Hybrid Auctions Revisited

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Abstract

We examine hybrid auctions with affiliated private values and risk-averse bidders, and show that the optimal hybrid auction trades off the benefit of information extraction in the ascending-bid phase and the cost of reduced competition in the sealed-bid phase.

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1 Introduction

The celebrated Revenue Equivalence Theorem only holds when values are independent and private, and the bidders are symmetric and risk neutral. When any of these conditions is violated, the revenue equivalence may break down. For example, when bidder values are affiliated, Milgrom and Weber (1982) show that the first-price sealed-bid auction is dominated by the second-price sealed-bid auction, which is in turn dominated by the English ascending-bid auction in terms of expected revenue. When bidders exhibit risk aversion, however, Maskin and Riley (1984) show that the seller favors first-price sealed-bid auctions over second-price or ascending-bid auctions. Finally, when bidders draw signals from different distributions (bidders are asymmetric), Maskin and Riley (2000a, b) show that a first-price sealed-bid auction again generates more expected revenue than a second-price sealed-bid or ascending-bid auction.

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Thus, considerations like bidder asymmetry or risk aversion favor the use of first-price sealed-bid auctions, while considerations like value affiliation favor the use of second-price sealed-bid or ascending-bid auctions. This insight is also demonstrated in other value settings – in particular, auctions with almost common values, where one bidder, the advantaged bidder, values the object slightly more than the other regular bidders. With only two bidders, a slight private value advantage is predicted to have an "explosive" effect on the outcome and revenue of an auction (Bikhchandani, 1988). The advantaged bidder always wins and revenue dramatically decreases relative to the pure common value auction. Ascending auctions, which reduce to two bidders, are thought to be particularly vulnerable to the explosive effect, which may discourage entry.[1] As such, it is reasonable to raise concerns about the use of ascending auctions. But ascending-bid phase is desirable in this setting, since the unraveling of private information through dropouts serves to diminish the private informational rent possessed by the bidders, which leads to higher expected revenue.

A solution to the dilemma of choosing between the ascending (English) and sealed-bid (Dutch) formats is first proposed by Klemperer (1995, 1998), who suggests combining the two in a hybrid, the so called "Anglo-Dutch auction."[2] The Anglo-Dutch auction works as follows. The auctioneer begins by raising the price continuously until all but two bidders have dropped out. The two remaining bidders are then required to make final sealed bids that are not lower than the current asking price, and the winner pays her bid. This ascending phase is meant to extract most of the information that would be revealed by a pure ascending auction, raising expected revenues if bidders' information is affiliated, while running a first-price sealed-bid auction in the final stage avoids, or at least mitigates, the concerns arising from an almost common value setting.[3]

In this note we examine Klemperer's insight and consider a generalized hybrid auction in which the number of ascending-bid stages (or the number of bidders being dropped in the ascending-bid phase) can be any between 1 and $N-2$. We analyze the performance of such generalized hybrid auctions in a model with affiliated private values and risk averse bidders. We demonstrate that, consistent with

[1] Levin and Kagel (2005) show that it does not necessarily extend to auctions that start with at least two regular bidders.

[2] See Klemperer (2002) for a more formal characterization of the Anglo-Dutch auction.

[3] Klemperer emphasizes the version with two bidders in the final sealed-bid phase mainly due to the concern with efficiency: efficiency is likely to be lower the more bidders in the sealed-bid phase, especially when bidders are asymmetric.

Klemperer's insight, the standard English ascending-bid auction is revenue-dominated by any hybrid auction in which at least one bidder is dropped in the ascending-bid phase. We also show that the optimal hybrid auction in our example requires that only one bidder be dropped during the ascending-bid phase, and the final sealed-bid auction is conducted among all but one bidders. This suggests that the hybrid auction with the number of ascending-bid stages being optimally determined can outperform the original version of Anglo-Dutch auction with two final bidders. Finally, we show that when bidders are sufficiently risk averse, even the optimal hybrid auction generates lower expected revenue than the first-price sealed-bid auction. This result suggests a caveat in applying hybrid auctions in practice; that is, under certain conditions, lengthening the ascending phase in a hybrid auction may hurt the seller by reducing competition in the final sealed-bid phase.

2 The model and main results

We consider a mechanism through which a single indivisible asset is offered for sale to N buyers. The values to the buyers are private and ex ante, they are independent draws conditional on a common "state of the world" V. In our example, we assume that $V \sim U[\underline{v}, \overline{v}]$. Conditional on $V = v$, we assume that the private value for buyer i, $x_i \sim U[v - \epsilon, v + \epsilon]$, $\quad 0 < \epsilon < \underline{v}$. Given a monetary income y, we also assume that the buyers' utility function follows CRRA: $u(y) = y^{1-\rho}$, where $\rho \in [0, 1)$.

We assume that a hybrid auction is employed in which the auctioneer begins by raising the price continuously until all but $N-k$ bidders have dropped out, where $1 \leq k \leq N-2$. So k bidders will be dropped during the ascending-bid phase. The $N - k$ remaining bidders will compete in a first-price sealed-bid auction.[4]

First, it is straightforward to see that in the ascending phase, each bidder will stay until the price reaches her private value x_i. Let x_k denote the k^{th} dropping price in the ascending phase. x_k serves as the lower bound of the values in the final sealed-bid phase.

Suppose all but bidder i follow a symmetric increasing equilibrium bid function $\beta(\cdot)$. Then the maximization problem for bidder i with value x, can be written as follows:

$$\max_{b \geq 0} (\beta^{-1}(b) - x_k)^{N-1-k}(x - b)^{1-\rho}$$

[4] In Klemperer's original proposal, two final bidders engage in the final sealed bid tender, so $k = N - 2$.

Differentiating with respect to b and imposing the equilibrium condition yields:

$$\beta'(x) = \frac{(N-1-k)(x-\beta(x))}{(1-\rho)(x-x_k)}$$

Solving the above differential equation with the initial condition $\beta(x_k) = x_k$, we have

$$\beta(x) = x_k + \left(\frac{N-1-k}{N-\rho-k}\right)(x-x_k)$$

Conditional on $V = v$, we can compute the expected revenue (net of v) under the hybrid auction.

$$\begin{aligned} REV_k^H - v &= E\left[x_k + \frac{N-1-k}{N-\rho-k}\left(x_{(1)} - x_k\right)|V=v\right] \\ &= v - \epsilon + \frac{2\epsilon}{N+1}k + \frac{N-1-k}{N-\rho-k}\frac{2\epsilon}{N+1}(N-k) \\ &= \frac{(N-3)(N-k)+\rho(N+1-2k)}{(N-\rho-k)(N+1)}\epsilon \end{aligned} \tag{1}$$

Since $\frac{\partial[REV_k^H - v]}{\partial k} = -\frac{2\rho\epsilon}{(N+1)}\frac{1-\rho}{(N-\rho-k)^2} \leq 0$ as $\rho \geq 0$, REV_k^H is maximized at $k = 1$, i.e., the optimal number of bidders to be dropped from the ascending-bid phase is 1. This is actually quite intuitive given a property specific to uniform distributions: in equilibrium a bidder presumes that her signal, x_i, is the highest. Thus, upon observing the first dropout point, x_1, each bidder behaves as if she were provided with the highest and lowest signals, a pair serving as sufficient statistics for the range of V. Any additional dropout does not add to the information extraction which helps the seller, but reduces the number of bidders in the sealed-bid phase which hurts the seller. As a result, the optimal hybrid requires that only one bidder be dropped during the ascending phase.

Note that in this example, the optimal hybrid auction can also be implemented with a two-stage sealed-bid auction: in the first stage, bidders submit sealed bids, and the lowest bidder is dropped from the second-stage auction. In the second stage, the first-stage lowest bid is revealed to the remaining bidders, which also serves as the lowest allowable bid.

Similarly we can compute the expected revenue (net of v) under the standard English ascending auction, and it can be easily verified that

$$REV^{Eng} - v = \frac{N-3}{N+1}\epsilon \tag{2}$$

By (1) and (2), we have

$$REV^{Eng} - REV_k^H = -\frac{2\rho\epsilon}{N+1}[(N-1)-k] \leq 0 \tag{3}$$

Note that $REV^{Eng} = REV_k^H$ only if $\rho = 0$ (the risk neutral case). We summarize the above results into the following proposition.

Proposition 1 *For any realization of v, the English ascending-bid auction is dominated by a hybrid auction in terms of the expected revenue. The optimal hybrid auction requires that exactly one bidder be eliminated from the ascending-bid phase; in effect, the optimal hybrid auction can be implemented by a two-stage sealed-bid auction.*

Our previous analysis focuses on the comparison between the English ascending-bid auction and hybrid auctions in which at least one bidder drops out before reaching the sealed-bid phase. Next we turn to the comparison between the standard first-price sealed-bid auction and hybrid auctions. To that end, we need to consider the equilibrium bid function under a standard first-price sealed-bid auction.[5]

When $x \in [\underline{v} - \epsilon, \underline{v} + \epsilon]$ (region 1), it can be easily verified that the equilibrium bid function is given by

$$\beta_I(x) = \frac{Nx + (1-\rho)(\underline{v} - \varepsilon)}{N+1-\rho}.$$

Next we consider the case in which $x \in [\underline{v} + \epsilon, \overline{v} - \epsilon]$ (region 2). By assuming that all but bidder i follow a symmetric increasing bid function $\beta(\cdot)$, the maximization problem faced by bidder i with signal x can be written as follows.

$$\max_{b \geq 0} (x-b)^{1-\rho} E[\beta^{-1}(b) - (V - \epsilon)]^{N-1} = (x-b)^{1-\rho} \int_{x-\epsilon}^{x+\epsilon} [\beta^{-1}(b) - (v-\epsilon)]^{N-1} dv$$

Imposing the equilibrium condition to the first order condition, and simplifying yields:

$$\beta'_{II}(x) = \frac{N(x - \beta_{II}(x))}{(1-\rho)2\epsilon}$$

[5] The equilibrium under risk neutrality is derived in Kagel, Harstad, and Levin (1987).

with boundary condition $\beta_{II}(\underline{v}+\epsilon)=\beta_I(\underline{v}+\epsilon)$. It can be verified that the solution for this case is given by

$$\beta_{II}(x)=x-\frac{(1-\rho)}{N}2\epsilon+\frac{(1-\rho)^2}{N(N+1-\rho)}2\epsilon\times\exp\{-\frac{N}{(1-\rho)2\epsilon}[x-(\underline{v}+\epsilon)]\}.$$

Finally, when $x\in[\overline{v}-\epsilon,\overline{v}+\epsilon]$ (region 3), it can be verified that the equilibrium is characterized by

$$\beta'_{III}(x)=\frac{x-\beta(x)}{1-\rho}\frac{N(2\epsilon)^{N-1}-N(x-\overline{v}+\epsilon)^{N-1}}{(2\epsilon)^N-(x-\overline{v}+\epsilon)^N}$$

with boundary condition $\beta_{III}(\overline{v}-\epsilon)=\beta_{II}(\overline{v}-\epsilon)$. Since there is no analytical solution to this ODE system, we are unable to compute the exact expected revenue under a first-price sealed-bid auction. Nevertheless we can obtain a lower bound for the expected revenue conditional on $v\in[\underline{v}+2\epsilon,\overline{v}-2\epsilon]$, in which case all signals lie in the interval $[\underline{v}+\epsilon,\overline{v}-\epsilon]$ so we can compute the expected revenue based on the equilibrium bid function in region 2. Using $x_{(1)}$ to denote the highest signal among all N bidders, we have

$$\begin{aligned}REV^{FPA} &= \int_{v-\epsilon}^{v+\epsilon}\beta_{II}(x)dF_{x_{(1)}|v}(x)\\ &> \int_{v-\epsilon}^{v+\epsilon}\left[x-\frac{1-\rho}{N}2\epsilon\right]d\left[\frac{x-(v-\epsilon)}{2\epsilon}\right]^N\\ &= \frac{N-1}{N+1}\epsilon-\frac{(1-\rho)2\epsilon}{N}+v. \end{aligned} \tag{4}$$

Using (4) and (1), we have

$$REV^{FPA}-REV_1^H>\frac{(1-\rho)2\epsilon}{N(N+1)(N-1-\rho)}[\rho(N+1)-(N-1)].$$

Proposition 2 *When $v\in[\underline{v}+2\epsilon,\overline{v}-2\epsilon]$, a sufficient condition for a standard first-price sealed-bid auction to generate higher expected revenue than the optimal hybrid auction is $\rho\geq\frac{N-1}{N+1}$.*

Proposition 2 is fairly intuitive. As the degree of risk aversion becomes sufficiently large, the benefit of information extraction through one dropout is more than offset by the cost of reduced competition with one less bidder competing in the final sealed-bid tender. This result has a similar flavor as in Bulow and Klemperer (1996), who show that the benefit of charging an optimal reserve price is more than offset by the cost of reduced competition with one less bidder.

As $\overline{v}$ becomes sufficiently large while holding ϵ constant, it is straightforward to see that regions 1 and 3 are negligible and Proposition 2 also holds for ex ante revenue comparison (without being conditional on v).

Though not covered in Proposition 2, it is clear that when ρ is sufficiently low, a first-price sealed-bid auction would be dominated by a hybrid auction: By (3), $REV^{Eng} \leq REV_k^H$ (with equality held only when $\rho = 0$). From Milgrom and Weber (1982), we have $REV^{FPA} \leq REV^{Eng}$ for $\rho = 0$. By continuity, we thus have $REV^{FPA} < REV_k^H$ when ρ is sufficiently low.

3 Conclusion

We examine a generalized hybrid auction in a simple model with affiliated private values and risk-averse bidders. We show that the hybrid auction generates higher expected revenue than the standard English ascending auction, and the optimal hybrid auction is characterized by an optimal number of ascending-bid stages (which is 1 in our example). We also demonstrate that when bidders are sufficiently risk averse, no hybrid auction can perform as well as a simple first-price sealed-bid auction. Our results suggest that the revenue-maximizing auction should optimally balance the benefit of information extraction in the ascending-bid phase with the cost of reduced competition in the sealed-bid phase.[6]

References

[1] Bikhchandani, S., 1988, Reputations in Repeated Second-Price Auctions, Journal of Economic Theory 46, 97-119.

[2] Bulow, J. and P. Klemperer, 1996, Auctions versus Negotiations, American Economic Review 86, 180-94.

[6] In the extreme case where signal affiliation is completely eliminated (i.e., when v is commonly known in our model), we should expect that the ascending phase does not have value and the optimal selling mechanism is simply a first-price sealed-bid auction. This is indeed the case, as we can show that the first-price sealed-bid auction generates more expected revenue than any hybrid auction (a formal proof is available upon request).

[3] Kagel, J., R. Harstad, and D. Levin, 1987, Information Impact and Allocation Rules in Auctions with Affiliated Private Valuations: An Experimental Study, Econometrica 55, 1275-1304.

[4] Klemperer, P., 1995, Comments on "Putting Auction Theory to Work", Discussant's Comments on Paul Milgrom's 1995 Churchill Lectures, Working Paper, Oxford University.

[5] Klemperer, Paul, 1998, Auctions with Almost Common Values: The "Wallet Game" and its Applications, *European Economic Review* 42, 759-69.

[6] Klemperer, P., 2002, What Really Matters in Auction Design, Journal of Economic Perspectives 16, 169-89.

[7] Levin, D. and J. Kagel, 2005, Almost Common-Value Auctions Revisited, European Economic Review 49, 1125-1136.

[8] Maskin, E. and J. Riley, 1984, Optimal Auctions with Risk Averse Buyers, Econometrica 52, 1473-1518.

[9] Maskin, E. and J. Riley, 2000a, Asymmetric Auctions, Review of Economic Studies 67, 413-38.

[10] Maskin, E. and J. Riley, 2000b, Equilibrium in Sealed High Bid Auctions, Review of Economic Studies 67, 439-54.

[11] Milgrom, P. and R. Weber, 1982, A Theory of Auctions and Competitive Bidding, Econometrica 50, 1089-1122.