indifferent between entry and staying out, a contradiction. Therefore, given that $0 < c_L < c_H < 1$, it must be the case that $0 < \tilde{v}_L < \tilde{v}_H \leq 1$ in any symmetric entry equilibrium.

When all the other bidders follow the pair of entry thresholds ($\tilde{v}_L$, $\tilde{v}_H$), the ex ante probability that a bidder enters the auction is given by:

$$\tilde{p}_E = 1 - qF(\tilde{v}_L) - (1 - q)F(\tilde{v}_H).$$

Let $\tilde{F}_{in}(x)$ be the probability that a bidder has a value less than $x$ conditional on entry. Then $\tilde{F}_{in}(x) = q[F(x) - F(\tilde{v}_L)]/\tilde{p}_E$ when $x \in [\tilde{v}_L, \tilde{v}_H)$ and $\tilde{F}_{in}(x) = [F(x) - qF(\tilde{v}_L) - (1 - q)F(\tilde{v}_H)]/\tilde{p}_E$

when $x \in [\tilde{v}_H, 1]$. Let $\tilde{f}_{in}(\cdot)$ be its associated density function. Similarly, let $\tilde{F}_{out}(x)$ be the probability that a bidder has a value less than $x$ conditional on staying out. Then $\tilde{F}_{out}(x) = F(x)/(1-\tilde{p}_E)$ when $x \in [0, \tilde{v}_L]$ and $\tilde{F}_{out}(x) = [qF(\tilde{v}_L) + (1-q)F(x)]/(1-\tilde{p}_E)$ when $x \in [\tilde{v}_L, \tilde{v}_H]$. Let $\tilde{f}_{out}(\cdot)$ be its associated density function.

We assume that each bidder plays the (weakly) dominant strategy to bid her value after entry.[9] We use $\tilde{\pi}(v, m, \tilde{v}_L, \tilde{v}_H)$ to denote a bidder's expected payoff conditional on entry, given her value $v$, the total number of entrants $m$, and the pair of entry thresholds followed by all the other bidders $(\tilde{v}_L, \tilde{v}_H)$. Then

$$\tilde{\pi}(v, m, \tilde{v}_L, \tilde{v}_H) = \int_{\tilde{v}_L}^{v} (v-x)\tilde{f}^{(1)}_{in,m-1}(x)dx = \int_{\tilde{v}_L}^{v} \tilde{F}^{(1)}_{in,m-1}(x)dx,$$

where $\tilde{f}^{(1)}_{in,m-1}(x)$ and $\tilde{F}^{(1)}_{in,m-1}(x)dx$ are the density function and cumulative distribution function, respectively, for the highest value possessed by all the other $m-1$ entrants.

Given that everyone else follow the symmetric entry thresholds, the number of other entrants follows a binomial distribution; that is, $\Pr(m-1$ entrants out of $n-1$ other bidders$) = C_{n-1}^{m-1}(\tilde{p}_E)^{m-1}(1-\tilde{p}_E)^{n-m}$. Therefore, if we use $\tilde{\pi}(v, \tilde{v}_L, \tilde{v}_H)$ to denote the expected total payoff conditional on entry, we have

$$\tilde{\pi}(v, \tilde{v}_L, \tilde{v}_H) = \sum_{m=1}^{n} C_{n-1}^{m-1}(\tilde{p}_E)^{m-1}(1-\tilde{p}_E)^{n-m}\tilde{\pi}(v, m, \tilde{v}_L, \tilde{v}_H).$$

It can be easily verified that given $(\tilde{v}_L, \tilde{v}_H)$, $\tilde{\pi}(v, \tilde{v}_L, \tilde{v}_H)$ strictly increases in $v$. Therefore, given that other bidders follow the entry thresholds $(\tilde{v}_L, \tilde{v}_H)$, no bidder has an incentive to deviate from following the same entry thresholds.

**Proposition 1** *In the benchmark case when resale is banned, there exists a unique symmetric entry equilibrium characterized by the entry thresholds $(\tilde{v}_L, \tilde{v}_H)$.*

**Proof.** See Appendix. ■

In particular, if the values of $(c_L, c_H)$ induce a pair of interior entry thresholds $(\tilde{v}_L, \tilde{v}_H)$, i.e., $0 < \tilde{v}_L < \tilde{v}_H < 1$, we must have the following indifference conditions:

$$\begin{cases} \tilde{\pi}(\tilde{v}_L, \tilde{v}_L, \tilde{v}_H) - c_L = 0 \\ \tilde{\pi}(\tilde{v}_H, \tilde{v}_L, \tilde{v}_H) - c_H = 0 \end{cases} \tag{1}$$

[9] Note that participation costs are sunk costs, which should not affect the bidding strategies (absent the resale opportunity).

That is, a bidder with an entry cost $c_L$ ($c_H$) will be indifferent between entering the auction and staying out if her value is $\tilde{v}_L$ ($\tilde{v}_H$).[10]

## 3.2 Entry with Resale

We now augment the model just analyzed by allowing a resale stage where the auction winner may resell the item. Similarly to the benchmark case without resale, we will focus on symmetric entry equilibria characterized by entry thresholds ($v_L$, $v_H$).

A bidder's ex ante probability of entry, $p_E$, the value distribution function conditional on entry, $F_{in}(\cdot)$, and the value distribution function conditional on staying out, $F_{out}(\cdot)$, are all defined analogously as in the no-resale case.

With resale opportunities, given that $0 < c_L < c_H < 1$, we can again rule out the case $v_L = v_H = 1$. However, unlike in the case without resale, now either $v_L = 0$ or $v_H = 0$ (but not both) is possible, given that the additional value from the opportunity of resale may more than offset the entry cost. Following a similar argument as in the no-resale case, we can conclude that in any symmetric entry equilibrium, $0 \leq v_L < v_H \leq 1$.

To characterize symmetric entry equilibria, we start our analysis from the last stage. Given the special features in our model, it is easily seen that in equilibrium, the initial auction winner can only possibly benefit from resale when her value $v_w \in [v_L, v_H)$. So resale can only be initiated when some type-$L$ bidder wins the initial auction, and the potential buyers participating in resale must be type-$H$ bidders with values in $[v_L, v_H]$ who stay out of the initial auction.[11] When resale is initiated, the resale mechanism will be announced (we assume that the reseller uses an optimal auction *a la* Myerson); in particular, the reserve price will be announced. Following the announcement, the potential buyers will come to the resale and participate in bidding.

So, in our model, resale provides an opportunity for bidders with low value and low cost to speculate: they can sell the item to a bidder with higher values who also has higher entry cost. In the resale, the seller sets an optimal reserve price to maximize her expected payoff, with the belief that potential buyers' values follow the distribution $F_{out}(\cdot)$. In equilibrium, a trade in resale occurs

[10] If $(c_L, c_H)$ cannot induce an interior solution to the equation system (1), it has to be the case that $\tilde{v}_L \in (0, 1)$ and $\tilde{v}_H = 1$. Note that $\tilde{v}_L = 0$ is impossible as we assume that $c_L > 0$.

[11] Note that resale will never be initiated in equilibrium whenever some bidder has a value higher than $v_H$, as the initial auction winner must possess the highest value among all the $n$ potential buyers.

only when the outside bidder with the highest value can meet the reserve price.

Define $w(v, m, v_L, v_H)$ to be the reseller's expected gross payoff from resale, given her value $v$, the number of entrants (including herself) $m$, and the entry thresholds that other bidders follow ($v_L$, $v_H$). We have

$$w(v, m, v_L, v_H) = \max_{r \in (v, v_H]} [F_{out}(r)]^{n-m} v + C^1_{n-m}[1 - F_{out}(r)][F_{out}(r)]^{n-m-1} r + \int_r^{v_H} x f^{(2)}_{out,n-m}(x) dx. \tag{2}$$

The first term reflects the reseller's payoff when no bidder bids higher than the reserve price, the second term reflects the reseller's payoff when only one bidder bids higher than the reserve price, and the third term reflects the reseller's payoff when at least two bidders bid above the reserve price.

The optimal reserve price, $r(v)$, is the solution to equation $x = v + (1 - F_{out}(x)) / f_{out}(x)$. When the truncated distribution of $F(\cdot)$ on $[0, a]$, $a \leq 1$, satisfies the monotone hazard rate property, there is only one solution to this equation.

Next we consider a bidder (with a value $v$) who stays out of the auction but participates in the resale. Such a bidder can only have a positive expected payoff when $v > r(v_L)$, as $r(v_L)$ is the lowest possible reserve price in the resale. Let $y$ be the highest value among the other $n - m - 1$ bidders in the resale and $z$ be the reseller's value (the highest value among $m$ entrants). Then this outsider's expected payoff from resale is given by:

$$L(v, m, v_L, v_H) = \begin{cases} 0 & \text{if} \quad v \leq r(v_L) \\ \int_{v_L}^{r^{-1}(v)} \int_{r(z)}^{v} (v - y) f^{(1)}_{out,n-m-1}(y) f^{(1)}_{in,m}(z) dy dz & \\ + \int_{v_L}^{r^{-1}(v)} \left[ \int_0^{r(z)} f^{(1)}_{out,n-m-1}(y) dy \right] [v - r(z)] f^{(1)}_{in,m}(z) dz & \text{if} \quad v > r(v_L) \end{cases}$$

Define

$$\beta(v, m, v_L, v_H) = \begin{cases} w(v, m, v_L, v_H) & \text{if} \quad v \leq v_H \\ v & \text{if} \quad v > v_H \end{cases} \tag{3}$$

Clearly, $\beta(v, m, v_L, v_H)$ is the expected payoff (gross of entry cost) conditional on winning the initial auction, taking into account the additional value coming from potential resale (for those who will initiate a resale auction with positive probability). Applying the envelope theorem in (2), we have

$$\frac{\partial w}{\partial v} = F_{out}(r(v))^{n-m} \tag{4}$$

Thus $w(\cdot, m, v_L, v_H)$ and hence $\beta(\cdot, m, v_L, v_H)$ given by (3) are strictly increasing. Below we will show that $\beta(\cdot, m, v_L, v_H)$ constitutes the unique equilibrium in the class of symmetric (strictly) increasing equilibria in the initial auction subgame, i.e., the (reduced) subgame after the entry decisions are made, with the resale stage being replaced by its correlated equilibrium payoffs.

**Proposition 2** *When resale is allowed, in the initial auction subgame there exists a unique symmetric (strictly) increasing equilibrium, in which entrant bidders follow the bid function $\beta(\cdot, m, v_L, v_H)$. Bidders bid above value for $v \in [v_L, v_H)$ and the magnitude of overbidding, $\beta(v, m, v_L, v_H) - v$, is decreasing in $v$.*

**Proof.** See Appendix. ∎

Given the specific information structure in our model, resale will never be initiated by a type-$H$ bidder, so, not surprisingly, resale has no effect on bidding for type-$H$ bidders. However, as suggested in Proposition 2, resale affects bidding for type-$L$ bidders. This is reflected by overbidding above value for those with valuations between $v_L$ and $v_H$ (who will possibly initiate resale after the initial auction). For those bidders, Proposition 2 shows that the lower the value, the more the magnitude of overbidding (or the stronger the effect of resale).

From Proposition 2, we can obtain the expected payoff for a bidder, denoted as $\pi(v, m, v_L, v_H)$, when she enters the auction with $m - 1$ other bidders:

$$\pi(v, m, v_L, v_H) = \begin{cases} \beta(v, 1, v_L, v_H) & \text{if} \quad m = 1 \\ \int_{v_L}^{v} \left[\beta(v, m, v_L, v_H) - \beta(x, m, v_L, v_H)\right] f_{in,m-1}^{(1)}(x) dx & \text{if} \quad m \geq 2 \end{cases}$$

Similarly as in the no-resale case, we can define the expected payoff for a bidder who enters the initial auction, $\pi(v, v_L, v_H)$, and the expected payoff for a bidder who stays out of the initial auction, $L(v, v_L, v_H)$.

$$\begin{aligned} \pi(v, v_L, v_H) &= \sum_{m=1}^{n} C_{n-1}^{m-1} (p_E)^{m-1} (1 - p_E)^{n-m} \pi(v, m, v_L, v_H) \\ L(v, v_L, v_H) &= \sum_{m=1}^{n-1} C_{n-1}^{m} (p_E)^{m} (1 - p_E)^{n-m-1} L(v, m, v_L, v_H) \\ &= \sum_{m=2}^{n} C_{n}^{m-1} (p_E)^{m-1} (1 - p_E)^{n-m} L(v, m - 1, v_L, v_H) \end{aligned}$$

It can be verified that given $(v_L, v_H)$, $\pi(v, v_L, v_H) - L(v, v_L, v_H)$ is strictly increasing in $v$, thus again the entry equilibria are characterized by entry thresholds.

**Proposition 3** *When resale is allowed, a symmetric entry equilibrium always exists, and is unique under certain conditions. In the case with multiple equilibria, for any two pairs of entry equilibrium thresholds $(v_{1L}, v_{1H})$ and $(v_{2L}, v_{2H})$ where $v_{1L} < v_{2L}$, it must be the case that $v_{1H} > v_{2H}$.*

**Proof.** See Appendix. ■

So in the case of multiple equilibria, there is a natural ordering of equilibria, which is in the increasing order of entry thresholds for type-$L$ bidders (or equivalently, in the decreasing order of entry thresholds for type-$H$ bidders).[12] In the proof we identify a necessary and sufficient condition for the uniqueness of the entry equilibrium. The condition is highly technical so we refer readers to the appendix for the details. For some limiting cases, we have identified the following sufficient conditions (each alone is sufficient to assure that the equilibrium is unique): (1) $c_L$ and $c_H$ are sufficiently close; (2) $q$ is sufficiently small or sufficiently large; (3) $n$ is sufficiently large; and (4) $c_L$ and $c_H$ are neither too close nor too far apart, while $n$ and $q$ are relatively small. Conditions (1) and (2) basically say that if bidders incur a common entry cost (or if the entry costs are almost the same), there will be no resale; hence, the entry equilibrium is also unique (a result consistent with Proposition 1). Condition (3) says that when $n$ is sufficiently large, some type-$H$ bidder or some type-$L$ bidder with a value higher than $v_H$ will win the item from the initial auction, which eliminates the possibility of resale. So the symmetric entry equilibrium will also be unique.

As in the benchmark case, for $c_L$ and $c_H$ to induce interior entry thresholds for both types, i.e., $0 < v_L < v_H < 1$, bidders with type $(v_L, c_L)$ or type $(v_H, c_H)$ should be indifferent between entering and staying out:

$$\begin{cases} \pi(v_L, v_L, v_H) - c_L & = 0 \\ \pi(v_H, v_L, v_H) - L(v_H, v_L, v_H) - c_H & = 0 \end{cases} \tag{5}$$

In our model, we assume that a second-price sealed-bid auction is conducted in the initial auction. This turns out to be without loss of generality, as in our setting the revenue equivalence holds among all standard auctions (in which the bidder with the highest bid wins the item, and the bidder with the lowest possible value earns zero expected payoff). The reason is that all standard auctions will induce the same entry thresholds, which guarantees the symmetric IPV setting for revenue equivalence to hold.

**Proposition 4** *Given that the reseller runs an optimal auction in the resale stage, revenue equivalence holds among all standard auctions conducted in the initial auction in the symmetric (entry and bidding) equilibrium.*

[12] This, we conjecture, may have to do with motives for speculative entry and resale hunting abstentions to be introduced later. A lower $v_L$ implies more speculative entry, and a higher $v_H$ implies more resale hunting abstentions. Thus, it seems intuitive that if $v_L$ decreases, $v_H$ should increase.

**Proof.** See Appendix. ∎

# 4 Resale versus No Resale: A Comparison

In general, it is very difficult to obtain closed-form solutions for the entry equilibrium in both the no-resale and resale cases; nonetheless we can establish the following comparison results.

**Proposition 5** *Given any equilibrium entry thresholds ($v_L$, $v_H$) when resale is allowed, we must have $\tilde{v}_L \geq v_L$ and $\tilde{v}_H \leq v_H$; in addition, if $(c_L, c_H)$ induces at least one pair of interior equilibrium thresholds in the no-resale and resale cases, we must have $\tilde{v}_L > v_L$ and $\tilde{v}_H < v_H$.*

**Proof.** See Appendix. ∎

While resale affects bidding only for type-$L$ bidders, it affects entry thresholds for both types. Proposition 5 suggests that when resale is allowed, the entry threshold is lower for type-$L$ bidders but higher for type-$H$ bidders. In other words, when resale is allowed, it is more likely for the low-cost type to enter the auction, while it is more likely for the high-cost type to stay away from the auction. For the low-cost type, bidders with $v \in (v_L, \tilde{v}_L)$ can be referred to as *entry speculators*, since they enter the auction simply because the resale opportunity is available. On the other hand, for the high-cost type, bidders with $v \in (\tilde{v}_H, v_H)$ can be referred to as *resale hunters*, as they stay out only because the resale opportunity is available and they might be able to obtain the item from the post-auction resale.

Closed-form solutions are generally unavailable in our model. In order to understand how resale opportunities affect entry, revenue, and efficiency, we turn to numerical computations by following a generalized uniform distribution family, $F(x) = x^{\alpha}$, where $\alpha \geq 1$. Our main numerical findings suggest that expected entry is higher when resale is allowed, but the effects of resale on expected revenue and efficiency are both ambiguous. So while resale induces more entry by type-$L$ bidders and less entry by type-$H$ bidders, overall expected entry increases with resale in our numerical examples. The ambiguous effect of resale on expected revenue can be explained intuitively in our setting. As implied in Propositions 2 and 5, resale induces the *competition effect* (type-$L$ bidders bid more aggressively) and *displacement effect* (high-valued type-$H$ bidders are replaced by low-valued type-$L$ bidders). While the competition effect pushes up expected revenue, the displacement effect works in the other direction. Thus, the net impact of resale on expected revenue depends on which effect dominates, and either effect may dominate in our setting. The ambiguous effect of resale on efficiency, however, goes against the notion that resale should help correct the inefficiency that resulted from the initial auction (although the correction may not be complete due to a nontrivial reserve price used in the resale auction). Such a notion is generally correct in settings with a fixed set of (symmetric) bidders, but fails in our setting where the set of bidders is endogenously determined by costly entry. The reason is that, while resale tends to increase the expected value realized from the sale (conditional on entry), the set of entrants with resale is not necessarily optimal in maximizing the efficiency measure in our setting (i.e., the expected value of the final item owner less the expected total entry cost incurred). This, we believe, has to do with the displacement effect induced by resale opportunities. Mainly due to the displacement effect, the expected value of the final item owner may go down with resale, making it possible that welfare may also go down. This intuition is basically confirmed in our identified cases where the welfare goes down with resale. In those cases, both the expected value and expected entry cost go down, but the reduction in the expected value is more than the savings in the expected entry cost, giving rise to lower efficiency when resale is allowed.

## 5 Discussion

Our analysis relies on several key assumptions. First, we assume that there is no participation cost for resale. The omission of entry costs for resale would be easier to justify in the contexts where the participation cost for resale is on a much smaller scale compared to the entry cost for the initial auction. This would be the case, for example, when the initial auction is conducted by some public authority that usually imposes stringent qualification requirements for bidding, while the resale is organized by some private parties, which is presumably more decentralized and less costly to participate.

This being said, we assume away the entry cost for resale, $c_R$, mainly for tractability; otherwise our analysis can easily get very involved. With $c_R > 0$, all the endogenously determined variables, including the equilibrium entry thresholds to the initial auction $v_L$ and $v_H$, will also depend on $c_R$. The other complication is that the resale mechanism is now an auction with costly entry as well. Since optimal auctions with costly entry are typically different from optimal auctions *a la* Myerson, the optimal reserve price set by the reseller should not only take into account the traditional tradeoff between efficiency and rent extraction, but should also induce the optimal entry, which is characterized by the entry threshold for the resale, $v_R$. To see how complicated the analysis of the resale subgame would be, note that $v_R$ depends on, in particular, $v_w$, the reseller's value, and $m$, the number of the entrants in the initial auction.[13]

Other than the technical complications, with costly entry to resale, the entry equilibrium should still be characterized by entry thresholds, and the motives for speculative entry and resale hunting should still be present. For this reason we believe that the main insights of our analysis should carry over, even if one can manage to work out a model with costly entry to resale.

Our analysis has also been focusing on symmetric entry equilibria, in which bidders of the same entry cost type follow the same entry threshold. As first demonstrated by Samuelson (1985) and Stegeman (1996), asymmetric entry equilibria, in which bidders of the same entry cost type follow different entry thresholds, may also exist.[14] While the previous work focuses on the case in which all the bidders have the same (and publicly known) entry cost, we focus on the case in which entry costs are private information and may differ among different bidders. However, the essence of their analysis should carry over, and we believe that asymmetric entry equilibria should also exist in our setting.

In the presence of multiple equilibria, coordination on entry is desirable. In fact, Stegeman (1996) shows that the second-price auction achieves the highest efficiency level as long as bidders can coordinate on an asymmetric entry equilibrium. Campbell (1998) further demonstrates that, if preplay communication is possible, bidders' expected payoffs can be improved in some cheap-talk equilibrium. We believe that pre-communication should be particularly effective for our case with multiple symmetric entry equilibria. For ease of illustration, we consider a simplest case where there are only two potential bidders, $n = 2$. Suppose there are two entry equilibria, characterized by $(v_{1L}, v_{1H})$ and $(v_{2L}, v_{2H})$, where, without loss of generality, $v_{1L} < v_{2L} < v_{2H} < v_{1H}$ (Proposition 3). Suppose the realized types are $(c_L, v_1)$ and $(c_H, v_2)$, where $v_{2L} < v_2 < v_{2H}$ and $v_{1L} < v_1 < v_{2L}$. If both bidders coordinate on the second equilibrium, no one enters and the payoffs are both zero. However, if bidder 2 signals that she will not enter in a preplay cheap-talk game (which is credible as doing so never hurts and can only benefit), then bidder 1 will enter for sure (who will then win the auction and resell the item to bidder 2). Clearly, in this example cheap talk allows bidders to coordinate on entry and secure the best outcome. As illustrated by this simple example, the resale

[13] When resale is initiated, the reseller announces a reserve price (based on $v_w$ and $m$). The potential buyers will then make their entry decisions simultaneously and independently.

[14] Following Stegeman (1996), conditions for the existence of asymmetric entry equilibria are identified by Campbell (1998) and Tan and Yilankaya (2006) under a second-price auction, and by Cao and Tian (2010) under a first-price auction.

opportunities reinforce the effectiveness of the type of cheap talk as proposed in Campbell (1998).[15]

We assume that entry costs and valuations are statistically independent. So, we do not allow for correlations between bidders' two-dimensional types. One justification is that there does not seem to be a consensus in the literature over whether the values and entry costs should be positively or negatively correlated. But if for some reason we believe that valuations and entry costs are positively correlated then this may be captured by a model where the values of type-$H$ bidders are drawn from $F(\cdot|c = c_H)$ and the values of type-$L$ bidders are drawn from $F(\cdot|c = c_L)$, where $F(\cdot|c = c_H)$ first-order stochastically dominates $F(\cdot|c = c_L)$. Such an environment should make a stronger case for our main results as it will strengthen both the speculative and resale hunting incentives.

Finally, a weakness of our analysis is that we fail to obtain analytical welfare comparison results. One may wonder that this intractability is due to the complication of maintaining incomplete information for the resale stage. With complete information at the resale stage, however, we can employ the standard Nash bargaining solution to determine the resale outcome, which may potentially simplify our welfare analysis. In Xu et al. (2010), we provide this very analysis by modeling resale under complete information. In addition, we further simplify the analysis by focusing on two bidders only ($n = 2$). With $n = 2$, it is clear that resale only affects entry but not bidding strategies.[16] Thus, in a sense, Xu et al. offer the simplest possible framework that models resale under complete information. With all those simplifications, however, we still fail to obtain analytical welfare results. Besides, given that resale is conducted under complete information and that the Nash bargaining solution is employed, there is no efficiency loss in the resale stage, which may potentially bias the efficiency comparison. This is likely the case as all the numerical examples in Xu et al. suggest that welfare can only go up with resale. Given that modeling resale as complete information does not really help with analytical welfare results and may potentially bias the welfare comparison, the incomplete information model of resale is adopted in this present paper, as it is more realistic and renders more complete picture of the welfare effects of the resale.

[15] We are not aware of any existing work on communicating entry costs via cheap talk. A closely related idea is to run a pre-auction mechanism to elicit truthful reports on entry costs, and the set of final bidders are then determined contingent on the reported entry cost types. Lu and Ye (2012) provide such an analysis in a model where bidders incur private entry costs before they learn their valuations.

[16] When both bidders enter the auction, the outcome will be efficient so there will be no resale; when only one bidder enters the auction, there will be no competition in bidding. In either case, bidding is not affected by the opportunity of resale.

# 6 Conclusion

In this paper we explicitly take into account the possibility of post-auction resale in an auction model with entry where bidders possess two-dimensional private information. We demonstrate that the symmetric entry equilibrium is characterized by entry thresholds, and we identify conditions under which such an equilibrium is unique. We show that compared to the no-resale case, resale opportunities make entry more likely for bidders with low entry costs and low values, and less likely for bidders with high entry costs and high values. In other worlds, resale introduces motivation for both speculative entry and resale hunting abstentions.

When valuations follow some generalized uniform distributions, our numerical results suggest that allowing resale always increases expected entry, but the effects on expected revenue and efficiency are both ambiguous. The implication is that a market regulator, whose objective is to maximize the social surplus, should exercise caution in suggesting whether or not resale should be permitted in auction settings similar to what is under our consideration.

Our analysis relies on several key assumptions. We have discussed the restrictions and argued that simplifications are made mainly for ease of analysis, without compromising main results and main insights of our analysis. Relaxation of those assumptions to allow for a more general analysis on entry and resale is left for future research.

# Appendix

**Proof of Proposition 1**: We will first consider the case with interior entry thresholds ($0 < v_L < v_H < 1$) and then the case with corner solution ($0 < v_L < v_H = 1$).

Define $\tilde{Y} = \{(c_L, c_H) : \tilde{\pi}(x_1, x_1, x_2) = c_L \text{ and } \tilde{\pi}(x_2, x_1, x_2) = c_H \text{ for some } x_1, x_2 \text{ where } 0 < x_1 < x_2 < 1\}$. That is, $\tilde{Y}$ is the set of all pairs of $(c_L, c_H)$ that can induce interior entry thresholds. When $(c_L, c_H) \in \tilde{Y}$, it is obvious that any $(\tilde{v}_L, \tilde{v}_H)$ that solves the equation system (1) is a pair of symmetric equilibrium entry thresholds because given $(\tilde{v}_L, \tilde{v}_H)$, the function $\tilde{\pi}(v, \tilde{v}_L, \tilde{v}_H)$ is strictly increasing in $v$. Next we show the uniqueness of equilibrium for this case. We can rewrite the equation system (1) as follows:

$$\begin{aligned} \tilde{\pi}(x_1, x_1, x_2) &= [qF(x_1) + (1-q)F(x_2)]^{n-1} x_1 = c_L \\ \tilde{\pi}(x_2, x_1, x_2) - \tilde{\pi}(x_1, x_1, x_2) &= \int_{x_1}^{x_2} [qF(z) + (1-q)F(x_2)]^{n-1} dz = c_H - c_L \end{aligned}$$

It can be easily verified that the signs of the first-order partial derivatives are given below:

$$\begin{aligned} \partial\tilde{\pi}(x_1,x_1,x_2)/\partial x_1 &> 0,\ \partial\tilde{\pi}(x_1,x_1,x_2)/\partial x_2 > 0, \\ \partial[\tilde{\pi}(x_2,x_1,x_2)-\tilde{\pi}(x_1,x_1,x_2)]/\partial x_1 &< 0,\ \partial[\tilde{\pi}(x_2,x_1,x_2)-\tilde{\pi}(x_1,x_1,x_2)]/\partial x_1 > 0. \end{aligned}$$

Let $\mathring{D} = \{(x_1,x_2) : 0 < x_1 \leq x_2 < 1\}$. Applying the implicit function theorem, there exists a unique, continuously differentiable function, $\phi(\cdot,c_L) : (0,1) \to (0,1)$, such that

$\{(x_1,\phi(\cdot,c_L)) : x_1 \in (0,1)\} = \{(x_1,x_2) \in \mathring{D} : \tilde{\pi}(x_1,x_1,x_2) = c_L\}$. In $\mathring{D}$, $\phi(x_1,c_L)$ is strictly decreasing in $x_1$. Similarly, there exists a unique, continuously differentiable function, $\psi(\cdot,c_H)$ : $(0,1) \to (0,1)$, such that

$\{(x_1,\psi(\cdot,c_H)) : x_1 \in (0,1)\} = \{(x_1,x_2) \in \mathring{D} : \tilde{\pi}(x_2,x_1,x_2) = c_H\}$ and $\psi(x_1,c_H)$ is strictly increasing in $x_1$.

To solve the equation system (1), we can solve from $\tilde{\pi}(\phi(x_1,c_L),x_1,\phi(x_1,c_L)) = c_H$ for $x_1$ first, and then obtain $x_2$ from $x_2 = \phi(x_1,c_L)$. Based on the signs of all the first-order partial derivatives, we have

$$\frac{d\tilde{\pi}(\phi(x_1,c_L),x_1,\phi(x_1,c_L))}{dx_1} = \frac{\partial\tilde{\pi}(x_2,x_1,x_2)}{\partial x_1}|_{x_2=\phi(x_1,c_L)} + \frac{\partial\tilde{\pi}(x_2,x_1,x_2)}{\partial x_2}|_{x_2=\phi(x_1,c_L)} \cdot \frac{\partial\phi(x_1,c_L)}{\partial x_1} > 0.$$

Therefore, as long as there exists at least one solution $x_1^*$ to the equation $\tilde{\pi}(\phi(x_1,c_L),x_1,\phi(x_1,c_L)) = c_H$, the solution is unique. A unique $x_1^*$ also induces a unique $x_2^*$ because $\partial\phi(x_1,c_L)/\partial x_1 < 0$.

Now we consider the case $(c_L,c_H) \notin \tilde{Y}$, where entry thresholds involve some corner solution. In this case, the following conditions must hold: $\tilde{\pi}(\tilde{v}_L,\tilde{v}_L,1) = c_L$, $\tilde{\pi}(1,\tilde{v}_L,1) \leq c_H$, and $0 \leq \tilde{v}_L < 1$. That is, all type-$H$ bidders prefer staying out, and no bidder has an incentive to deviate from the entry thresholds $(\tilde{v}_L,1)$ (as given $\tilde{v}_L$, $\tilde{\pi}(v,\tilde{v}_L,1)$ is strictly increasing in $v$).

The existence of $\tilde{v}_L$ in this case is obvious given that $\tilde{\pi}(0,0,1) = 0 < c_L$ and $\tilde{\pi}(1,1,1) = 1 > c_L$. The uniqueness is also trivial as the solution of $\tilde{v}_L$ is unique given that $\partial\tilde{\pi}(x_1,x_1,x_2/\partial x_1 > 0$. ■

**Proof of Proposition 2**: First we show that $\beta(\cdot,m,v_L,v_H)$ constitutes a symmetric (strictly) increasing equilibrium bid function in the initial auction subgame. Given that everyone else bid according to $\beta(\cdot,m,v_L,v_H)$, we will show that there is no incentive for a bidder with value $v$ to deviate from bidding $\beta(v,m,v_L,v_H)$.

1. No incentive to overbid $\beta(v,m,v_L,v_H)$. Clearly, over-bidding affects the outcome only when the deviating bidder wins against some other bidder whose bid is higher than $\beta(v,m,v_L,v_H)$. Suppose that bidder's value is $u$. Then the deviating bidder's payment in the initial auction

is $\beta(u, m, v_L, v_H)$, and the value of $u$ is fully revealed to her (since $\beta(\cdot, m, v_L, v_H)$ is strictly increasing, $u$ can be inverted from the bid). The deviating bidder can first try to resell the item to the outsiders using an optimal auction. If the resale fails, she can sell the item to the bidder with value $u$ at the price $u$ (the overbidding opens the possibility of selling the item to a losing bidder). This procedure leads to an expected payoff of $\beta(u, m, v_L, v_H)$, which is exactly the payment she made to obtain the item from the initial auction. This shows that she does not find it profitable to overbid.

2. No incentive to underbid $\beta(v, m, v_L, v_H)$. First of all, bidding less than $\beta(v_L, m, v_L, v_H)$ is not profitable, as this amount is the lowest possible rival bid. Assume that this bidder mimics type $\hat{v}$, $v_L \leq \hat{v} < v$ and that $z$ is the value of the highest rival. Under-bidding affects the outcome only when $\hat{v} < z < v$, namely, when she loses while she is not supposed to.

   (a) When $z \geq v_H$, there is no resale. The bidder is worse off by deviating.

   (b) When $v_H \geq z \geq r^{-1}(\min\{v, v_H\})$, the deviant's contingent payoff from resale (as a buyer) will be 0 as $v \leq r(z)$ almost surely,[17] while her equilibrium payoff will be positive.

   (c) When $z < r^{-1}(\min\{v, v_H\})$, $w(z, m, v_L, v_H)$ is the highest rival's bid. We will argue that given any $z$, there is no benefit from deviation. Note that if the bidder in question does not deviate, the expected payoff conditional on entry should be $\beta(v, m, v_L, v_H) - w(z, m, v_L, v_H) \geq v - w(z, m, v_L, v_H)$. If she deviates, the expected payoff is:

$$\begin{aligned}
& [F_{out}(r(z))]^{n-m}(v - r(z)) + C^1_{n-m}[F_{out}(v) - F_{out}(r(z))][F_{out}(r(z))]^{n-m-1}[v - E(x|r(z) \leq x \leq v)] \\
& + \int_{r(z)}^{\min\{v, v_H\}} (v - x) f^{(1)}_{out,n-m}(x)dx \\
< \; & [F_{out}(r(z))]^{n-m}(v - z) + C^1_{n-m}[1 - F_{out}(r(z))][F_{out}(r(z))]^{n-m-1}(v - r(z)) \\
& + \int_{r(z)}^{v_H} (v - x) f^{(2)}_{out,n-m}(x)dx \\
= \; & v - w(z, m, v_L, v_H).
\end{aligned}$$

So there is no profitable deviation for underbidding either. This shows that $\beta(\cdot, m, v_L, v_H)$ is a symmetric equilibrium bid function in the initial auction subgame.

Next, we establish the uniqueness of the equilibrium in the claimed class. Given any symmetric (strictly) increasing equilibrium bid function $\tilde{\beta}(\cdot, m, v_L, v_H)$, we claim that $\tilde{\beta}(\cdot, m, v_L, v_H) \geq$

[17] If $v > v_H$, we have $v_H \geq z \geq r^{-1}(v_H) = v_H$, which holds with probability zero.

$\beta(\cdot, m, v_L, v_H)$. Suppose, in negation, $\tilde{\beta}(v, m, v_L, v_H) < \beta(v, m, v_L, v_H)$ for some $v \in (v_L, v_H)$. Then a bidder with such a value $v$ will be strictly better off by bidding $\beta(v, m, v_L, v_H)$ instead. To see this, the deviation only matters in the event that the bidder wins rather than loses the auction simply because of the deviation. When she wins with the bid $\beta(v, m, v_L, v_H)$, she pays an amount strictly less than $\beta(v, m, v_L, v_H)$. Since her expected payoff from resale is at least $\beta(v, m, v_L, v_H)$, the proposed deviation is profitable.[18] Thus we must have $\tilde{\beta}(\cdot, m, v_L, v_H) \geq \beta(\cdot, m, v_L, v_H)$. Now we show that $\tilde{\beta}(\cdot, m, v_L, v_H) = \beta(\cdot, m, v_L, v_H)$. Suppose, in negation, $\tilde{\beta}(v, m, v_L, v_H) > \beta(v, m, v_L, v_H)$ for some $v \in (v_L, v_H)$. But then a bidder with such a value $v$ will be strictly better off by bidding $\beta(v, m, v_L, v_H)$: this deviation only matters in the event that she loses rather than wins the auction simply because of the deviation. However, in that case she would have won with a payment higher than $\beta(v, m, v_L, v_H)$, which is her expected payoff from resale.[19] By deviating to $\beta(v, m, v_L, v_H)$ to lose the auction, her expected payoff is at least zero: with a positive probability, her value is higher than the winner's optimal reserve price in resale (since $\tilde{\beta}(\cdot, m, v_L, v_H) > \beta(\cdot, m, v_L, v_H)$). So the deviation is profitable. This shows that $\beta(v, m, v_L, v_H)$ is the unique symmetric (strictly) increasing equilibrium bid function in the initial auction.

Finally, note that $w(v_H, m, v_L, v_H) = v_H$ (when the initial auction winner's value is $v_H$, there is no scope for resale). Moreover, by (4), we have $\partial w / \partial v = F_{out}(r(v))^{n-m} < 1$. Thus, $w(v_H, m, v_L, v_H) > v$ for $v \in [v_L, v_H)$, and $w(v_H, m, v_L, v_H) - v$ is decreasing in $v$. ■

**Proof of Proposition 3**: Define $Y = \{(c_L, c_H) : \pi(x_1, x_1, x_2) = c_L$ and $\pi(x_2, x_1, x_2) - L(x_2, x_1, x_2) = c_H$ for some $x_1, x_2$ where $0 < x_1 < x_2 < 1\}$. That is, $Y$ is the set of all pairs of $(c_L, c_H)$ that can induce interior entry thresholds.

We first show that when $(c_L, c_H) \in Y$, any solution to the equation system (5) characterizes a pair of equilibrium thresholds for entry.

When a bidder's entry cost is $c_L$ and her value is lower than $v_L$, she does not benefit from entry because given that everyone else follows the entry thresholds $(v_L, v_H)$, her expected payoff from entry, $(1 - p_E)^{n-1} w(v, 1, v_L, v_H)$, is strictly increasing in $v$. When a bidder's entry cost is $c_L$ and her value is higher than $v_L$, her expected payoff from staying out, $L(v, v_L, v_H)$, is lower than her

[18] When the item is auctioned among the outsiders only, the expected payoff from resale is $\beta(v, m, v_L, v_H)$. Since $\tilde{\beta}(v, m, v_L, v_H) < \beta(v, m, v_L, v_H)$, the highest losing bidder from the initial auction may also demand the item. Hence the expected payoff is at least $\beta(v, m, v_L, v_H)$.

[19] If she bids $\tilde{\beta}(v, m, v_L, v_H)$ and wins, all the other entrant bidders have values lower than $v$ (as $\tilde{\beta}(\cdot, m, v_L, v_H)$ is strictly increasing). So the demand in the resale auction will only come from the outsiders.

equilibrium payoff, $\pi(v, v_L, v_H) - c_L$ (as $\pi(v, v_L, v_H) - L(v, v_L, v_H)$ is strictly increasing in $v$ and $\pi(v_L, v_L, v_H) - L(v_L, v_L, v_H) = c_L$). Similarly, bidders with entry cost $c_H$ do not have incentive to deviate from the entry threshold $v_H$.

When $(c_L, c_H) \in (0,1) \times (0,1) \backslash Y$, i.e., when $(c_L, c_H)$ fails to induce interior entry thresholds for both types $H$ and $L$, the equilibrium is determined by one of the following three cases (involving at least one corner threshold):

1. All type-$L$ bidders prefer entry ($v_L = 0$): $\pi(0, 0, v_H) \geq c_L$, $\pi(v_H, 0, v_H) - L(v_H, 0, v_H) = c_H$, and $0 < v_H \leq 1$. No bidder has an incentive to deviate because $\pi(v, 0, v_H) - L(v, 0, v_H)$ strictly increases in $v$.

2. All type-$H$ bidders prefer staying out ($v_H = 1$): $\pi(v_L, v_L, 1) = c_L$, $\pi(1, v_L, 1) - L(1, v_L, 1)$, $0 \leq v_L < 1$. No bidder has an incentive to deviate because $\pi(v, v_L, 1)$ strictly increases in $v$.

3. $v_L = 0$ and $v_H = 1$, i.e., all type-$L$ bidders enter the initial auction and all type-$H$ bidders stay out.

Hence an entry equilibrium involving corner solution always exists.[20] The uniqueness of equilibrium is also obvious.

We next turn to the uniqueness of the equilibrium with interior thresholds for both types. We will identify conditions under which the equilibrium is unique.

To simplify notation, we substitute $v_L = x_1$ and $v_H = x_2$ into the equation system (5) to obtain

$$\begin{cases} \pi(x_1, x_1, x_2) - c_L & = 0 \\ \pi(x_2, x_1, x_2) - L(x_2, x_1, x_2) - c_H & = 0 \end{cases} \tag{6}$$

As in the proof of Proposition 1, we will employ the implicit function theorem. Based on the two indifference conditions in (6), we will first solve $x_2$ in terms of $x_1$ to obtain $\eta(x_1, c_L)$ and $\varphi(x_1, c_H)$, respectively. The solution(s) for $x_1$ is then obtained by equating $\eta(x_1, c_L)$ and $\varphi(x_1, c_H)$. We will identify conditions under which the solution for $x_1$ (and hence the solution for the equilibrium) is unique.

In (6), it is obvious that $\partial\pi(x_1, x_1, x_2)/\partial x_1 > 0$ and $\partial\pi(x_1, x_1, x_2)/\partial x_2 > 0$ when $x_1 \neq x_2$. The reason is as follows: $\pi(x_1, x_1, x_2)$ is the expected payoff for a bidder with value $x_1$, when the pair

[20] The characterization is due to the conditions $\partial\pi(v_L, v_L, v_H)/\partial v_L > 0$ and $\partial\left[\pi(v_H, v_L, v_H) - L(v_H, v_L, v_H)\right]/\partial v_H > 0$ when $v_L \neq v_H$, which are demonstrated below.

of thresholds is $(x_1, x_2)$. Fixing $x_2$, when $x_1$ increases to $x_1 + \varepsilon$, we have $\pi(x_1 + \varepsilon, x_1 + \varepsilon, x_2) > \pi(x_1, x_1 + \varepsilon, x_2) > \pi(x_1, x_1, x_2)$. The second inequality is due to two effects: first, the chance for a bidder to enter alone is higher when the lower threshold is $x_1 + \varepsilon$; second, conditional on entering alone, a bidder's expected payoff from resale is higher: even if she still uses the optimal reserve price (in the resale) when the lower threshold is $x_1$, her payoff from resale is higher than before (with the lower threshold $x_1$, the potential buyers in resale are those with $c_H$ and $v \in (x_1,\, x_2)$; with threshold $x_1 + \varepsilon$, in addition to those bidders, potential buyers also include the bidders with $c_L$ and $v \in (x_1,\, x_1 + \varepsilon)$). In the resale, the previously set reserve price can generate an expected payoff which cannot be higher than the current maximal payoff. Therefore, we have $\pi(x_1, x_1 + \varepsilon, x_2) > \pi(x_1, x_1, x_2)$. Fixing $x_1$, when $x_2$ increases, $\pi(x_1, x_1, x_2)$ also increases: the probability for one to enter alone increases and the expected payoff from resale conditional on entering alone also increases. We also have

$$
\begin{aligned}
\partial[\pi(x_2, x_1, x_2) - L(x_2, x_1, x_2)]/\partial x_1 &= (n-1)qf(x_1)\{[qF(x_1) + (1-q)F(x_2)]^{n-2}x_2 \\
&\quad + \int_{x_1}^{x_2} [qF(x_1) + (1-q)F(r(x))]^{n-2}[r'(x) - 1]dx\} \\
&> (n-1)qf(x_1)\{[qF(x_1) + (1-q)F(x_2)]^{n-2}x_2 \\
&\quad -[qF(x_1) + (1-q)F(x_2)]^{n-2}(x_2 - x_1)\} \\
&= (n-1)qf(x_1)[qF(x_1) + (1-q)F(x_2)]^{n-2}x_1 \\
&> 0.
\end{aligned}
$$

$$
\begin{aligned}
\partial[\pi(x_2, x_1, x_2) - L(x_2, x_1, x_2)]/\partial x_2 &= (n-1)(1-q)f(x_2)[qF(x_1) + (1-q)F(x_2)]^{n-2}x_2 \\
&\quad +[qF(x_1) + (1-q)F(x_2)]^{n-1} \\
&> 0
\end{aligned}
$$

By the implicit function theorem, there exists a unique, continuously differentiable function $\eta(\cdot, c_L)$, such that $\{(x_1, \eta(x_1, c_L)) : x_1 \in (0,1)\} = \{(x_1, x_2) \in \mathring{D} : \pi(x_1, x_1, x_2) = c_L\}$ and a unique, continuously differentiable function $\varphi(\cdot, c_H)$, such that $\{(x_1, \varphi(x_1, c_H)) : x_1 \in (0,1)\} = \{(x_1, x_2) \in \mathring{D} : \pi(x_2, x_1, x_2) - L(x_2, x_1, x_2) = c_H\}$.

Applying the chain rule, given $c_L$ and $c_H$, both $\eta(x_1, c_L)$ and $\varphi(x_1, c_H)$ are decreasing in $x_1$. Since $x_1$ is obtained by $\eta(x_1, c_L) = \varphi(x_1, c_H)$, without further information, the uniqueness for $x_1$ is not straightforward. However, in the case with multiple equilibria, for any two entry equilibrium thresholds $(v_{1L}, v_{1H})$ and $(v_{2L}, v_{2H})$, if $v_{1L} < v_{2L}$, it must be the case that $v_{1H} = \eta(v_{1L}, c_L) > \eta(v_{2L}, c_L) = v_{2H}$, that is, there is a natural ranking for the multiple equilibria.

Next we derive sufficient conditions under which the pair of equilibrium entry thresholds is unique. As mentioned above, to solve the equation system (6) is equivalent to identifying the intersection(s) between $\eta(x_1, c_L)$ and $\varphi(x_1, c_H)$ (the existence has already been established). Since both $\eta(x_1, c_L)$ and $\varphi(x_1, c_H)$ are, given $c_L$ and $c_H$, decreasing in $x_1$, an immediate sufficient condition is that $\partial\eta(x_1, c_L)/\partial x_1 < \partial\varphi(x_1, c_L)/\partial x_1$ or $\partial\eta(x_1, c_L)/\partial x_1 > \partial\varphi(x_1, c_L)/\partial x_1$ for any $x_1 \in (0, 1)$.

If we define

$$\begin{aligned}\Delta(x_1, q, n, c_L, c_H) \triangleq\; & \frac{\partial\pi(x_1, x_1, x_2)}{\partial x_1}|_{x_2=\eta(x_1,c_L)} \cdot \frac{\partial[\pi(x_2, x_1, x_2) - L(x_2, x_1, x_2)]}{\partial x_2}|_{x_2=\varphi(x_1,c_H)} \\ & - \frac{\partial\pi(x_1, x_1, x_2)}{\partial x_2}|_{x_2=\eta(x_1,c_L)} \cdot \frac{\partial[\pi(x_2, x_1, x_2) - L(x_2, x_1, x_2)]}{\partial x_1}|_{x_2=\varphi(x_1,c_H)},\end{aligned}$$

the necessary and sufficient condition for the uniqueness is that $\Delta(x_1, q, n, c_L, c_H)$ does not change the sign at any $x_1$ where $\eta(x_1, c_L) = \varphi(x_1, c_H)$ (otherwise there would be multiple solutions for $x_1$). Hence any condition under which

$$\begin{aligned}\bar{\Delta}(x_1, x_2, q, n) \triangleq\; & \frac{\partial\pi(x_1, x_1, x_2)}{\partial x_1} \cdot \frac{\partial[\pi(x_2, x_1, x_2) - L(x_2, x_1, x_2)]}{\partial x_2} \\ & - \frac{\partial\pi(x_1, x_1, x_2)}{\partial x_2} \cdot \frac{\partial[\pi(x_2, x_1, x_2) - L(x_2, x_1, x_2)]}{\partial x_1}\end{aligned}$$

does not change sign for any $(x_1, x_2) \in \mathring{D}$ is a sufficient condition for the uniqueness. Moreover, if $\bar{\Delta}(x_1, x_2, q, n)$ does not change sign for any $(x_1, x_2) \in \hat{D}$, where $\hat{D} \subset \mathring{D}$, we have that the corresponding set of $(c_L, c_H)$ generates a unique pair of equilibrium thresholds.

We do not give the expression for $\bar{\Delta}(x_1, x_2, q, n)$ here because doing so would be tedious. Nevertheless we can evaluate the following cases:

1. When $c_L = c_H$, $\eta(x_1, c_L) = \varphi(x_1, c_H) = x_1$ and $x_2 = x_1$:

$$\begin{aligned}\bar{\Delta}(x_1, x_2, q, n) &= (1 - p_E)^{n-1}\frac{\partial\pi(x_1, x_1, x_2)}{\partial x_1} \\ &\quad + (n-1)(1-q)f(x_2)(1-p_E)^{n-2}[qF(x_1) + (1-q)F(r(x_1))]^{n-1}x_2 \\ &> 0.\end{aligned}$$

   This condition implies that when $c_L$ and $c_H$ are sufficiently close, the solution to the equation system (5) is unique.

2. We also have

$$\begin{aligned}\lim_{q\to 0}\bar{\Delta}(x_1,x_2,q,n) &> (n-1)f(x_2)(1-p_E)^{n-2}[qF(x_1)+(1-q)F(r(x_1))]^{n-1}x_2>0\\ \lim_{q\to 1}\bar{\Delta}(x_1,x_2,q,n) &= \lim_{q\to 1}(1-p_E)^{n-1}\frac{\partial\pi(x_1,x_1,x_2)}{\partial x_1}>0\end{aligned}$$

These two conditions imply that when $q$ is sufficiently small or sufficiently large, $\Delta(x_1,q,n,c_L,c_H)>0$.

3. Taking the limit, we have[21]

$$\begin{aligned}\lim_{n\to\infty}\bar{\Delta}(x_1,x_2,q,n) &= \lim_{n\to\infty}-(n-1)^3q(1-q)f(x_1)f(x_2)(1-p_E)^{n-2}x_2\\ &\quad\cdot\int_{x_1}^{x_2}[qF(x_1)+(1-q)F(r(x))]^{n-2}\,r'(x)\left[\frac{(1-p_E)}{qF(x_1)+(1-q)F(r(x))}-\frac{n-2}{n-1}\right]dx\\ &= 0_-\end{aligned}$$

This condition implies that when $n$ is sufficiently large, $\bar{\Delta}(x_1,x_2,q,n)<0$.

4. Define $M(x_1,x_2,n)=\int_{r(x_1)}^{x_2}\left[(n-1)\frac{qF(x_1)+(1-q)F(x_2)}{qF(x_1)+(1-q)F(r(x))}-(n-2)\right]\frac{f(x_1)}{f(x)}dx$. When the following inequality holds:

$$\begin{aligned}&\int_{x_1}^{x_2}[qF(x_1)+(1-q)F(r(x))]^{n-2}[1-r'(x)]dx\\ \le\;& (1-p_E)^{n-2}x_2\int_{x_1}^{x_2}\left[\frac{qF(x_1)+(1-q)F(x_2)}{qF(x_1)+(1-q)F(r(x))}-\frac{n-2}{n-1}\right]dx,\end{aligned}$$

$\bar{\Delta}(x_1,x_2,q,n)>0$ as long as $(1-p_E)^{n-1}/[qF(x_1)+(1-q)F(r(x_1))]^{n-1}\le(1-q)\,/\,[qM(x_1,x_2,n)]+1$. These two conditions can be satisfied when $x_1$ and $x_2$ are not too close nor too far apart (which also means that $c_L$ and $c_H$ are not too close nor too far apart), and $q,n$ are relatively small. ■

**Proof of Proposition 4**: We will show that in the symmetric (entry and bidding) equilibrium, all standard auctions will lead to the same expected payoffs (revenue equivalence).

First we claim that given the same induced entry thresholds, say, $(v_L,v_H)$, revenue equivalence holds. To see this, given $m$, the number of entrants induced by the thresholds $(v_L,v_H)$, the reseller faces $n-m$ outside bidders with values independently drawn from $F_{out}(\cdot)$. As the resale mechanism is fixed (which is an optimal auction), the reseller's expected payoff from resale is the same. Similarly,

[21] Several other terms are omitted below, as those terms converge to 0 much faster.

for outside bidders, the expected payoff conditional on staying out is also invariant across different (standard) auction formats. In the initial auction, the $m$ entrants' values are independently drawn from $F_{in}(\cdot)$. Hence the symmetric IPV paradigm applies and the revenue equivalence follows. Since revenue equivalence holds given any realized $m$, it also holds *ex ante* (before $m$ is realized); in other words, an auction induces the same expected payoffs as long as it induces the same entry thresholds.

But auctions with the same expected payoffs (for the seller and bidders) will indeed induce the same symmetric entry thresholds. Thus the claim established in the preceding paragraph and the same entry thresholds are self-enforcing, implying that revenue equivalence holds in the symmetric (entry and bidding) equilibrium among all standard auctions. ■

**Proof of Proposition 5**: First we show that if at least one pair of equilibrium thresholds are interior, we have $\tilde{v}_L > v_L$ and $\tilde{v}_H < v_H$. We will proceed by ruling out all the other cases:

1. $\tilde{v}_L \leq v_L$ and $\tilde{v}_H \leq v_H$. If this is true, we immediately have $1-\tilde{p}_E \leq 1-p_E$. This implies that $c_L = (1-\tilde{p}_E)^{n-1}\tilde{v}_L \leq (1-p_E)^{n-1}v_L < (1-p_E)^{n-1}w(v_L, 1, v_L, v_H) = c_L$, a contradiction.

2. $\tilde{v}_L \geq v_L$ and $\tilde{v}_H \geq v_H$. Because $\tilde{v}_H \geq v_H$, we know the bidder with type $(c_H, v_H)$ has a higher payoff in the resale than in the no-resale case. However, this cannot happen when $\tilde{v}_L \geq v_L$ and $\tilde{v}_H \geq v_H$ both hold. To see why, if $\tilde{v}_L \geq v_L$ and $\tilde{v}_H \geq v_H$, there will be more entrants with resale; Moreover, type-$H$ bidders bid their values, which is the same as in the no-resale case, while type-$L$ bidders bid more aggressively (Proposition 2). Therefore, the expected payoff conditional on entry for the bidder with $v_H$ can only be lower in the resale case, a contradiction.

3. $\tilde{v}_L \leq v_L$ and $\tilde{v}_H \geq v_H$. Since $\tilde{v}_L \leq v_L$, we have $\pi(\tilde{v}_L, v_L, v_H) \leq c_L$. Since $\pi(\tilde{v}_L, v_L, v_H) = (1-p_E)^{n-1}w(\tilde{v}_L, 1, v_L, v_H)$ and $w(\tilde{v}_L, 1, v_L, v_H) > \tilde{v}_L$, we have $(1-p_E)^{n-1}\tilde{v}_L < \pi(\tilde{v}_L, v_L, v_H) \leq c_L = (1-\tilde{p}_E)^{n-1}\tilde{v}_L$. We thus have $p_E > \tilde{p}_E$. Next we consider the equilibrium payoff for a bidder of type $(\tilde{v}_H, c_H)$ in the resale case. Her expected payoff conditional on entry is $\tilde{\pi}(\tilde{v}_H, \tilde{v}_L, \tilde{v}_H)$ in the no-resale case and is $\pi(\tilde{v}_H, v_L, v_H)$ in the resale case, while $\pi(\tilde{v}_H, v_L, v_H) \geq c_H$ and $\tilde{\pi}(\tilde{v}_H, \tilde{v}_L, \tilde{v}_H) = c_H$. This implies that by entering the auction, this bidder will have a higher expected payoff in the resale case. However, we will argue that bidders bid more aggressively in the resale case. First, in the resale case type-$L$ bidders with $v \in (v_L,\ \tilde{v}_H)$ bid their expected payoff from resale instead of their own values, and these bidders also enter the auction in the no-resale case. Second, type-$L$ bidders with $v \in (\tilde{v}_L,\ v_L)$ stay out in the

resale case. However, this effect of reduced competition is more than offset by the entry of type-$H$ bidders with $v \in (v_H, \tilde{v}_H)$, as in a probability sense, there will be more type-$H$ bidders with $v \in (v_H, \tilde{v}_H)$ than type-$L$ bidders with $v \in (\tilde{v}_L, v_L)$, given $p_E > \tilde{p}_E$ as established above. Therefore, the bidder with $\tilde{v}_H$ faces more entry by other bidders and will make a strictly higher expected payment in every winning event, contradicting $\pi(\tilde{v}_H, v_L, v_H) \geq \tilde{\pi}(\tilde{v}_H, \tilde{v}_L, \tilde{v}_H)$.

Next we show that when equilibrium thresholds involve some corner solution(s), we have $\tilde{v}_L \geq v_L$ and $\tilde{v}_H \leq v_H$. As demonstrated in the proof of Proposition 3, when $(c_L, c_H)$ fails to induce interior entry thresholds for both types $H$ and $L$, the equilibrium is determined by three cases (involving at least one corner threshold).

In the first case ($v_L = 0$ and $v_H \in (0, 1]$), $0 = v_L \leq \tilde{v}_L$ holds trivially, and $v_H \geq \tilde{v}_H$ is also true. If we suppose not, similar arguments paralleling those in the second case above lead to contradiction.

In the second case ($v_H = 1$ and $v_L \in [0, 1)$), $1 = v_H \geq \tilde{v}_H$ holds trivially, and $v_L \leq \tilde{v}_L$ is also true. If we suppose not, similar arguments paralleling those in the first case above lead to contradiction.

In the third case ($v_L = 0$ and $v_H = 1$), $0 = v_L \leq \tilde{v}_L$ and $1 = v_H \geq \tilde{v}_H$ hold trivially. ■

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