Bad News Can Be Good News: Early Dropouts in an English Auction with Multi-dimensional Signals

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Abstract

In an English auction with multi-dimensional signals, a "reversal" may arise: an earlier dropout may be better news to the seller than a later dropout, and the expected revenue can be decreasing over some range of clock prices.

Keywords: English auction; Multi-dimentional signals; Reversal

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1 Introduction

In this note we demonstrate that "bad news" can be "good news" in auctions with multi-dimensional signals. More specifically, we show that in an English auction with multi-dimensional signals, an earlier dropout can *raise*, rather than *reduce* the expected revenue received by the seller.[1] In fact, as the clock price rises before the first dropout, expected revenue can be decreasing over some range of prices. We refer to this phenomenon as a "reversal."

Our paper contributes to the small, but growing literature on auctions with multi-dimensional

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[1] Jackson and Peck (1999) show that good news can be bad news in a different context.

signals.[2] Our paper is most closely related to Goeree and Offerman (2003), who study an auction model with both common-value and private-value signals. Based on a scalar index they are able to characterize equilibria under the logconcavity condition. Our model is similar to theirs, except that logconcavity fails in our setting. Nevertheless we show that similar index can be constructed to characterize the equilibrium in our model. Our example is well behaved, in the sense that all the regularity conditions introduced in Milgrom and Weber (1982), including affiliation and monotonicity, are satisfied. Yet in our example, an early dropout can be good news, not only for the remaining buyers who now face less competition, but also for the seller. As a direct consequence, the expected revenue can be decreasing in the clock price for some range, before the first dropout occurs. A similar reversal arises in timing games analyzed by Levin and Peck (2006), who show that more investment during a given round may be bad news.[3]

2 The Model

An English ascending auction is employed to sell a single, indivisible item. There are three bidders. Each bidder receives a three-dimensional signal vector (w_i, x_i, y_i), $i = 1, 2, 3$. All signals of all bidders are independent, where

$$\begin{aligned} w_i &= \begin{cases} 0 & \text{with probability } \frac{1}{2} \\ 1 & \text{with probability } \frac{1}{2} \end{cases} \\ x_i &\sim U[0,1] \\ y_i &= \begin{cases} 0 & \text{with probability } \frac{1}{2} \\ 1 & \text{with probability } \frac{1}{2} \end{cases} \end{aligned}$$

The first two signals are private-value signals, where w_i indicates whether the bidder is in the low-value category or the high-value category, and x_i specifies the relative position within that category. The third signal provides the interdependence, and can be thought of as a signal about the common

[2] See, e.g., Harstad et al. (1996), Dasgupta and Maskin (2000), Zheng (2000), Jehiel and Moldovanu (2001), Goeree and Offerman (2002, 2003), Asker and Cantillon (2004), Jackson (2005), and Fang and Morris (2006).

[3] Kirchkamp and Moldovanu (2004) demonstrate that bidding functions may shift upwards following a dropout in an English auction with single-dimensional signals, though such "reversal" does not occur for expected revenues.

value. Given the signals, the total value of the object to bidder i is given by the following:

$$u_i(w_i, x_i, y_1, y_2, y_3) = 4w_i + x_i + \sum_{j=1}^{3} y_j.$$

The total private value component, $4w_i + x_i$, has a natural analog in Goeree and Offerman (2003), corresponding to their private cost. Except for the fact that signals are multi-dimensional, all the assumptions of Milgrom and Weber (1982) are satisfied for our example. The value function is continuous and nondecreasing in all the arguments, and affiliation is trivially satisfied due to the independence. Milgrom and Weber show that the symmetric equilibrium is for bidder i to drop out at a price equal to the expected value of the item to her, conditional on being the "marginal winner" at that point. From their analysis, it follows immediately that the event of a dropout causes the remaining bidders to shift their bidding functions downward, because the conditional expected value of the item is lowered. It must therefore be the case that (i) the event of a dropout lowers the expected revenue, and (ii) for a given history, expected revenue increases as the clock price increases with no intervening dropouts.

We show below that we can construct a scalar index, based on which Milgrom and Weber's equilibrium characterization can be applied to our example. However, the regularity properties of this index are not inherited from the underlying signals, and the above properties about seller revenue do not hold.

3 The Analysis

Our equilibrium analysis for the English auction follows Milgrom and Weber (1982), and more closely, Goeree and Offerman (2003). We first define the following index:

$$s_i = 4w_i + x_i + y_i.$$

s_i can be regarded as the effective "type" for each bidder. In Table 1 below, $B_0(s)$ denotes the dropout price for a bidder with $S_i = s$ when no one has yet dropped, and $B_1(s, [a, b])$ denotes the dropout price for a bidder with $S_i = s$ when the first dropout occurs at a price within the interval, $[a, b]$.

Table 1: English Auction Bidding Strategies

	$s \in [0,1]$	$s \in [1,2]$	$s \in [4,5]$	$s \in [5,6]$
$B_0(s)$	s	$s+2$	s	$s+2$
$B_1(s,[0,1])$	s	$s+1$	s	$s+1$
$B_1(s,[1,3])$	drop	$s+1$	s	$s+1$
$B_1(s,[3,4])$	drop	$s+2$	$s+1$	$s+2$
$B_1(s,[4,5])$	drop	drop	s	$s+1$
$B_1(s,[5,7])$	drop	drop	drop	$s+1$
$B_1(s,[7,8])$	drop	drop	drop	$s+2$

Proposition 1 *The strategies specified in Table 1 characterize a symmetric perfect Bayesian equilibrium in our English auction game. After any history in which a player should have dropped according to Table 1, that player immediately drops.*

Proof. Beliefs are determined by Bayes' rule on the equilibrium path, and if a bidder drops at a price within $[1,3]$ or $[5,7]$, that player is believed to have the low common-value signal, $y=0$. It can be verified that the bids in the table are computed according to the following formula.[4]

$$\begin{aligned} B_0(s) &= s + 2E(Y|S=s) \\ B_1(s;p) &= s + E(Y|S=s) + E(Y|B_0(S)=p) \end{aligned} \tag{1}$$

As is obvious from the table, all bid functions are strictly increasing in s. Therefore, if the other two bidders adopt the equilibrium specified in the table and bidder i wins the auction, the price she will pay is $s_{(1)} + E(Y|S=s_{(1)}) + E(Y|S=s_{(2)})$, where $s_{(1)}$ and $s_{(2)}$ are, respectively, the first and second highest indices among the other two bidders (given the monotonicity of the bid functions, $s_{(1)}$ and $s_{(2)}$ can be inferred from the dropout points). The expected value of the item to bidder i is given by $s_i + E(Y|S=s_{(1)}) + E(Y|S=s_{(2)})$. So the expected payoff for bidder i is nonnegative if and only if $s_i \geqslant s_{(1)}$. Using the strategies as specified in the table, bidder i will win if and only if $s_i > s_{(1)}$.[5] Hence following the strategy specified in the table is also optimal for bidder i. ■

[4] Detailed derivations can be viewed in our online appendix: http://www.econ.ohio-state.edu/lixinye/Reversal/appendix.pdf.

[5] We ignore the null event $s_i = s_{(1)}$.

Note that in our case, the monotonicity of $E(Y|S=s)$, which holds in Goeree and Offerman, is not satisfied for some interval of s. Nevertheless, the strategies as specified in (1) are equilibrium strategies in our case, because monotonicity of the entire strategy holds.

Based on the equilibrium characterized in Proposition 1, we can compute the expected revenue evaluated when the first dropout just occurs at p, denoted as $ER_1(p)$, and the expected revenue as a function of the clock price (while no one has dropped), denoted as $ER(p)$. Our finding is as follows.[6]

Proposition 2 *Both $ER_1(p)$ and $ER(p)$ fail to be monotonic over some range near $p = 4$.*

The schedules of $ER_1(p)$ and $ER(p)$ over the price interval $[3, 5]$ are plotted in the figure below. As illustrated, $ER_1(p)$ (represented by the solid line) has a jump at $p = 4$, while $ER(p)$ (represented by the dotted line) is decreasing for a range of prices below $p = 4$. We thus identify an example in which expected revenue fails to be increasing in both the dropout price and the clock price for some range.

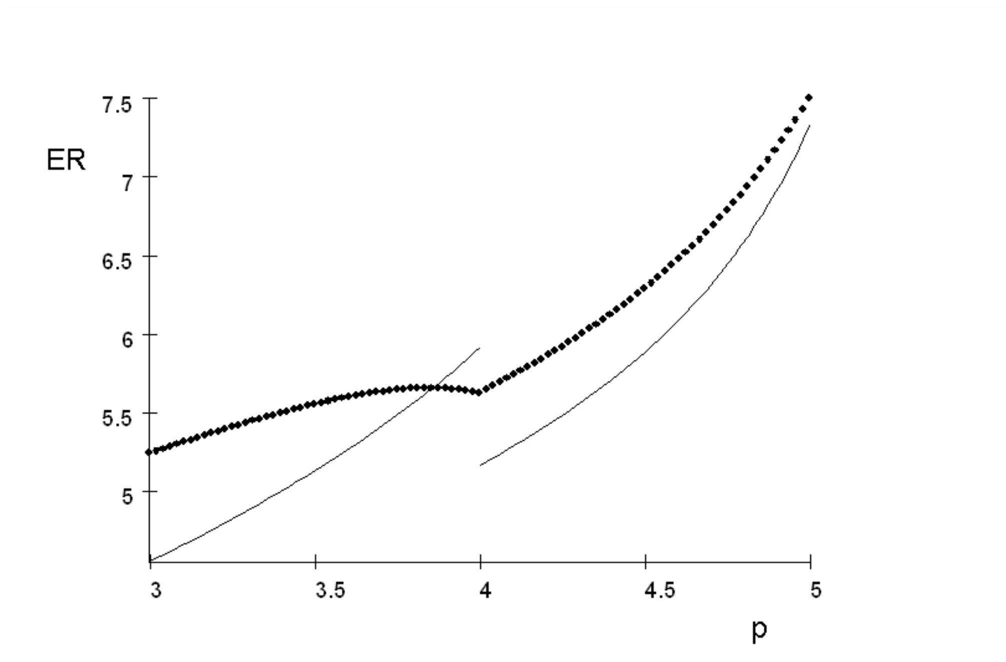

Figure 1: Expected Revenue Functions

[6]The proof can be found in http://www.econ.ohio-state.edu/lixinye/Reversal/appendix.pdf.

The discontinuity of $ER_1(p)$ is driven by the following observation. When bidder i drops out at a price in (3, 4), this indicates $y_i = 1$ (with $w_i = 0$ and $s_i \in [1, 2]$). When bidder i drops out at a *higher* price in (4, 5), this indicates $y_i = 0$ (with $w_i = 1$ and $s_i \in [4, 5]$). Thus, the lower dropout can be better news about the valuations for the other bidders, so bidding by the remaining two bidders is more aggressive following a dropout in $(3, 4)$ than following a dropout in $(4, 5)$, which results in the discontinuity at 4.

Due to the discontinuity of $ER_1(p)$, it is now less surprising to see that the schedule of $ER(p)$ exhibits some non-monotonicity: the expected revenue as a function of the current clock price, conditional on all bidders remaining in the auction, is decreasing for prices in (3.85, 4).

4 Discussion

Given that reversals occur in the English auction, one might conjecture that the English auction does not generate the same expected revenue as the other auctions do. This is, however, not true. It can be verified that for the other standard auctions, the symmetric equilibrium can be characterized based on the same index, $s = w + x + y$, and is strictly increasing. As a result, the bidder with the highest summary statistic always wins, which fixes the same allocation rule. In view of the envelope theorem, revenue equivalence follows.

We now demonstrate that regularity conditions may not aggregate in auctions with multi-dimensional signals. Using the scalar index, we can re-write the value function for bidder 1 as follows.

$$U_1(s_1, s_2, s_3) = \begin{cases} s_1 & \text{for} \quad s_2, s_3 \in [0,1] \cup [4,5] \\ s_1 + 1 & \text{for} \quad s_2 \in [0,1] \cup [4,5] \quad \text{and} \quad s_3 \in [1,2] \cup [5,6] \\ s_1 + 1 & \text{for} \quad s_2 \in [1,2] \cup [5,6] \quad \text{and} \quad s_3 \in [0,1] \cup [4,5] \\ s_1 + 2 & \text{for} \quad s_2, s_3 \in [1,2] \cup [5,6] \end{cases}$$

It can be easily seen that the monotonicity is violated. For example, we have $U_1(s_1, 1.8, 4.2) = s_1 + 1$, while $U_1(s_1, 4.2, 4.2) = s_1$.

Our analysis in the previous section suggests that the monotonicity of $E(Y|S = s)$ is quite crucial. In fact, the discontinuity of $ER_1(p)$ at $p = 4$ is exactly because $E(Y|S = s)$ fails to be increasing over the entire range of s.[7] It is intuitive to see that if $E(Y|S = s)$ is increasing in s, then $ER_1(p)$, and hence $ER(p)$, will also be increasing. Thus the monotonicity of $E(Y|S = s)$ is one sufficient condition to rule out the reversal. Note that reversals cannot arise in Goeree and Offerman's analysis, because their logconcavity condition is sufficient to guarantee the monotonicity of $E(Y|S = s)$. In multi-dimensional applications, one of the dimensions might reflect a binary variable, such as whether a buyer is a local or global player, whether a buyer has a secondary use for the object, and so on. For these natural applications, logconcavity is unlikely to be satisfied.

Since our only departure from the traditional auction model is the introduction of the multi-dimensional signals, we thus conclude that stronger regularity conditions, such as the logconcavity condition identified in Goeree and Offerman, are needed with multiple dimensions in order to restore all the usual properties of auction outcomes.

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