Monopolistic Nonlinear Pricing with Costly Information Acquisition*

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Abstract

We consider costly information acquisition in the canonical monopolistic nonlinear pricing model (Mussa and Rosen, 1978; Maskin and Riley, 1984) so that consumers need to incur privately known entry costs in order to learn their preference "intensities" for the quality of the product. We show that by taking into account costly information acquisition, the nature of optimal nonlinear pricing contracts changes dramatically: both quality distortion and market exclusion are reduced compared to the Mussa-Rosen benchmark, and the monopolistic solution may even achieve the first best. We identify conditions under which the monopolistic solution involves quality distortion or no distortion. Also, when the monopolist can charge entry fees, we show that the optimal monopolistic solution is always the first best. Despite the possibility of production efficiency (no quality distortion), the monopolist always induces insufficient entry compared to socially efficient entry.

Keywords: Nonlinear Pricing, Information Acquisition, Consumer Entry, Contract, Quality Distortion.

JEL Classification: D82, D23, L12, L15

1 Introduction

Since the influential work of Mussa and Rosen (1978) and Maskin and Riley (1984), there has been a growing literature on nonlinear pricing. In a typical nonlinear pricing model with ver-

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tically differentiated products, the varieties of a product are indexed by *quality*, q, which summarizes the underlying attributes of the product.[1] A central task in this literature is how to construct a sorting nonlinear pricing contract in which different "types" of consumers are induced to sort themselves to different varieties of products (so as to extract the maximal possible private informational rents from the consumers).

An implicit assumption in this well-developed literature is that consumers are endowed with their "types." For example, in the canonical model of Mussa and Rosen (1978, henceforth referred to as simply MR), a type, θ, is the preference "intensity" that "measures intensity of a consumer's taste for quality" (MR, page 303). More precisely, θ is the marginal utility of quality, or the consumer's marginal rate of substitution, which completely determines a consumer's preference over q and money. A fundamental assumption in MR's analysis, as well as in the overall nonlinear pricing literature, is that consumers know their θ's at the outset of the game, and make purchase decisions based on their known types.

For highly familiar products or services (e.g., electricity, telephone service, newspaper subscriptions, etc.), it is reasonable to assume that consumers are well aware of their preference intensities. However, for some relatively new products or services, it may be less reasonable to assume that one is endowed with her preferences for free. For example, without actually watching 3D televisions with two different displaying technologies, one may never know about her "incremental value" of watching a model that does not require to wear eyeglasses over watching one that does;[2] after Smartphones were introduced, many users have been confused over which data plan to subscribe for, reflecting the uncertainty about their preferences over different data capacities needed;[3] even when purchasing a standard product like a new car, one may not settle down with a specific model (say, Mercedes-Benz C350 or E350) until after some test driving.

The above examples suggest that consumers often need to make efforts to discover their preferences (e.g., through trying the product or test driving). We believe that many other products or services share this common feature, and for those markets, it would be more sensible to assume that it is costly for the consumers to learn about their preferences, as trying the product or simply spending some time to learn about its different features is demanding in both effort and time. We believe this is particularly true for new products. According to Clay Christensen

[1] The term "nonlinear pricing" is more accurate in settings where q is the quantity as in, e.g., Maskin and Riley (1984). However, they are mathematically equivalent to models in which q is interpreted as quality. We thus follow the literature to use the phrase "nonlinear pricing" throughout this paper.

[2] The basic requirement for creating the 3D perception is to display offset images that are filtered separately to the left and right eyes. Two technologies are currently available: having the viewer wear eyeglasses to filter the separately offset images to each eye, or having the light source split the images directionally into the viewer's eyes (no glasses are required).

[3] "A Comprehensive Guide To All Those Confusing Smartphone Data Plans" by Steve Kovach, March 06, 2012: http://articles.businessinsider.com/2012-03-06/tech/31126289_1_unlimited-data-data-plan-unlimited-plans

at Harvard Business School, 30,000 new consumer products are launched each year.[4] Given this astounding number of the new products, it is unrealistic to assume that consumers know their preferences over all of them.

In this paper, we explicitly take into account the opportunity costs in learning one's preferences in a standard monopolistic nonlinear pricing model that is otherwise identical to the original MR model. More specifically, we add a costly information acquisition stage to the MR model, so that each consumer needs to incur a privately known information acquisition cost, c_i, in order to learn her preference type, θ_i. Since consumers make their purchase decisions based on their preference types, information acquisition is equivalent to an entry decision in our model.[5] In the traditional nonlinear pricing setting, where consumers are passively endowed with private information about their preference types, the analysis usually focuses on optimal elicitation of that private information. When costly entry is taken into account and the information acquisition costs are privately known, the nonlinear pricing contract has to additionally take into account entry in the information acquisition stage. Optimal nonlinear pricing in such a setting is thus potentially challenging as it has to balance information acquisition and information elicitation, which are interdependent: the nonlinear pricing contract has a direct effect on the set of entrants to be induced (and hence the actual market base for the product), and entry in the information acquisition stage imposes restrictions on the nonlinear pricing contracts to be offered.

Nevertheless, using standard techniques in calculus of variations,[6] we are able to completely characterize the optimal monopolistic nonlinear pricing contract in this setting. We show that the optimal nonlinear pricing contract either involves downward quality distortion or no quality distortion at all (i.e., the first-best allocation). This result marks a striking difference from the MR benchmark, where the first best never occurs. Even when the optimal solution takes the form of quality distortion, it is shown that quality distortion and market exclusion are both smaller compared to the MR benchmark. These results are intuitive: as the monopolist now needs to take into account consumer entry, it becomes optimal to reduce quality distortion and market exclusion, and sometimes even provide efficient quality provision without any market exclusion (anyone who enters the sale will be served).

We identify exact conditions under which no distortion and downward distortion occur. The

[4] "Clay Christensen's Milkshake Marketing," by Carmen Nobel, Harvard Business School Working Knowledge, February 14, 2011.

[5] More generally speaking, c_i in our model also includes the participation cost, e.g., the opportunity cost of making a trip to a store or a car dealership office. We rule out the case in which consumers make purchase directly without paying a trip to a store or learning their preference types.

[6] Given the more complicated structure of the objective function in our new setting, we can no longer use the more conventional piece-wise maximization technique (*a la* Myerson, 1981) or Hamiltonian approach in our analysis.

condition identified can be explained intuitively. In our model the monopolistic nonlinear pricing contract affects the consumers' equilibrium payoffs only through its effect on the entry cutoff. So in a sense the consumers' rent extraction is taken care of by endogenous entry. As such, the monopolist has an incentive to provide as efficient as possible quality provision. So by fixing the first-best quality provision, whenever optimal entry can be induced by adjusting a (nondistortionary) common rent provision (which is uniformly applied to all types of consumers covered), then the first best will indeed emerge in the monopolistic solution. On the other hand, by fixing the first-best quality provision, if optimal entry cannot be induced by adjusting the common rent provision alone, then the first-best solution is not *feasible* and the quality provision has to be distorted downward. In the latter case, we show that the *infeasibility* issue can be completely overcome by the monopolist's ability to charge an entry fee, as charging an entry fee effectively relaxes the *ex post* individual rationality (IR) constraint (imposed after entry occurs) to the *ex ante* IR constraint (imposed before entry occurs) – we show that in our setting, when the monopolist can charge entry fees, the first-best quality provision is always *feasible*, and hence optimal as well.

Lastly, we show that even when the optimal nonlinear pricing involves no quality distortion (production efficiency is achieved), the presence of a monopoly always induces insufficient entry in our model, compared to the socially efficient benchmark. In other words, entry distortion is always present in our model.

Related Literature

The role of information acquisition has been examined in several papers in the context of principal-agent settings (e.g, Cremer and Khalil, 1992; Cremer, Khalil, and Rochet, 1998a, 1998b).[7] Cremer and Khalil (1992) incorporate a costly information acquisition stage to Baron and Myerson's (1982) model of regulating a monopolist with unknown cost. They show that, although the firm does not acquire information in equilibrium, the ability to acquire information decreases the downward distortion at the production stage, which is consistent with our result. Cremer, Khalil, and Rochet (1998a) modify this setting so that all information about the cost structure has to be acquired at some fixed cost. They show that when the cost is not too small,[8] distortion is reduced for low cost types but increased for high cost types in the optimal contract. This turns out to be the most efficient way to increase rents for the firm, hence providing incentive for the firm to acquire the information. Since the principal fails to claim the entire surplus, the produc-

[7]Also see Bergemann and Välimäki (2006) for an excellent survey on information acquisition in the context of mechanism design

[8]Otherwise the optimal contract remains the same as in Baron and Myerson.

tion level is lower than the ex-ante efficient level. Cremer, Khalil, and Rochet (1998b) further modify the setting so that the firm's information acquisition decision is taken covertly before the contract is offered. This reversal in timing introduces strategic uncertainty for the principal as the firm may randomize over information acquisition.

To the extent that information acquisition is usually modeled as entry, our paper is also related to several papers that take into account participation constraints in the principal-agent setting (e.g., Lewis and Sappington, 1989; Biglaiser and Mezzetti, 1993; and Maggi and Rodriguez, 1995). Jullien (2000) provides a general analysis in which the agent's participation constraint is type-dependent. In this setting, the informational rent exhibits non-monotonicity such that exclusion may occur for interior types. Interestingly, upward distortions can also arise in this environment with one-dimensional type-dependent participation constraints, which is completely absent in our model.

In an important contribution, Rochet and Stole (2002) introduce a random participation component into the traditional monopolistic nonlinear pricing framework, effectively making the participation constraints two-dimensional type dependent. More specifically, they extend the MR setting by modeling the participation constraint as a random variable, x, which is also private information to the consumer. The preferences of a consumer of type (θ, x) over q and money is given by $\theta q - P(q)$ as in MR's setting, but unlike in MR, this consumer now makes a purchase if and only if $\max_q \{\theta q - P(q)\} \geq x$. In the case with continuous types and under the regularity assumption about distributions, they show that there is either bunching or no quality distortion at the bottom. In our setting, we show that either quality distortion is maximal at the bottom, or no distortion for all the types; in particular, under regularity conditions pooling never occurs in our model. This suggests a qualitative difference between the two entry models, arising due to the difference between costly participation and costly information acquisition.

Our results are very much similar to those obtained from the competitive nonlinear pricing literature (see, for example, Spulber, 1989; Champsaur and Rochet, 1989; Wilson, 1993; Gilbert and Matutes, 1993; Stole, 1995; Verboven, 1999; Villas-Boas and Schmidt-Mohr, 1999; Epstein and Peters, 1999; Peters, 2001; Armstrong and Vickers, 2001, 2010; Martimort and Stole, 2002; Rochet and Stole, 2002; Ellison, 2005; and Yang and Ye, 2008). By reinterpreting x as a consumer's horizontal type in the duopoly setting, Rochet and Stole (2002) show that under full-market coverage quality distortions disappear and the equilibrium is characterized by the cost-plus-fee pricing feature. A similar result is also obtained in Armstrong and Vickers (2001). When partial market coverage (along vertical dimension) is allowed, Yang and Ye (2008) show that quality distortion is reduced and market coverage is increased under a duopoly compared to a monopoly benchmark. Our results from this current research share many of these flavors, suggesting that entry has a similar effect as competition on nonlinear pricing schedules, although the exact workings are quite different.

Our approach in this research is also related to a growing literature on auctions with costly information acquisition (e.g., McAfee and McMillan, 1987; Levin and Smith, 1994; Bergemann and Välimäki, 2002). In particular, Lu (2010) and Moreno and Wooders (2011) study an auction setting that would otherwise be very similar to what is examined in this paper. They characterize the equilibrium in which each bidder enters the auction if and only if her information acquisition cost is lower than some endogenously determined entry threshold. While Moreno and Wooders (2011) show that the optimal auction involves a distortionary reserve price when entry fees are not feasible, Lu (2010) demonstrates that no distortionary reserve price should be used whenever entry fees are feasible. Their results are thus consistent with our findings in spirit, suggesting some intimate connection between the two seemingly unrelated literatures.

Finally, our model belongs to the general framework of sequential screening (e.g., Courty and Li, 2000). However, note that in our setting there is no benefit for the monopolist to run an additional mechanism to screen consumers at the information acquisition or entry stage. This is due to the following reasons. First, in our setting the entry cost c and preference type θ are independent, so learning about c does not help in the nonlinear pricing mechanism; second, in our setting entry of an individual consumer does not impose externality on the rest of the consumers who enter (as the allocation and transfer in a nonlinear pricing mechanism are only functions of one's report on her own "types"). This feature is very different from, say, the auction settings with sequential screening (e.g., Ye, 2007 and Lu and Ye, 2012), where running a prescreening mechanism to shortlist bidders can improve the seller's revenue.

The rest of the paper is organized as follows. Section 2 lays out the model, Section 3 provides the detailed analysis, and Section 4 concludes. All long proofs are relegated to Appendix A, and Appendix B provides an analysis on second-order conditions for optimality.

2 The Model

We start with a review of the well known MR model. In their setting, a monopolist offers to sell a single good at various levels of quality and price, which can be represented by a nonlinear pricing schedule, $P(q)$. The consumer's preference is completely determined by the "intensity" of taste, θ, with associated gross utility $\theta q - P(q)$, where θ is the marginal utility of quality, or the marginal rate of substitution of quality for money. The consumer's outside option is normalized to be zero. Ex ante, θ follows distribution $F(.)$ with strictly positive density $f(\cdot)$ over its support $\left[\underline{\theta}, \overline{\theta}\right]$. Under the regularity condition about $F(.)$, MR show that the optimal nonlinear pricing contract is characterized by quality distortion for all but the highest type and market exclusion of some low-type consumers. More specifically, the optimal monopolistic solution is given by

$$q^{MR}(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)} \text{ for } \theta \in [\theta^*_{MR}, \overline{\theta}], \tag{1}$$

where the lowest type served in the market, θ^*_{MR}, is determined by the following condition:

$$\theta^*_{MR} = \begin{cases} \underline{\theta} & \text{if} \quad \underline{\theta} - 1/f(\underline{\theta}) \geq 0 \\ \left[1-F(\theta^*_{MR})\right]/f(\theta^*_{MR}) & \text{if} \quad \underline{\theta} - 1/f(\underline{\theta}) < 0 \leq \overline{\theta} \end{cases} \tag{2}$$

We are now ready to describe our model. We introduce costly information acquisition to the monopolistic nonlinear pricing model in MR described above. Formally, there are a continuum of consumers with measure 1. Consumers are heterogeneous in their information acquisition costs, or simply put, their entry costs, c_i, which is private information to consumer i. Ex ante, c_i follows distribution $G(.)$ with strictly positive density function $g(\cdot)$ on $[\underline{c}, \overline{c}]$.[9] After entry, consumers draw θ's from the distribution $F(.)$ on $\left[\underline{\theta}, \overline{\theta}\right]$. We assume that θ_i and c_i are independent (so consumers are symmetric in terms of preference "intensities" even after they learn their c_i's). We will maintain the following regularity assumptions:

Assumption 1. $g(c)/G(c)$ is strictly decreasing in $c \in [\underline{c}, \overline{c}]$ and $f(\theta)/(1-F(\theta))$ is strictly increasing in $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$.

The firm's production cost is given by $c(q) = q^2/2$.[10] The monopolist's objective is to maximize its expected profit from the sale. It is easily verified that, under complete information about θ, the first-best solution is given by $q^{fb}(\theta) = \theta$.

Formally, the timeline is as follows:

1. The monopolist offers the (nonlinear) pricing schedule, $P(q)$, or equivalently, the menu of quality-price contracts, $\{q(\theta), p(\theta)\}$;

2. The consumers make simultaneous and independent entry decisions. Once a consumer participates, she incurs a cost c_i and learns her preference type θ_i ;

3. Consumers who have learned their θ_i's make purchase decisions, and the sale is realized.

[9] More generally, the entry cost c_i in our model may include the participation cost as modeled in Rochet and Stole (2002).

[10] We assume this quadratic cost function for ease of analysis. Our main results should be robust to convex cost functions.

The firm offers the (nonlinear) pricing schedule $p(q): R_+ \to R_+$, which is equivalent to offering a menu of (direct) contracts of the form $\{q(\theta), p(\theta)\}$, where $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. Given the menu of contracts $\{q(\theta), p(\theta)\}$, the utility obtained by a consumer with type θ, when choosing the offer $\{q(\widehat{\theta}), p(\widehat{\theta})\}$, is given by

$$x(\widehat{\theta}, \theta) = \theta q(\widehat{\theta}) - p(\widehat{\theta}).$$

Let $x(\theta) = x(\theta, \theta)$. Incentive compatibility (IC) implies that

$$x(\theta) = \max_{\widehat{\theta}} \theta q(\widehat{\theta}) - p(\widehat{\theta})$$

By the envelope theorem, we have $x'(\theta) = q(\theta)$. The following lemma is standard:

Lemma 1. The IC condition is satisfied if and only if the following two conditions hold: (i)

$$x(\theta) = x(\theta^*) + \int_{\theta^*}^{\theta} q(\tau) d\tau \text{ for all } \theta \in (\theta^*, \theta], \tag{3}$$

where $\theta^* \in [\underline{\theta}, \overline{\theta}]$ is the lowest type that purchases from the firm; (ii) $q(\theta)$ is increasing in θ.

By (3), the equilibrium rent provision for a type-θ consumer ($x(\theta)$) is completely determined by the rent for the lowest type covered ($x(\theta^*)$) and the quality provision schedule ($q(\cdot)$). Since $x(\theta^*)$ is provided to all consumers who are covered, we also refer to it as *the common rent provision*.

Note that $\{q(\theta), p(\theta)\}$ can be recovered from $x(\theta)$ as follows:

$$q(\theta) = x'(\theta) \text{ and } p(\theta) = \theta x'(\theta) - x(\theta)$$

Thus any menu of IC nonlinear pricing contract can be characterized by the rent provision schedules $x(\cdot)$. For this reason we will work with $x(\cdot)$ in characterizing the optimal monopolistic nonlinear pricing contract.

Given that our objective function contains a term of demand, we will demonstrate that in our case with entry, the individual rationality constraint (IR) may not bind for type θ^* (i.e., it is possible that $x(\theta^*) > 0$). This marks the first departure from the standard screening model. Given the definition of θ^*, it cannot be the case that $x(\theta^*) > 0$ while $\theta^* \in (\underline{\theta}, \overline{\theta}]$; otherwise a type-$\theta^*$ consumer would not be indifferent from accepting a contract and staying out (from being served). Thus we have either (1) $x(\theta^*) = 0$ and $\theta^* \in (\underline{\theta}, \overline{\theta}]$ or (2) $x(\theta^*) \geq 0$ and $\theta^* = \underline{\theta}$.

Given the IC menu of contracts offered in the final sale, the expected utility (gross of entry cost) for a consumer who enters the sale is given by

$$Ex = \int_{\theta^*}^{\overline{\theta}} x(\theta) dF(\theta) = \int_{\theta^*}^{\overline{\theta}} [\theta q(\theta) - p(\theta)] dF(\theta),$$

where $\theta^* \in \left[\underline{\theta}, \overline{\theta}\right]$ is the lowest type who purchases the product. Using Lemma 1, we have

$$\begin{aligned} Ex &= \int_{\theta^*}^{\overline{\theta}} \left[x(\theta^*) + \int_{\theta^*}^{\theta} q(\tau) d\tau \right] dF(\theta) \\ &= [1 - F(\theta^*)] x(\theta^*) + \int_{\theta^*}^{\overline{\theta}} [1 - F(\theta)] q(\theta) d\theta. \end{aligned} \quad (4)$$

In equilibrium, a consumer with cost type c_i enters the sale if and only if $c_i \le c^* \equiv Ex$. In other words, given $\{q(\theta), p(\theta)\}$, a total measure of $G(c^*)$ consumers will enter the sale. Hence $G(c^*)$ can also be interpreted as the *actual* market base of the product.

The firm's problem is thus given by

$$\begin{aligned} \max_{q(\cdot), p(\cdot)} \quad & G(c^*) \cdot \int_{\theta^*}^{\overline{\theta}} \{p(\theta) - c(q(\theta))\} dF(\theta) \\ \text{s.t.} \quad & x(\theta^*) \ge 0,\ \theta^* \in [\underline{\theta}, \overline{\theta}) \\ & x(\theta) = x(\theta^*) + \int_{\theta^*}^{\theta} q(\tau) d\tau \end{aligned}$$

Since $p(\theta) - c(q(\theta)) = \theta q(\theta) - c(q(\theta)) - x(\theta)$, using standard technique from the mechanism design literature, we can transform the above program into the following:

$$\begin{aligned} \max_{\theta^*, x(\cdot)} \quad & G\left(\int_{\theta^*}^{\overline{\theta}} x(\theta) dF(\theta)\right) \cdot \int_{\theta^*}^{\overline{\theta}} \left[\theta x'(\theta) - c(x'(\theta)) - x(\theta)\right] dF(\theta) \\ = \quad & G\left(\int_{\theta^*}^{\overline{\theta}} x(\theta) dF(\theta)\right) \cdot \left\{ \int_{\theta^*}^{\overline{\theta}} \left[\left(\theta - \frac{1 - F(\theta)}{f(\theta)}\right) x'(\theta) - c(x'(\theta))\right] dF(\theta) - [1 - F(\theta^*)] x(\theta^*) \right\} \\ \text{s.t.} \quad & x''(\theta) \ge 0, x(\theta^*) \ge 0, \theta^* \in [\underline{\theta}, \overline{\theta}) \end{aligned}$$

We will ignore the monotonicity constraint, $x''(\theta) = q'(\theta) \ge 0$, and obtain the solution first. After solving for the solution, we will then check for consistency.

Define

$$H(\theta, x(\theta), x'(\theta)) = \theta x'(\theta) - c(x'(\theta)) - x(\theta) \quad (5)$$

The firm's problem can be rewritten as follows:

$$\begin{aligned} \max_{\theta^*,x(\cdot)} \quad & G\left(\int_{\theta^*}^{\overline{\theta}} x(\theta)dF(\theta)\right)\cdot\int_{\theta^*}^{\overline{\theta}} H(\theta,x(\theta),x'(\theta))dF(\theta) \\ \text{s.t.} \quad & \theta^*\in[\underline{\theta},\overline{\theta}), x(\theta^*)\geq 0, x''(\theta)\geq 0, \text{and } x(\overline{\theta}) \text{ is free} \end{aligned}$$

Let θ^* and $x^*(\theta)$, $\theta^*\leq\theta\leq\overline{\theta}$ be optimal and consider a comparison function $x(\theta)$, $\theta^*-\delta\theta^*\leq\theta\leq\overline{\theta}$. The domains of the two functions may differ slightly with $\delta\theta^*$ small in absolute value but of any sign. Since the domains may not be identical, either x^* or x can be extended to have a common domain.[11] Let $h(\theta)=x(\theta)-x^*(\theta)$ be an admissible and arbitrary deviation function, and let $\delta\theta^*$ be a small fixed number.

The objective function evaluated at $\left\{x^*(\theta):\theta\in\left[\theta^*,\overline{\theta}\right]\right\}$ is as follows:

$$J^*=G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta)dF(\theta)\right)\cdot\int_{\theta^*}^{\overline{\theta}} H(\theta,x^*(\theta),x^{*\prime}(\theta))dF(\theta), \tag{6}$$

and the objective function evaluated at $\left\{x(\theta):\theta\in\left[\theta^*-\delta\theta^*,\overline{\theta}\right]\right\}$ is as follows:

$$\begin{aligned} J &= G\left(\int_{\theta^*-\delta\theta^*}^{\overline{\theta}} x(\theta)dF(\theta)\right)\cdot\int_{\theta^*-\delta\theta^*}^{\overline{\theta}} H(\theta,x(\theta),x'(\theta))dF(\theta) \\ &= G\left(\int_{\theta^*-\delta\theta^*}^{\theta^*} x(\theta)dF(\theta)+\int_{\theta^*}^{\overline{\theta}} x(\theta)dF(\theta)\right)\left\{\int_{\theta^*-\delta\theta^*}^{\theta^*} H(\theta,x(\theta),x'(\theta))dF(\theta)+\int_{\theta^*}^{\overline{\theta}} H(\theta,x(\theta),x'(\theta))dF(\theta)\right\} \end{aligned} \tag{7}$$

Using standard procedure to evaluate the first variation, $J-J^*$, we can obtain the following result:

Lemma 2. Under Assumption 1, the firm's optimal nonlinear pricing contract exhibits perfect sorting, which involves either no quality distortion or downward quality distortion.

Proof. See Appendix A. □

Next we identify conditions under which the two cases, with and without quality distortions, arise as the monopolistic optimal solution. To that end, we start with the expression of *virtual surplus* (Myerson, 1981) from the sale, which is given by

$$v(q,\theta)=\theta q(\theta)-c(q(\theta))-\frac{1-F(\theta)}{f(\theta)}q(\theta).$$

[11] This can be done, e.g., via linear extrapolations as introduced in Kamien and Schwartz (1991, page 57).

$v(q,\theta)$ can also be interpreted as the *marginal revenue (profit)* contributed from a type-θ buyer (Bulow and Roberts, 1989). We can compute the expected value of $v(q,\theta)$ when the first-best solution is offered (coupled with $x\left(\underline{\theta}\right)=0$), as follows:

$$
\begin{aligned}
Ev(q^{fb}) &= \int_{\underline{\theta}}^{\overline{\theta}} \left[(\theta - \frac{1-F(\theta)}{f(\theta)}) \cdot \theta - \frac{1}{2}\theta^2 \right] dF(\theta) \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \left[\frac{1}{2}\theta^2 - \frac{1-F(\theta)}{f(\theta)}\theta \right] dF(\theta) \\
&= \frac{1}{2}\left[\int_{\underline{\theta}}^{\overline{\theta}} \theta^2 dF(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} (1-F(\theta))\, d\theta^2 \right] \\
&= -\frac{1}{2}(1-F(\theta))\theta^2 |_{\underline{\theta}}^{\overline{\theta}} \\
&= \frac{1}{2}\underline{\theta}^2.
\end{aligned}
$$

So if the first-best solution is offered (coupled with $x\left(\underline{\theta}\right)=0$), then the expected profit a buyer who enters the sale contributes to the firm is equal to $\underline{\theta}^2/2$; On the other hand, under the first-best allocation with $x\left(\underline{\theta}\right)=0$, the entry cutoff type induced is given by

$$
c^{*fb} = \int_{\underline{\theta}}^{\overline{\theta}} x^{fb}(\theta) dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} q^{fb}(\tau) d\tau dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta^2 - \underline{\theta}^2}{2} dF(\theta). \tag{8}
$$

It turns out that the first best will arise as the monopolistic nonlinear pricing solution in our setting if and only if

$$
\frac{1}{2}\underline{\theta}^2 \geq \frac{G\left(c^{*fb}\right)}{G'\left(c^{*fb}\right)}, \tag{9}
$$

where c^{*fb} is given in (8).

Proposition 1. The monopolistic nonlinear pricing achieves the first best if and only if condition (9) holds; otherwise the monopolistic nonlinear pricing contract involves downward quality distortion for all but the highest type.

Proof. See Appendix A. □

While the proof in the appendix provides the formal arguments, Proposition 1 can be understood intuitively as follows. In our setting, the rent provision schedule $x(\cdot)$ determines the entry cutoff type c^* (or the actual market base, $G(c^*)$). Note that once c^* is determined, the expected payoff to each buyer is also determined. The reason is as follows: The cutoff type c^* makes zero expected payoff in equilibrium; Before entry, an entrant with a type $c_i < c^*$ differs from the cutoff type only in her entry cost; Thus, the equilibrium expected payoff to a buyer with type c_i is given by $c^* - c_i$. So in our setting, the monopolist extracts the rents from the buyers via the induced entry cutoff c^*, regardless of the specific nonlinear pricing contract offered in the sale – in this sense consumer rents are extracted by endogenous entry in our model.

It is thus clear that, given any targeted c^*, the monopolist has an incentive to make the quality provision as efficient as possible (since the expected rent to the buyers is fixed given c^*). An implication is, whenever feasible, the monopolist would offer the first-best quality provision, and let the targeted entry cutoff be induced by $x(\underline{\theta})$, the (nondistortionary) common rent provision to all types.

To check such "feasibility", we first consider the monopolist's profit maximization problem by fixing the quality provision schedule, $q(\cdot)$. Using (3), we can derive the expected payoff to a consumer before entry (or the cutoff entry type) as follows:

$$c^* = \int_{\underline{\theta}}^{\overline{\theta}} x(\theta) dF(\theta) = x(\theta^*)\left(1 - F(\theta^*)\right) + \int_{\underline{\theta}}^{\overline{\theta}} \int_{\theta^*}^{\theta} q(\tau) d\tau dF(\theta). \tag{10}$$

So given $q(\cdot)$, the monopolistic problem is to choose $x(\theta^*)$ to maximize the expected profit

$$\Pi = G(c^*) \cdot \int_{\underline{\theta}}^{\overline{\theta}} [\theta q(\theta) - c(\theta) - x(\theta)] dF(\theta) = G(c^*) \cdot \left[\int_{\underline{\theta}}^{\overline{\theta}} [(\theta q(\theta) - c(\theta))] dF(\theta) - c^* \right]. \tag{11}$$

Note that when $q(\cdot)$ is fixed, choosing $x(\theta^*)$ is equivalent to choosing c^*. Taking the derivative of (11) with respect to c^*, we derive an expression of marginal (expected) profit:

$$\begin{aligned} \frac{d\Pi}{dc^*} &= G'(c^*) \cdot \int_{\underline{\theta}}^{\overline{\theta}} [\theta q(\theta) - c(\theta) - x(\theta)] dF(\theta) - G(c^*) \\ &= G'(c^*) \cdot \left[\int_{\underline{\theta}}^{\overline{\theta}} [\theta q(\theta) - c(\theta) - x(\theta)] dF(\theta) - \frac{G(c^*)}{G'(c^*)} \right] \end{aligned} \tag{12}$$

The marginal profit expression derived above is fairly intuitive. By raising the expected rent provision by dc^*, an additional measure $G'(c^*)dc^*$ of consumers will enter the sale, which brings additional expected gain of $G'(c^*) \cdot \int_{\underline{\theta}}^{\overline{\theta}} [\theta q(\theta) - c(\theta) - x(\theta)] dF(\theta) \cdot dc^*$; on the other hand, the additional rent provision dc^* applies to all the consumers who enter, leading to a total additional "cost" of $G(c^*)dc^*$. Therefore marginal expected profit is given by (12).

Now we drop the assumption that $q(\cdot)$ is fixed and consider the optimal $q(\cdot)$. When (9) fails, $d\Pi/dc^* < 0$ at $q(\cdot) = q^{fb}(\cdot)$ and $x\left(\underline{\theta}\right) = 0$. This suggests that the first-best solution is not feasible; otherwise the "optimal" $x\left(\underline{\theta}\right)$ would have to be strictly negative, which violates the individual rationality constraint (IR) for type-$\underline{\theta}$ consumers after entry. In this case, the optimal quality provision schedule $q^*(\cdot)$ is determined by $d\Pi/dc^* = 0$, i.e.,

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\theta q^*(\theta) - c(\theta) - x^*(\theta)\right] dF(\theta) = \frac{G(c^*)}{G'(c^*)},$$

where $x^*(\theta^*) = 0$ and $c^* = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\theta^*}^{\theta} q^*(\tau) d\tau dF(\theta)$. This can be regarded as the "interior" solution case in which downward quality distortion occurs ($q^*(\cdot) \le q^{fb}(\cdot)$).[12]

Conversely, when (9) holds, the first-best solution is achieved ($q(\cdot) = q^{fb}(\cdot)$), and the optimal $x\left(\underline{\theta}\right)$ (and hence c^*) is chosen such that

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\theta q^{fb}(\theta) - c(\theta) - x^{fb}(\theta)\right] dF(\theta) = \frac{G(c^*)}{G'(c^*)},$$

where $c^* = x\left(\underline{\theta}\right) + \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} q^{fb}(\tau) d\tau dF(\theta)$. This can be regarded as the "corner" solution case in which the optimal solution achieves the first best (the "corner").

To further understand condition (9), we present the following two corollaries:

Corollary 1. The monopolist never achieves the first best if $\underline{\theta} = 0$.

Proof. (9) never holds with $\underline{\theta} = 0$. □

This result has an intuitive explanation. Note that the expected virtual surplus, $\underline{\theta}^2/2$, is strictly positive only if $\underline{\theta} > 0$. The expression of the expected virtual surplus also suggests that the higher $\underline{\theta}$, the more likely that the firm will offer the first-best contract.

Next we consider the case where θ is distributed uniformly. Let $\Delta = \left(\overline{\theta} - \underline{\theta}\right)$ be the range of the support, and define $\eta(c) \equiv G(c)/G'(c)$ for all $c \in \left[\underline{c}, \overline{c}\right]$. Then η^{-1} is well defined given Assumption 1.

Corollary 2. If θ is distributed uniformly over $\left[\underline{\theta}, \overline{\theta}\right]$, the monopoly solution achieves the first best if $0 < \Delta \le \Delta^*$, and involves downward quality distortion if $\Delta > \Delta^*$, where $\Delta^* = \left(\sqrt{\left(3\underline{\theta}\right)^2 + 24\eta^{-1}\left(\underline{\theta}^2/2\right)} - 3\underline{\theta}\right)/2$.

Proof. See Appendix A. □

Corollary 2 can be interpreted rather intuitively. If the range of support Δ is not too big, sorting via the first-best quality provision is optimal. However, if the range of support Δ is sufficiently large, sorting via the first-best quality provision is too costly for the monopolist: recall that the higher Δ, the larger the rent provision for the consumers by (3).

[12]Note that given Assumption 1, the solution determined by the first-order condition $d\Pi/dc^* = 0$ is indeed optimal.

Suppose the entry cost c is also distributed uniformly over, say, $[0, \overline{c}]$. Define the relative measure of consumers' vertical type heterogeneity $\gamma = \overline{\theta}/\underline{\theta}$, and let $\gamma^* = \left(\sqrt{21}-1\right)/2$. It can be verified that:

- the solution is the first best with full-market coverage if $\gamma \in (1, \gamma^*]$;
- the solution involves downward quality distortion and full coverage if $\gamma \in (\gamma^*, 4]$;
- the solution involves downward quality distortion and partial coverage if $\gamma > 4$.

So the smaller the relative measure of consumers' vertical type heterogeneity, the more likely that the first-best quality provision will be offered or the more likely that the market will be fully covered.

Our Proposition 1 also marks a striking difference from MR, as the first best can never occur in MR. More generally, we can show the following comparison result:

Proposition 2. Compared to the MR benchmark, in our model both quality distortion and market exclusion are smaller.

Proof. See Appendix. □

As discussed above, the nonlinear pricing contract affects consumers' payoffs only through its effect on the entry cutoff, which in turn determines the market base in the final sale. Unlike in the traditional monopolistic nonlinear pricing setting where quality distortion is necessary for rent extraction, in our setting rent extraction is pretty much taken care of by endogenous entry and the monopolist has an incentive to make the quality provision as efficient as possible. Intuitively, taking costly information acquisition into account, the monopolist has to balance entry (market share) and profit conditional on consumer entry. By reducing quality distortion and increasing market coverage (conditional on entry), the monopolist makes the product more attractive and induces an optimal set of entrants to maximize the expected profit.

It turns out that, even when Condition (9) fails, the monopolist can still achieve the first-best solution when given the ability to charge entry fees (before entry occurs), such as in the form of club membership fees.

Proposition 3. When the monopolist can charge entry fees, the optimal solution is always the first best.

Proof. See Appendix A. □

As the discussion following Proposition 1 reveals, when condition (9) fails, optimal entry under the first-best quality provision cannot be induced by any positive $x\left(\underline{\theta}\right)$. This is the sense in which the first-best solution is not *feasible*: if $x\left(\underline{\theta}\right)$ could be made negative, the first-best solution would still be justified. However, a negative $x\left(\underline{\theta}\right)$ would be a direct violation of the post-entry IR constraints for consumers with type $\underline{\theta}$ and also those with types sufficiently close to $\underline{\theta}$ (by continuity of the payoff function). This problem can be overcome with entry fee payments, which are collected before entry occurs. In a sense, by charging entry fees, the *ex post* IR constraint (imposed after buyers learn their θ_i's) is replaced by the *ex ante* IR constraint (imposed before buyers learn their θ_i's). It is clear that the requirement for *ex ante* IR is less stringent than that for the *ex post* IR, making the first-best solution easier to emerge. In fact, as Proposition 3 shows, the first best is always achieved when the option of charging entry fees is available for the monopolist.

So when condition (9) fails, charging entry fees (e.g., club membership fees) proves to be a superior instrument for the monopolist to maximize profit, as it does not distort production efficiency and is less constrained compared to the option of charging a higher price. With the ability to charge entry fees, the monopolist simply maximizes the production efficiency in the final sale by completely removing quality distortion, leaving the market base to be entirely adjusted by entry fees. It appears that the monopolistic profit maximization can be done sequentially: $q^{fb}(\cdot)$ is chosen to maximize expected surplus in the final sale, followed by an optimal entry fee s^* to induce an "optimal" set of entrants. Note that if (9) holds, the first best quality provision (along with an appropriate positive $x\left(\underline{\theta}\right)$) is optimal even when entry fees are not feasible.

In competitive nonlinear pricing settings with both vertically and horizontally differentiated products, Armstrong and Vickers (2001) and Rochet and Stole (2002) show that quality distortion disappears and the equilibrium is characterized by the cost-plus-fee pricing feature. The similar finding in our Proposition 3 thus suggests that entry has a similar effect as competition on nonlinear pricing schedules.[13] The exact workings, however, are quite different. In Armstrong and Vickers (2001) and Rochet and Stole (2002), each firm's competitive advantage is due to (horizontal) location rather than quality provision (with market being fully covered in the vertical dimension); thus there is no reason to distort quality. In our setting, on the other hand, consumer rents are "extracted" by endogenous entry (possibly along with entry fee payments); thus the firm does not have incentive to distort quality merely for rent extraction purpose.

As demonstrated above, even in the presence of a monopoly, the first-best solution can be

[13]It is clear that nonlinear pricing with entry fees is different from the standard two-part tariff scheme: under a two-part tariff, one does not need to pay the fixed fee unless the purchase is actually made; in our setting with entry fees, however, everyone who enters the sale has to pay the entry fee even if no purchase is made after entry (e.g., in the case of market exclusion).

achieved, either due to condition (9) or the ability to charge entry fees. So production distortion does not have to be associated with the existence of a monopoly, which may appear inconsistent with the basic wisdom from a microeconomics textbook. However, as the following proposition illustrates, even when production inefficiency is absent in our setting with costly information acquisition, a distortion will arise in the form of inefficient entry.

Proposition 4. The monopoly induces insufficient entry compared to the socially optimal entry level.

Proof. See Appendix A. □

In the socially efficient benchmark, the social planner maximizes the expected total social surplus, which is equal to the expected total surplus generated from the sale less the expected total information acquisition cost. We show in the appendix that the socially optimal solution is characterized by first-best quality provision coupled with full entry subsidization: all the rents are given back to the consumers in the form of entry subsidies. While formal arguments are provided in the proof in the appendix, this result can be explained intuitively. In equilibrium, a potential buyer will enter the sale if and only if her expected profit from entry is larger than her entry cost. Such entry is socially efficient as (1) there is no production inefficiency due to the first-best quality provision, and (2) a potential buyer enters if and only if the expected profit, and hence her contribution to the social surplus, is greater than zero.

In general, in our setting monopolistic inefficiency takes the form of both production distortion and entry distortion. Even when production distortion is absent (under condition (9) or with the ability to charge entry fees), entry distortion persists. Our model thus suggests a subtle implication for anti-trust experts in nonlinear pricing settings with costly information acquisition.

4 Conclusion

To our knowledge, this paper is the first to explicitly model costly information acquisition on consumer preferences in a nonlinear pricing framework. Despite the potential complication with sequential screening under two-dimensional private information, we are able to completely characterize the optimal monopolistic nonlinear pricing contracts. We demonstrate that the optimal nonlinear pricing in our setting involves either downward quality distortion or no distortion; even in the former case, as long as the monopolist can charge entry fees, the first best always emerges as the optimal monopolistic solution. It is worth noting that the first-best solution is belief-free, as it does not depend on the firm's knowledge about the underlying distributions.

In this sense we have presented an analysis where, despite the presumably more complicated setup, the solution may turn out to be relatively simple.

We also show that, compared to the benchmark without costly information acquisition, quality distortion is unambiguously reduced and in some cases completely disappears. Furthermore, in our setting market coverage has unambiguously increased. These results are intuitive, and, we believe, are potentially testable empirically. In particular, our model predicts that quality distortion should be smaller for less familiar or relatively new products or services and bigger for more familiar or better established products or services.

By and large, this research produces some key results that are very similar to those obtained from the existing literature on competitive nonlinear pricing, suggesting that the effect of entry on monopolistic nonlinear pricing is very similar to the effect of competition on nonlinear pricing in the context of multiple firms. However, in our setting with a monopoly, even when production distortion is absent, entry distortion persists. This points to a subtle policy implication for antitrust experts.

With the increasing number of new products introduced every year, consumer entry is becoming increasingly critical for the market viability of the new products. The analysis of this research sheds some light on how a firm will adjust its nonlinear pricing schedule in response to consumer entry.

Our analysis relies on some simplifying assumptions. First of all, we assume that the two-dimensional private information, c and θ, are independent, as there does not seem to be a consensus in the literature over whether they should be positively or negatively correlated. Nevertheless, a more general analysis allowing for correlation between c and θ may potentially produce more insights. Second of all, our current analysis is restricted to a monopoly regime. A more general analysis should extend costly information acquisition to a competitive setting with more than one firm. Note that the MR model is inappropriate for such an extension, as the post-entry competition between firms would result in an unrealistic Bertrand outcome. Therefore it is not trivial to incorporate information acquisition in a competitive nonlinear pricing framework. Given the challenge in extending our current analysis along those directions, we leave these questions for future research.

Appendix A: Long Proofs

Proof of Lemma 2: Following (7), we have

$$
\begin{aligned}
J &\approx G\left(x(\theta^*)f(\theta^*)\delta\theta^* + \int_{\theta^*}^{\overline{\theta}} x^*(\theta)f(\theta)d\theta + \int_{\theta^*}^{\overline{\theta}} h(\theta)f(\theta)d\theta\right) \\
&\cdot \left\{ \begin{array}{c} H\left(\theta, x^*(\theta), x^{*\prime}(\theta)\right) f(\theta^*)\delta\theta^* \\ + \int_{\theta^*}^{\overline{\theta}} \left[H\left(\theta, x^*(\theta), x^{*\prime}(\theta)\right) + H_x\left(\theta, x^*(\theta), x^{*\prime}(\theta)\right) h(\theta) + H_{x'}\left(\theta, x^*(\theta), x^{*\prime}(\theta)\right) h'(\theta)\right] dF(\theta) \end{array} \right\} \\
&\approx \left[G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta)f(\theta)d\theta\right) + G'\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta)f(\theta)d\theta\right) \cdot \left(\int_{\theta^*}^{\overline{\theta}} h(\theta)f(\theta)d\theta + x(\theta^*)f(\theta^*)\delta\theta^*\right)\right] \\
&\cdot \left[\begin{array}{c} \int_{\theta^*}^{\overline{\theta}} H(\theta, x^*(\theta), x^{*\prime}(\theta))dF(\theta) + H(\theta, x^*(\theta), x^{*\prime}(\theta))f(\theta^*)\delta\theta^* \\ + \int_{\theta^*}^{\overline{\theta}} \{H_x(\theta, x^*(\theta), x^{*\prime}(\theta))h(\theta) + H_{x'}(\theta, x^*(\theta), x^{*\prime}(\theta))h'(\theta)\} dF(\theta) \end{array} \right]
\end{aligned} \tag{13}
$$

Using (6) and (13), we have from the optimality of x^*,

$$
\begin{aligned}
\frac{J - J^*}{G} &\approx \int_{\theta^*}^{\overline{\theta}} \left\{1 - b + \left[H_x\left(\theta, x^*(\theta), x^{*\prime}(\theta)\right) - \frac{d\left[H_{x'}\left(\theta, x^*(\theta), x^{*\prime}(\theta)\right) f(\theta)\right]}{f(\theta)d\theta}\right]\right\} h(\theta)dF(\theta) \\
&\quad + H_{x'}\left(\overline{\theta}, x^*(\overline{\theta}), x^{*\prime}(\overline{\theta})\right) f(\overline{\theta})h(\overline{\theta}) + \left[\frac{1}{2}(x'(\theta^*))^2 - b \cdot x\left(\theta^*\right)\right] f(\theta^*) \cdot \delta\theta^* \\
&\quad - \left[\theta^* - c'(x'(\theta^*))\right] f(\theta^*) \cdot \delta x\left(\theta^*\right) \\
&\le 0,
\end{aligned} \tag{14}
$$

where

$$
\begin{aligned}
b &= 1 - \frac{G'}{G} \cdot \int_{\theta^*}^{\overline{\theta}} H\left(\theta, x^*(\theta), x^{*\prime}(\theta)\right) dF(\theta) \\
&= 1 - \frac{G'}{G} \left\{ \int_{\theta^*}^{\overline{\theta}} \left[\theta q(\theta) - \frac{1}{2}(q(\theta))^2 - \frac{1 - F(\theta)}{f(\theta)} q(\theta)\right] dF(\theta) - \left[1 - F(\theta^*)\right] x\left(\theta^*\right) \right\} \\
G &= G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta)f(\theta)d\theta\right)
\end{aligned}
$$

Note that in the optimal solution, $\int_{\theta^*}^{\overline{\theta}} H\left(\theta, x^*(\theta), x^{*\prime}(\theta)\right) dF(\theta)$ cannot be strictly negative, as the monopolist can guarantee zero profit by not offering any contract. Thus we must have $b \le 1$.

Substituting (5) into (14) and simplifying, we have

$$
\begin{aligned}
\frac{J - J^*}{G} &\approx \int_{\theta^*}^{\overline{\theta}} \left\{-b - \frac{d\left[(\theta - q(\theta))f(\theta)\right]}{f(\theta)d\theta}\right\} h(\theta)dF(\theta) + \left[\overline{\theta} - q(\overline{\theta})\right] f(\overline{\theta})h(\overline{\theta}) \\
&\quad + \left[\frac{1}{2}q(\theta^*)^2 - bx\left(\theta^*\right)\right] f(\theta^*) \cdot \delta\theta^* - \left[\theta^* - q(\theta^*)\right] f(\theta^*) \cdot \delta x\left(\theta^*\right)
\end{aligned}
$$

$$\leq \quad 0, \tag{15}$$

where

$$G = G\left(\int_{\theta^*}^{\overline{\theta}} \frac{1-F(\theta)}{f(\theta)} q(\theta) dF(\theta) + \left[1-F(\theta^*)\right] x\left(\theta^*\right)\right).$$

Since $h(\overline{\theta})$ is free, we have immediately $q(\overline{\theta}) = \overline{\theta}$ (efficiency at the top). Since $h(\theta)$ is an arbitrary admissible deviation, we also have

$$b = -\frac{d\left[(\theta - q(\theta)) f(\theta)\right]}{f(\theta) d\theta},$$

which, combined with $q(\overline{\theta}) = \overline{\theta}$, gives

$$q(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)} \cdot b \tag{16}$$

Substituting (16) into (15), we have

$$\frac{J-J^*}{G} \approx \left[\frac{1}{2} q(\theta^*)^2 - b \cdot x\left(\theta^*\right)\right] f(\theta^*) \cdot \delta\theta^* - \left[\theta^* - q(\theta^*)\right] f(\theta^*) \cdot \delta x\left(\theta^*\right) \leq 0$$

If $x(\theta^*) > 0$, $\delta x(\theta^*)$ can be both positive and negative. We then have $q(\theta^*) = \theta^*$ which, by (16), implies that $b = 0$. This in turn implies that the solution is first best. In this case, we also have $\theta^* = \underline{\theta}$ (otherwise θ^* cannot be the lowest type served in the market).

If $x(\theta^*) = 0$, $\delta x(\theta^*)$ can only be positive, thus we have $q(\theta^*) \leq \theta^*$, which implies $b \geq 0$. When $\theta^* > \underline{\theta}$, $\delta\theta^*$ can be both positive and negative, hence we have $q(\theta^*) = 0$ and $b = \theta^* f(\theta^*)/(1-F(\theta^*))$; when $\theta^* = \underline{\theta}$, $\delta\theta^*$ can only be negative, and we have $q(\underline{\theta}) \geq 0$, which implies $\underline{\theta} f\left(\underline{\theta}\right) \geq b \geq 0$.

We thus show that $b \geq 0$ in all cases. Under Assumption 1, $q(\theta)$ given in (16) is strictly increasing in θ, so the optimal monopolistic nonlinear pricing contract is perfect sorting (which also justifies the relaxation of the monotonicity constraint).

Since we have argued previously that $b \leq 1$, we conclude that $0 \leq b \leq 1$, i.e., the firm's optimal solution involves either no quality distortion ($b = 0$) or downward quality distortion ($0 < b \leq 1$).

Proof of Proposition 1: We first show that the first best is achieved if and only if (9) holds. We start with sufficiency. By Assumption 1 and continuity, (9) implies that there exists a unique $\underline{x} \in \left[0, \underline{\theta}^2/2\right]$, such that

$$\frac{1}{2}\underline{\theta}^2 = \underline{x} + \frac{G}{G'},$$

where $G = G\left(\int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta^2 - \underline{\theta}^2}{2} dF(\theta) + \underline{x}\right)$.

Thus we can choose $x\left(\underline{\theta}\right) = \underline{x} \geq 0$, such that

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\frac{1}{2}\theta^2 - \frac{1-F(\theta)}{f(\theta)}\theta\right] dF(\theta) - x\left(\underline{\theta}\right) = \frac{1}{2}\underline{\theta}^2 - x\left(\underline{\theta}\right) = \frac{G}{G'},$$

where $G = G\left(\int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta^2 - \underline{\theta}^2}{2} dF(\theta) + x\left(\underline{\theta}\right)\right)$. This implies

$$b = 1 - \frac{G'}{G}\left\{\int_{\underline{\theta}}^{\overline{\theta}} \left[\theta q^{fb}(\theta) - \frac{1}{2}(q^{fb}(\theta))^2 - \frac{1-F(\theta)}{f(\theta)} q^{fb}(\theta)\right] dF(\theta) - x\left(\underline{\theta}\right)\right\} = 0,$$

where $G = G\left(\int_{\underline{\theta}}^{\overline{\theta}} \frac{1-F(\theta)}{f(\theta)} q^{fb}(\theta) dF(\theta) + x\left(\underline{\theta}\right)\right)$.

As such, the schedule $x(\cdot)$ defined by $x(\theta) = \int_{\underline{\theta}}^{\theta} t dt + x\left(\underline{\theta}\right)$, where $x\left(\underline{\theta}\right)$ is chosen above, verifies the variational inequalities (16): $q(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)} b$, and

$$\begin{aligned} & \left[\frac{1}{2}q^2(\theta^*) - b \cdot x\left(\theta^*\right)\right] f(\theta^*) \cdot \delta\theta^* - \left[\theta^* - q(\theta^*)\right] f(\theta^*) \cdot \delta x\left(\theta^*\right) \\ = & \left[\frac{1}{2}\underline{\theta}^2 - b \cdot x\left(\underline{\theta}\right)\right] f(\underline{\theta}) \cdot \delta\underline{\theta} - \left[\underline{\theta} - q(\underline{\theta})\right] f(\underline{\theta}) \cdot \delta x\left(\underline{\theta}\right) \\ = & \frac{1}{2}\underline{\theta}^2 f(\underline{\theta}) \cdot \delta\underline{\theta} \leq 0 \end{aligned}$$

Given Assumption 1, the above variational inequalities are also sufficient for optimality (Appendix B). Therefore, we have shown that under condition (9), the monopolistic solution is the first best, and $\theta^* = \underline{\theta}$, $x\left(\underline{\theta}\right) \geq 0$.

Next we show necessity. Suppose that the monopolistic solution achieves the first best, $q(\theta) = \theta$, $\theta \in [\underline{\theta}, \overline{\theta}]$ and $\theta^* = \underline{\theta}$, $x\left(\underline{\theta}\right) \geq 0$, then $x(\theta) = \int_{\underline{\theta}}^{\theta} t dt + x\left(\underline{\theta}\right)$, $\theta \in [\underline{\theta}, \overline{\theta}]$, and $x\left(\underline{\theta}\right) \geq 0$. Substituting this into $b = 0$, we have

$$\int_{\underline{\theta}}^{\overline{\theta}} \left[\frac{1}{2}\theta^2 - \frac{1-F(\theta)}{f(\theta)}\theta\right] dF(\theta) - x\left(\underline{\theta}\right) = \frac{G\left(\int_{\underline{\theta}}^{\overline{\theta}} \frac{1-F(\theta)}{f(\theta)}\theta dF(\theta) + x\left(\underline{\theta}\right)\right)}{G'\left(\int_{\underline{\theta}}^{\overline{\theta}} \frac{1-F(\theta)}{f(\theta)}\theta dF(\theta) + x\left(\underline{\theta}\right)\right)},$$

which implies (9), given Assumption 1 and continuity.

The second part of the proposition follows by invoking Lemma 2.

Proof of Corollary 2: Plugging $F(\theta) = \left(\theta - \underline{\theta}\right)/\left(\overline{\theta} - \underline{\theta}\right)$ into (9), we have

$$\begin{aligned} \frac{\underline{\theta}^2}{2} &\geq \frac{G\left(\frac{\overline{\theta}+2\underline{\theta}}{3}\left(\overline{\theta}-\underline{\theta}\right)\right)}{G'\left(\frac{\overline{\theta}+2\underline{\theta}}{3}\left(\overline{\theta}-\underline{\theta}\right)\right)}, \text{ or} \\ \eta^{-1}\left(\frac{\underline{\theta}^2}{2}\right) &\geq \frac{\overline{\theta}+2\underline{\theta}}{3}\left(\overline{\theta}-\underline{\theta}\right), \end{aligned}$$

which gives rise to $0 < \Delta \leq \left(\sqrt{\left(3\underline{\theta}\right)^2 + 24\eta^{-1}\left(\underline{\theta}^2/2\right)} - 3\underline{\theta}\right)/2$. When $\Delta > \left(\sqrt{\left(3\underline{\theta}\right)^2 + 24\eta^{-1}\left(\underline{\theta}^2/2\right)} - 3\underline{\theta}\right)/2$, downward distortion follows.

Proof of Proposition 2: This is trivial under condition (9). When (9) is violated, the firm's optimal solution is given by $q(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)}b$, $\theta \in [\theta^*, \overline{\theta}]$, $1 \geq b > 0$, and $\theta^* = \underline{\theta}$, or $b\frac{1-F(\theta^*)}{f(\theta^*)}$ (the proof of Lemma 2).

In MR (1978), the solution is given by (1) and (2). Thus we have

$$q(\theta) - q^{MR}(\theta) = \frac{1-F(\theta)}{f(\theta)}(1-b) \geq 0,$$

which means that quality distortion is smaller than in MR.

The lowest type being served is trivially (weakly) lower when $\theta^* = \underline{\theta}$ compared to the MR benchmark. When $\theta^* = b\frac{1-F(\theta^*)}{f(\theta^*)}$,

$$\begin{aligned} \theta^* - \frac{1-F(\theta^*)}{f(\theta^*)} &= -(1-b)\frac{1-F(\theta^*)}{f(\theta^*)} \\ &\leq 0 \\ &\leq \theta^*_{MR} - \frac{1-F(\theta^*_{MR})}{f(\theta^*_{MR})} \end{aligned}$$

By Assumption 1, $\theta^* \leq \theta^*_{MR}$.

Proof of Proposition 3: We consider a more general setting where the monopolist can provide an entry subsidy s to every consumer who enters, where s can be both positive and negative (a negative s is an *entry fee*). The firm's problem can be formulated as follows:

$$\begin{aligned} \max_{s,x(\theta)} \quad & G\left(\int_{\theta^*}^{\overline{\theta}} x(\theta)dF(\theta) + s\right) \cdot \left\{\int_{\theta^*}^{\overline{\theta}} H(\theta, x(\theta), x'(\theta))dF(\theta) - s\right\} \\ & \text{s.t. } x(\theta^*) \geq 0, \theta^* \geq \underline{\theta}, x''(\theta) \geq 0 \end{aligned}$$

Suppose that the optimal solution to the maximization problem above exists and is given by $\left\{s^*, x^*(\theta) : \theta \in \left[\theta^*, \overline{\theta}\right]\right\}$. Let $\left\{s, x(\theta) : \theta \in \left[\theta^* - \delta\theta^*, \overline{\theta}\right]\right\}$ denote some other admissible function. Then $x(\theta) = x^*(\theta) + h(\theta)$ and $s = s^* + \delta s$.[14]

The values of the objective function evaluated at $\{s^*, x^*(\theta)$: $\theta \in \left[\theta^*, \overline{\theta}\right]\}$ and $\{s, x(\theta)$: $\theta \in \left[\theta^* - \delta\theta^*, \overline{\theta}\right]\}$ are given by, respectively,

$$
\begin{aligned}
J^* &= G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta) dF(\theta) + s^*\right) \cdot \left\{\int_{\theta^*}^{\overline{\theta}} H\left(\theta, x^*(\theta), x^{*\prime}(\theta)\right) dF(\theta) - s^*\right\}, \\
J &= G\left(\int_{\theta^* - \delta\theta^*}^{\overline{\theta}} x(\theta) dF(\theta) + s\right) \cdot \left\{\int_{\theta^* - \delta\theta^*}^{\overline{\theta}} H(\theta, x(\theta), x'(\theta)) dF(\theta) - s\right\} \\
&= G\left(\int_{\theta^* - \delta\theta^*}^{\theta^*} x(\theta) dF(\theta) + \int_{\theta^*}^{\overline{\theta}} x(\theta) dF(\theta) + s\right) \cdot \left\{ \begin{array}{c} \int_{\theta^* - \delta\theta^*}^{\theta^*} H(\theta, x(\theta), x'(\theta)) dF(\theta) \\ + \int_{\theta^*}^{\overline{\theta}} H(\theta, x(\theta), x'(\theta)) dF(\theta) - s \end{array} \right\}
\end{aligned}
$$

Following similar steps as in the proof of Proposition 1, we have

$$
\begin{aligned}
J - J^* \approx\; & \int_{\theta^*}^{\overline{\theta}} \left\{ \begin{array}{c} G'\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta) dF(\theta) + s^*\right) \cdot \{\int_{\theta^*}^{\overline{\theta}} H\left(\theta, x^*(\theta), x^{*\prime}(\theta)\right) dF(\theta) - s^*\} \\ + G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta) dF(\theta) + s^*\right) \left[H_x\left(\theta, x^*(\theta), x^{*\prime}(\theta)\right) - \frac{d\{H_{x'}(\theta, x^*(\theta), x^{*\prime}(\theta)) f(\theta)\}}{f(\theta) d\theta}\right] \end{array} \right\} h(\theta) dF(\theta) \\
& + G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta) dF(\theta) + s^*\right) \cdot H_{x'}\left(\overline{\theta}, x^*(\overline{\theta}), x^{*\prime}(\overline{\theta})\right) f(\overline{\theta}) h(\overline{\theta}) \\
& + \left\{ \begin{array}{c} G'\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta) dF(\theta) + s^*\right) \cdot \left(\int_{\theta^*}^{\overline{\theta}} H\left(\theta, x^*(\theta), x^{*\prime}(\theta)\right) dF(\theta) - s^*\right) \\ - G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta) dF(\theta) + s^*\right) \end{array} \right\} \cdot \delta s \\
& - G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta) dF(\theta) + s^*\right) \cdot H_{x'}\left(\theta^*, x^*(\theta^*), x^{*\prime}(\theta^*)\right) f(\theta^*) h(\theta^*) \\
& + G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta) dF(\theta) + s^*\right) \cdot H\left(\theta^*, x^*(\theta^*), x^{*\prime}(\theta^*)\right) f(\theta^*) \delta\theta^* \\
& + G'\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta) dF(\theta) + s^*\right) \cdot \left[\int_{\theta^*}^{\overline{\theta}} H\left(\theta, x^*(\theta), x^{*\prime}(\theta)\right) dF(\theta) - s^*\right] \cdot x(\theta^*) f(\theta^*) \delta\theta^* \\
\leq\; & 0
\end{aligned}
$$

Since $h(\theta)$ is an arbitrary admissible deviation, we have $H_{x'}\left(\overline{\theta}, x^*(\overline{\theta}), x^{*\prime}(\overline{\theta})\right) = 0$, which leads to $q(\overline{\theta}) = \overline{\theta}$ (i.e., efficient quality provision at the top), and

$$
\left\{ \begin{array}{c} G'\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta) dF(\theta) + s^*\right) \cdot \{\int_{\theta^*}^{\overline{\theta}} H\left(\theta, x^*(\theta), x^{*\prime}(\theta)\right) dF(\theta) - s^*\} \\ + G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta) dF(\theta) + s^*\right) \left[H_x\left(\theta, x^*(\theta), x^{*\prime}(\theta)\right) - \frac{d\{H_{x'}(\theta, x^*(\theta), x^{*\prime}(\theta)) f(\theta)\}}{f(\theta) d\theta}\right] \end{array} \right\} = 0,
$$

[14]Again, since the domains may not be identical, either x^* or x can be extended to have a common domain (e.g., via linear extrapolations).

or

$$\frac{d\{H_{x'}\left(\theta,x^*(\theta),x^{*\prime}(\theta)\right)f(\theta)\}}{f(\theta)d\theta} = H_x\left(\theta,x^*(\theta),x^{*\prime}(\theta)\right) + \frac{G'}{G}\left\{\int_{\theta^*}^{\overline{\theta}} H\left(\theta,x^*(\theta),x^{*\prime}(\theta)\right)dF(\theta) - s^*\right\},$$

$$\frac{d\{(\theta-q(\theta))f(\theta)\}}{f(\theta)d\theta} = -1 + \frac{G'}{G}\left\{\int_{\theta^*}^{\overline{\theta}} H\left(\theta,x^*(\theta),x^{*\prime}(\theta)\right)dF(\theta) - s^*\right\} \quad (17)$$

where $G = G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta)f(\theta)d\theta + s^*\right)$.

Since δs can be both positive and negative,[15] we must have

$$G' \cdot \left(\int_{\theta^*}^{\overline{\theta}} H\left(\theta,x^*(\theta),x^{*\prime}(\theta)\right)dF(\theta) - s^*\right) - G = 0, \quad (18)$$

where $G = G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta)f(\theta)d\theta + s^*\right)$. Substituting (18) into (17), and using the boundary condition $q(\overline{\theta}) = \overline{\theta}$, we have

$$q(\theta) = \theta \text{ for all } \theta \in \left[\theta^*, \overline{\theta}\right] \quad (19)$$

That is, efficient quality is provided for all consumers served.

Next we identify the optimal θ^*. Note that

$$\delta x\left(\theta^*\right) = x\left(\theta^* - \delta\theta^*\right) - x^*\left(\theta^*\right) \approx x\left(\theta^*\right) - x^{*\prime}\left(\theta^*\right)\delta\theta^* - x^*\left(\theta^*\right) = h\left(\theta^*\right) - x^{*\prime}\left(\theta^*\right)\delta\theta^*. \quad (20)$$

Substituting (18)-(20) into the expression of $(J - J^*)$, and manipulating terms, we have

$$\begin{aligned}\frac{J-J^*}{f(\theta^*)G} &\approx \left\{\begin{array}{c} H\left(\theta^*,x^*(\theta^*),x^{*\prime}(\theta^*)\right) - x^{*\prime}(\theta^*)\cdot H_{x'}\left(\theta^*,x^*(\theta^*),x^{*\prime}(\theta^*)\right) \\ +\frac{G'}{G}\cdot x(\theta^*)\cdot\left[\int_{\theta^*}^{\overline{\theta}} H\left(\theta^*,x^*(\theta^*),x^{*\prime}(\theta^*)\right)dF(\theta) - s^*\right]\end{array}\right\}\cdot\delta\theta^* \\ &\quad -H_{x'}\left(\theta^*,x^*(\theta^*),x^{*\prime}(\theta^*)\right)\delta x\left(\theta^*\right) \\ &\leq 0,\end{aligned}$$

where $G = G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta)f(\theta)d\theta + s^*\right)$.

Substituting $H(\theta,x(\theta),x'(\theta)) = \theta x'(\theta) - (x'(\theta))^2/2 - x(\theta)$ into the above equation, we have

$$\frac{J-J^*}{f(\theta^*)G} \approx \frac{1}{2}(\theta^*)^2 \cdot \delta\theta^* \leq 0$$

So $\delta\theta^*$ cannot be positive, which suggests that we must have $\theta^* = \underline{\theta}$ (full coverage).

Substituting the first-best solution (i.e., $x(\theta) = x\left(\underline{\theta}\right) + \int_{\underline{\theta}}^{\theta} \tau d\tau$) into (18), we can derive the

[15] Note that this is the difference between using an entry subsidy and supplying $x(\theta^*) = x_*$: x_* has to be nonnegative.

following condition that determines the optimal entry subsidy, s^*:

$$z = \frac{\underline{\theta}^2}{2} - \frac{G\left(\int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta^2-\underline{\theta}^2}{2} dF(\theta) + z\right)}{G'\left(\int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta^2-\underline{\theta}^2}{2} dF(\theta) + z\right)},$$

where $z = x\left(\underline{\theta}\right) + s^*$, which is the sum of the common rent for all types ($x\left(\underline{\theta}\right)$) and a net entry subsidy (s^*). z can be regarded as the *compound subsidy*. By Assumption 1 and continuity, we have

$$\begin{cases} z \geq 0 & \text{if} \quad \frac{\underline{\theta}^2}{2} \geq G/G'\left(\int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta^2-\underline{\theta}^2}{2} dF(\theta)\right) \\ z < 0 & \text{if} \quad \frac{\underline{\theta}^2}{2} < G/G'\left(\int_{\underline{\theta}}^{\overline{\theta}} \frac{\theta^2-\underline{\theta}^2}{2} dF(\theta)\right) \end{cases} \tag{21}$$

Apparently, the optimal (net) subsidy $s^* = z - x\left(\underline{\theta}\right)$ can be both negative or positive: when condition (9) holds, $z \geq 0$ and the optimal subsidy can be positive (or simply zero); when condition (9) is violated, $z < 0$ and the optimal subsidy has to be negative (which is meant for entry fees). It is the latter case that the ability of charging entry fees is required for the first-best solution to emerge in the optimal nonlinear pricing contract.

Thus when the monopolist can charge entry fees, it is always optimal to offer the first-best quality provision to all the consumers who enter the sale.

Proof of Proposition 4: First, we consider the case where the social planner can use entry subsidies. The social planner's objective is to maximize the expected total social surplus, which is the expected total surplus generated from the sale less the expected total information acquisition cost, by choosing an entry subsidy level, s, and the nonlinear pricing contract represented by $x(\cdot)$. Letting $c^* = \int_{\theta^*}^{\overline{\theta}} x(\theta) dF(\theta) + s$, the social planner's maximization program is as follows:

$$\begin{aligned} \max_{s,x(\theta)} \quad & G\left(c^*\right) \cdot \int_{\theta^*}^{\overline{\theta}} \left[\theta x'(\theta) - c(x'(\theta))\right] dF(\theta) - \int_{\underline{c}}^{c^*} c dG(c) \\ & \text{s.t. } x(\theta^*) \geq 0,\, \theta^* \geq \underline{\theta},\, x''(\theta) \geq 0,\, s \leq \int_{\theta^*}^{\overline{\theta}} \left[\theta x'(\theta) - c(x'(\theta)) - x(\theta)\right] dF(\theta) \end{aligned}$$

Note that the last constraint is the "resource" constraint: the total entry subsidy cannot exceed the total expected profit. We will ignore all the constraints to obtain the solution first. We will perform the consistency check later.

Suppose that the optimal solution to the maximization problem above exists and is given by $(s^*, x^*(\theta))$. Let $x(\theta)$ denote any given admissible function, and $h(\theta) = x(\theta) - x^*(\theta)$ be the resulting admissible deviation. Let

$$\begin{aligned} y(\theta) &= x^*(\theta)+ah(\theta), \text{ for some constant } a, \text{ and} \\ s &= s^* + a\cdot\delta s. \end{aligned}$$

Finally, let $k(a)$ be the value of the objective function evaluated at $(s, y(\theta))$. Then

$$\begin{aligned} k(a) &= G\left(\int_{\theta^*}^{\overline{\theta}} y(\theta)dF(\theta)+s\right)\cdot\int_{\theta^*}^{\overline{\theta}}\left[\theta y'(\theta)-c(y'(\theta))\right]dF(\theta)-\int_{\underline{c}}^{\int_{\theta^*}^{\overline{\theta}} y(\theta)dF(\theta)+s} cdG(c) \\ &= G\left(\int_{\theta^*}^{\overline{\theta}}\left(x^*(\theta)+ah(\theta)\right)dF(\theta)+s^*+a\delta s\right)\cdot\int_{\theta^*}^{\overline{\theta}}\left[\theta\left(x'^*(\theta)+ah'(\theta)\right)-c(x'^*(\theta)+ah'(\theta))\right]dF(\theta) \\ &\quad -\int_{\underline{c}}^{\int_{\theta^*}^{\overline{\theta}}(x^*(\theta)+ah(\theta))dF(\theta)+s^*+a\delta s} cdG(c) \end{aligned}$$

By the optimality of $(s^*, x^*(\theta))$, $k(a)$ achieves its maximum at $a=0$. This then implies that $k'(0)=0$. Tedious but straightforward calculation leads to

$$\begin{aligned} k'(0) &= \int_{\theta^*}^{\overline{\theta}}\left\{\begin{array}{c} G'\cdot f(\theta)\cdot\int_{\theta^*}^{\overline{\theta}}\left[\theta x'^*(\theta)-c(x'^*(\theta))\right]dF(\theta)-G\frac{d[\theta-c'(x'^*(\theta))]}{d\theta}f(\theta) \\ -\left[\int_{\theta^*}^{\overline{\theta}} x^*(\theta)f(\theta)d\theta+s^*\right]\cdot f(\theta)\cdot G' \end{array}\right\}h(\theta)d\theta \\ &\quad +G\cdot\left[\theta-c'(x'^*(\theta))\right]f(\theta)h(\theta)|_{\theta=\overline{\theta}} \\ &\quad +\left\{G'\cdot\int_{\theta^*}^{\overline{\theta}}\left[\theta x'^*(\theta)-c(x'^*(\theta))\right]dF(\theta)-\left[\int_{\theta^*}^{\overline{\theta}} x^*(\theta)f(\theta)d\theta+s^*\right]\cdot G'\right\}\cdot\delta s \end{aligned}$$

where $G = G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta)dF(\theta)+s^*\right)$.

Since $h(\cdot)$ and δs are arbitrarily given deviations, the necessary conditions for optimality are characterized by the following equation system:

$$\begin{aligned} G'\cdot\int_{\theta^*}^{\overline{\theta}}\left[\theta x'^*(\theta)-c(x'^*(\theta))\right]dF(\theta)-G\frac{d\left[\theta-c'(x'^*(\theta))\right]}{d\theta}-\left[\int_{\theta^*}^{\overline{\theta}} x^*(\theta)f(\theta)d\theta+s^*\right]\cdot G' &= 0 \\ \left[\theta-c'(x'^*(\theta))\right]|_{\theta=\overline{\theta}} &= 0 \\ G'\cdot\int_{\theta^*}^{\overline{\theta}}\left[\theta x'^*(\theta)-c(x'^*(\theta))\right]dF(\theta)-\left[\int_{\theta^*}^{\overline{\theta}} x^*(\theta)f(\theta)d\theta+s^*\right]\cdot G' &= 0 \end{aligned}$$

Solving, the above necessary conditions lead to (1) $\theta^*=\underline{\theta}$, $q(\theta)=\theta$, $\theta\in[\underline{\theta},\overline{\theta}]$, i.e., the socially efficient nonlinear pricing is the first-best; and (2) the optimal subsidy is given by

$$\begin{aligned} s^* &= \int_{\theta^*}^{\overline{\theta}} \left[\theta x'^*(\theta) - c(x'^*(\theta)) - x^*(\theta)\right] dF(\theta) \\ &= \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta^2 - \frac{1}{2}\theta^2 - \frac{1-F(\theta)}{f(\theta)}\theta\right] dF(\theta) - x\left(\underline{\theta}\right) \\ &= \frac{1}{2}\underline{\theta}^2 - x\left(\underline{\theta}\right). \end{aligned} \tag{22}$$

It is easily verified that all the ignored constraints are satisfied in the above solution.

Now we consider the case when entry subsidies are not feasible, i.e., $s = 0$. In this situation the social planner can choose $x\left(\underline{\theta}\right) = \underline{\theta}^2/2$ so that (22) holds (which means that the social planner returns all the expected profit to the consumers who enter the sale). So regardless of whether the social planner can use entry subsidies or not, the first best is achieved in the socially optimum solution. For this reason we can, without loss of generality, simply assume that the socially optimum solution is achieved by setting $x\left(\underline{\theta}\right) = \underline{\theta}^2/2$ and $s = 0$.

We are now ready to compare the entry levels induced by the monopolist and the social planner. Let c^* and c^{**} be entry cutoffs under monopolistic and socially optimum solutions, respectively, and $x^*(\theta)$ and $x^{**}(\theta)$ be the rent provisions under monopolistic and socially optimum solutions, respectively. First we consider the case when condition (9) is satisfied. By (21), the first best is achieved by the monopolist without charging entry fees. In this case, $x^*(\underline{\theta}) = \underline{\theta}^2/2 - G/G' < \underline{\theta}^2/2 = x^{**}(\underline{\theta})$, which implies that $x^*(\theta) < x^{**}(\theta)$. Thus $c^* = \int_{\underline{\theta}}^{\overline{\theta}} x^*(\theta) dF(\theta) < \int_{\underline{\theta}}^{\overline{\theta}} x^{**}(\theta) dF(\theta) = c^{**}$.

Next we consider the case when condition (9) is violated. By (21) again, the monopolistic solution involves downward quality distortion when the monopolist cannot charge entry fees, in which case it is obvious that $c^* < c^{**}$. When the monopolist can charge entry fees, the first best is achieved, in which case $x^*(\theta) = s^* + x^*(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} q^{fb}(\tau) d\tau < \int_{\underline{\theta}}^{\theta} q^{fb}(\tau) d\tau < x^{**}(\theta)$, as $s^* + x^*(\underline{\theta}) < 0$. Thus again, $c^* = \int_{\underline{\theta}}^{\overline{\theta}} x^*(\theta) dF(\theta) < \int_{\underline{\theta}}^{\overline{\theta}} x^{**}(\theta) dF(\theta) = c^{**}$.

In summary, the monopoly induces insufficient entry compared to socially efficient entry in all the cases.

Appendix B: Second-Order Conditions for Optimality

We will show that the strict monotonicity of G'/G in Assumption 1 is sufficient to guarantee that the second-order condition for optimality is satisfied. There are three cases: (1) $x(\theta^*) = 0$ and $\theta^* > \underline{\theta}$, (2) $x(\theta^*) = 0$ and $\theta^* = \underline{\theta}$, and (3) $x(\theta^*) > 0$ and $\theta^* = \underline{\theta}$. We start with the first case, in which the firm's problem is given as follows:

$$\begin{array}{ll} \max_{\theta^*, x(\cdot)} & G\left(\int_{\theta^*}^{\overline{\theta}} x(\theta) dF(\theta)\right) \cdot \int_{\theta^*}^{\overline{\theta}} H(\theta, x(\theta), x'(\theta)) dF(\theta) \\ \text{s.t.} & x(\theta^*) = 0, x''(\theta) \geq 0, \text{and } x(\overline{\theta}) \text{ is free} \end{array}$$

Suppose that the optimal solution to the maximization problem above exists and is given by $x^*(\theta)$. Let $x(\theta)$ denote some arbitrary admissible function. Define $h(\theta) = x(\theta) - x^*(\theta)$. Given that both $x(\theta)$ and $x^*(\theta)$ are admissible, we have $h(\theta^*) = 0$. Furthermore, we know that the function $y(\theta) = x^*(\theta) + ah(\theta)$, for some constant a, is also admissible, and $y(\theta^*) = 0$.

The objective function evaluated at $y(\theta)$ is then given by:

$$\begin{aligned} k(a) &= G\left(\int_{\theta^*}^{\overline{\theta}} y(\theta) f(\theta) d\theta\right) \cdot \int_{\theta^*}^{\overline{\theta}} H(\theta, y(\theta), y'(\theta)) d\theta \\ &= G\left(\int_{\theta^*}^{\overline{\theta}} [x^*(\theta) + ah(\theta)] f(\theta) d\theta\right) \cdot \int_{\theta^*}^{\overline{\theta}} H(\theta, x^*(\theta) + ah(\theta), x'^*(\theta) + ah'(\theta)) d\theta \end{aligned} \tag{23}$$

Taking the first and second order derivatives of $k(a)$ with respect to a, and evaluating at $a = 0$, we can obtain:

$$\begin{aligned} k'(0) &= G'\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta) f(\theta) d\theta\right) \cdot \int_{\theta^*}^{\overline{\theta}} h(\theta) f(\theta) d\theta \cdot \int_{\theta^*}^{\overline{\theta}} H(\theta, x^*(\theta), x'^*(\theta)) d\theta \\ &\quad + G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta) f(\theta) d\theta\right) \int_{\theta^*}^{\overline{\theta}} \left\{H_x\left(\theta, x^*(\theta), x'^*(\theta)\right) h(\theta) + H_{x'}\left(\theta, x^*(\theta), x'^*(\theta)\right) h'(\theta)\right\} d\theta \end{aligned}$$

and

$$\begin{aligned} k''(0) &= G'' \cdot \left(\int_{\theta^*}^{\overline{\theta}} h(\theta) f(\theta) d\theta\right)^2 \cdot \int_{\theta^*}^{\overline{\theta}} H(\theta, x^*(\theta), x'^*(\theta)) d\theta \\ &\quad + 2G' \cdot \int_{\theta^*}^{\overline{\theta}} h(\theta) f(\theta) d\theta \cdot \int_{\theta^*}^{\overline{\theta}} \left\{H_x\left(\theta, x^*(\theta), x'^*(\theta)\right) h(\theta) + H_{x'}\left(\theta, x^*(\theta), x'^*(\theta)\right) h'(\theta)\right\} d\theta \\ &\quad + G \cdot \int_{\theta^*}^{\overline{\theta}} \left\{ \begin{array}{c} H_{xx}\left(\theta, x^*(\theta), x'^*(\theta)\right) h^2(\theta) + H_{xx'}\left(\theta, x^*(\theta), x'^*(\theta)\right) h(\theta) h'(\theta) \\ + H_{x'x'}\left(\theta, x^*(\theta), x'^*(\theta)\right) h'(\theta) h'(\theta) \end{array} \right\} d\theta, \end{aligned} \tag{24}$$

where $G = G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta) f(\theta) d\theta\right)$.

From $k'(0) = 0$, we have

$$\begin{aligned} & \int_{\theta^*}^{\overline{\theta}} \left\{ H_x\left(\theta, x^*(\theta), x'^*(\theta)\right) h(\theta) + H_{x'}\left(\theta, x^*(\theta), x'^*(\theta)\right) h'(\theta) \right\} d\theta \\ = \quad & -\frac{G'}{G} \cdot \int_{\theta^*}^{\overline{\theta}} h(\theta) f(\theta) d\theta \cdot \int_{\theta^*}^{\overline{\theta}} H(\theta, x^*(\theta), x'^*(\theta)) d\theta, \end{aligned} \tag{25}$$

where $G = G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta) f(\theta) d\theta\right)$. We also have

$$H_{xx}\left(\theta, x^*(\theta), x'^*(\theta)\right) \quad = \quad H_{xx'}\left(\theta, x^*(\theta), x'^*(\theta)\right) = 0 \tag{26}$$

$$H_{x'x'}\left(\theta, x^*(\theta), x'^*(\theta)\right) \quad = \quad -c''\left(x'^*(\theta)\right) f(\theta) = -f(\theta) \tag{27}$$

Substituting equations (25)-(27) into (24), we have

$$k''(0) = \left(G'' - 2\frac{G'^2}{G}\right) \cdot \left(\int_{\theta^*}^{\overline{\theta}} h(\theta) f(\theta) d\theta\right)^2 \cdot \int_{\theta^*}^{\overline{\theta}} H(\theta, x^*(\theta), x'^*(\theta)) d\theta \; - G \cdot \int_{\theta^*}^{\overline{\theta}} [h'(\theta)]^2 f(\theta) d\theta,$$

where $G = G\left(\int_{\theta^*}^{\overline{\theta}} x^*(\theta) f(\theta) d\theta\right)$.

Thus $k''(0) \le 0$ if $G'' - 2\frac{G'^2}{G} \le 0$, or $G'' \cdot G \le 2G'^2$. Assumption 1 states that G'/G is strictly decreasing, which implies that $G''G \le G'^2 \le 2G'^2$. Thus Assumption 1 is sufficient to guarantee that the second-order condition for optimality holds.

Following exactly the same procedure above, we can demonstrate that for the remaining two cases, a sufficient condition is also given by $G'' \cdot G \le 2G'^2$, which is guaranteed by Assumption 1.

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