Bad News Can Be Good News: Early Dropouts in an English Auction with Multi-dimensional Signals

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Abstract

In this note we provide an example of an English auction with multi-dimensional signals, which satisfies the regularity conditions in Milgrom and Weber (1982), including affiliation and monotonicity. However, we demonstrate that a "reversal" may arise: an earlier dropout may be better news to the seller than a later dropout, and the expected revenue can be decreasing over some range of clock prices (conditional on no dropout). Our example suggests that regularity conditions may not aggregate in the analysis of auctions with multi-dimensional signals, and stronger conditions are needed to eliminate the possibility of reversals.

1 Introduction

With the appropriate regularity conditions, auctions with single-dimensional signals are well behaved. Milgrom (1981) shows that higher signals are always more favorable news than lower signals if the monotone likelihood ratio property is satisfied. The result is applied to common value second-price auctions to illustrate the winner's curse. If others bid below you, then their low signals are bad news, which must be taken into account. Milgrom and Weber (1982) consider a general model within the single-dimensional signal framework that allows for both common and private values. Under affiliation, monotonicity of the value function, and some other assumptions, this "bad news" feature extends. In English auctions, for example, the act of a bidder dropping out of the auction

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reveals her signal to be at the lower endpoint of an interval rather than anywhere else in the interval, which is bad news.

The purpose of this note is to demonstrate that "bad news" can be "good news" in auctions with multi-dimensional signals.[1] We provide an example of an English auction with multi-dimensional signals, where an earlier dropout can *raise*, rather than *reduce* the expected revenue received by the seller. In fact, as the clock price rises before the first dropout, expected revenue (conditional on current clock price) can be decreasing over some range of prices. We will refer to this phenomenon as a "reversal." Our example satisfies all of the regularity conditions, in particular affiliation and monotonicity, that are sufficient to rule out these abnormalities in single-dimensional auctions.

Despite the vast literature on auctions, there is little work on auctions with multi-dimensional signals. Auctions with multi-dimensional signals are far more difficult to analyze than the single-dimensional case. One problem is that it may be impossible to aggregate the signals into a scalar index or summary statistic from which equilibrium bidding functions can be derived. Jackson (2005) presents an example in which bidders receive a signal about the common value component and a signal about the private value component. Equilibrium exists for the (single-dimensional) polar cases of pure common values and pure private values. However, equilibrium does not exist whenever bidders place positive utility weight on both components. Goeree and Offerman (2003) study a model with common-value signals and private-value signals, in which the signals can be aggregated into a scalar index (the "*surplus*" in their language), based on which they characterize the equilibrium under the first-price sealed-bid, second-price sealed-bid, and English ascending bid auctions. They show that under the logconcavity condition,[2] the equilibrium bidding functions are increasing in the surplus, and revenue equivalence holds among all standard auctions.[3]

[1] Jackson and Peck (1999) describe a phenomenon in which good news can be bad news. The framework is a market game in which the price can overshoot the value, so there is no connection to the current auction model.

[2] That is, the density functions of the private value signal and common value signal are both log concave.

[3] Also see Goeree and Offerman (2002) for an experimental testing of their model. Among other existing work, Jehiel and Moldovanu (2001) show that (ex post) efficiency is unlikely to be achieved in the interdependent valuation setting with multi-dimensional private information, though for the single good case Dasgupta and Maskin (2000) show that (constrained) efficiency is attainable. Zheng (2000) analyzes scoring rules based on which multi-dimensional bids are evaluated. Asker and Cantillon (2004) demonstrate that a supplier's multi-dimensional types can be effectively reduced to a *pseudotype* in the analysis of a procurement auction equipped with a scoring rule, and Fang and Morris (2006) study an auction model with correlated multi-dimensional private signals, the analysis of which cannot be based on

In this note we work with an English auction model with both private value and common value components, which is most closely related to that in Goeree and Offerman (2003), except that the logconcavity condition fails in our setting. Since the logconcavity assumption is not satisfied, the equilibrium established in Goeree and Offerman cannot be directly applied to our case. Nevertheless we show that the same type of equilibrium as in Goeree and Offerman exists, which is increasing in a scalar index aggregated over both private and common value signals. The interdependent value formulation adopted here is also a special case covered in the framework of Milgrom and Weber (1982). Our example is well behaved, in the sense that all of the regularity conditions introduced in Milgrom and Weber are satisfied, including affiliation and monotonicity. Yet in our example, an early dropout can be good news, not only for the remaining buyers who now face less competition, but also for the expected revenue for the seller. As a direct consequence, the expected revenue can be decreasing in the clock price for some range, before the first dropout occurs.[4]

Our conclusion is that when moving from auctions with single-dimensional signals to auctions with multi-dimensional signals, stronger regularity conditions are needed to eliminate the possibility of reversals. Despite the fact that we can characterize equilibrium in terms of an aggregated index, and thus reduce the problem to a single dimension, imposing regularity conditions like affiliation and monotonicity on the original problem does not guarantee that these properties carry over to the aggregated index. For our example, when we aggregate the signals into a single-dimensional index, the monotonicity of value function is violated.

Reversals cannot arise in Goeree and Offerman's setting, because their logconcavity condition is sufficient to rule out any adverse inference about the underlying common value component, based on the scalar index revealed through a dropout. A side product of our example is to highlight the role of logconcavity in the analysis of the Goeree-Offerman type auction model. While logconcavity is quite strong and not necessary to ensure the existence of an equilibrium, it is sufficient to rule out the reversals documented in our example.

Our example is also related to a phenomenon that arises in timing games. Levin and Peck (2006)

single dimensional indices.

[4]Kirchkamp and Moldovanu (2004) have an example with single-dimensional signals and asymmetric valuations, in which the bidding function of one bidder can be decreasing in the dropout price of another bidder. However, in their case, a dropout is bad news for the seller's expected revenue.

study a dynamic investment game in which potential investors observe a signal correlated with the common investment return and a signal representing their private cost of investing. Usually, more investment during a given round is good news about investment returns, but reversals are possible, in which more investment during a given round is bad news. Because firms with the high common-value signal have already invested unless their cost is relatively high, investment is more likely to come from a firm with the low common-value signal after certain histories. In our auction example, the first dropout within a certain price range can only come from a bidder with a favorable common-value signal and a low private value signal, so this is good news for the other bidders and the seller.

2 The Model

An English ascending auction is employed to sell a single, indivisible item. There are three bidders, indexed by $i = 1, 2, 3$. Each bidder receives a three-dimensional signal vector (w_i, x_i, y_i). All signals of all bidders are independent, where

$$\begin{aligned} w_i &= \begin{cases} 0 & \text{with probability } \frac{1}{2} \\ 1 & \text{with probability } \frac{1}{2} \end{cases} \\ x_i &\sim U[0,1] \\ y_i &= \begin{cases} 0 & \text{with probability } \frac{1}{2} \\ 1 & \text{with probability } \frac{1}{2} \end{cases} \end{aligned}$$

The first two signals are private-value signals, where w_i indicates whether the bidder is in the low-value category or the high-value category, and x_i specifies the relative position within that category. The third signal provides the interdependence, and can be thought of as a signal about the common value. Given the signals, the total value of the object to bidder i is given by the following:

$$u_i(w_i, x_i, y_1, y_2, y_3) = 4w_i + x_i + \sum_{j=1}^{3} y_j.$$

Note that the total private value component, $4w_i + x_i$, has a natural analog in Goeree and Offerman (2003), which corresponds to their private cost. Except for the fact that signals are multi-dimensional, all of the assumptions of Milgrom and Weber (1982) are satisfied for our example. The value function is continuous and nondecreasing in all the arguments, and affiliation is trivially satisfied, due to the independence of all signals. For all single-dimensional examples satisfying these assumptions, Milgrom and Weber (1982) show that the symmetric equilibrium bidding strategy is for bidder i to drop out at a price equal to the expected value of the item to her, conditional on being the "marginal winner" at that point (i.e., all remaining bidders having i's signal while those who have previously dropped out having the signals implied by their dropout prices). From their analysis, it follows immediately that the event of a dropout causes the remaining bidders to shift their bidding functions downward, because the conditional expected value of the item is lowered. It must therefore be the case that (i) the event of a dropout lowers the seller's expected revenue, and (ii) for a given history, expected revenues increase as the clock price increases with no intervening dropouts. We show below that we can construct a scalar index for which Milgrom and Weber's (1982) equilibrium characterization applies to our example. However, the regularity properties of this index are not inherited from the underlying signals, and the above properties about seller revenue do not hold.

3 The Analysis

Our equilibrium analysis for the English auction follows Milgrom and Weber (1982), and more closely, Goeree and Offerman (2003). We first define the following index, or summary statistic:

$$s_i = 4w_i + x_i + y_i.$$

As in Goeree and Offerman (2003), s_i is the effective "type" for each bidder. In Table 1 below, $B_0(s)$ denotes the dropout price for a bidder with $S_i = s$ when no one has yet dropped, and $B_1(s, [a, b])$ denotes the dropout price for a bidder with $S_i = s$ when the first dropout occurs at a price within the interval, $[a, b]$.

Table 1: English Auction Equilibrium Strategies

	$s \in [0,1]$	$s \in [1,2]$	$s \in [4,5]$	$s \in [5,6]$
$B_0(s)$	s	$s+2$	s	$s+2$
$B_1(s,[0,1])$	s	$s+1$	s	$s+1$
$B_1(s,[1,3])$	drop	$s+1$	s	$s+1$
$B_1(s,[3,4])$	drop	$s+2$	$s+1$	$s+2$
$B_1(s,[4,5])$	drop	drop	s	$s+1$
$B_1(s,[5,7])$	drop	drop	drop	$s+1$
$B_1(s,[7,8])$	drop	drop	drop	$s+2$

Proposition 1 *The strategies specified in Table 1 characterize a symmetric perfect Bayesian equilibrium in our English auction game. After any history in which a player should have dropped according to Table 1, that player immediately drops.*

Proof. Beliefs are determined by Bayes' rule on the equilibrium path, and if a bidder drops at a price within $[1,3]$ or $[5,7]$, that player is believed to have the low common-value signal, $y = 0$.[5] It can be verified that the bids in the table are computed according to the following formula.[6]

$$
\begin{aligned}
B_0(s) &= s + 2E(Y|S=s) \\
B_1(s;p) &= s + E(Y|S=s) + E(Y|B_0(S)=p)
\end{aligned}
\tag{1}
$$

As is obvious from the table, all bid functions are strictly increasing in s. Therefore, if the other two bidders adopt the equilibrium specified in the table and bidder i wins the auction, the price she will pay is $s_{(1)} + E(Y|S=s_{(1)}) + E(Y|S=s_{(2)})$, where $s_{(1)}$ and $s_{(2)}$ are, respectively, the first and second highest summary statistics among the other two bidders (given the monotonicity of the bid functions, $s_{(1)}$ and $s_{(2)}$ can be inferred from the dropout points). The expected value of the item to bidder i is given by $s_i + E(Y|S=s_{(1)}) + E(Y|S=s_{(2)})$. So the expected payoff for bidder i is nonnegative if and only if $s_i \geqslant s_{(1)}$. Using the strategies as specified in the table, bidder i will win if and only if $s_i > s_{(1)}$.[7] Hence following the strategy specified in the table is also optimal for bidder i when the other two bidders follow the same strategies. ■

[5] We omit the details for unimportant measure-zero cases of a drop on one of the endpoints.

[6] Detailed derivations are relegated to the technical appendix.

[7] We ignore the null event $s_i = s_{(1)}$.

The intuition behind Proposition 1 is standard in the literature. When no one has dropped out, a bidder with type s should remain, up to the price equal to the expected value of the object, conditional on being the "marginal winner," given by $B_0(s)$. When the first dropout occurs at a price p, $s_{(2)}$ can be inferred from p, and the expected value of the object to a "marginal winner" is now given by $B_1(s;p)$.[8]

Based on the equilibrium characterized in Proposition 1, we can compute the expected revenue evaluated when the first dropout just occurs at p, denoted as $ER_1(p)$, and the expected revenue as a function of the clock price (while no one has dropped), denoted as $ER(p)$. Our finding is as follows.

Proposition 2 *Both $ER_1(p)$ and $ER(p)$ fail to be monotonic over some range near $p = 4$.*

Proof. See Appendix. ■

The schedules of $ER_1(p)$ and $ER(p)$ over the price interval $[3, 5]$ are plotted in the figure below. As illustrated, $ER_1(p)$ (represented by the solid line) has a jump at $p = 4$,[9] while $ER(p)$ (represented by the dotted line) is decreasing for a range of prices below $p = 4$. We thus identify an example in which expected revenue fails to be increasing in both the dropout price and the clock price for some range.

[8] Note that in our case, the monotonicity of $E(Y|S = s)$, which holds in Goeree and Offerman, is not satisfied for some interval of s. Nevertheless, the strategies as specified in (1) are equilibrium strategies in our case, because monotonicity of the entire strategy holds.

[9] The size of the jump, at $p = 4$, is .75.

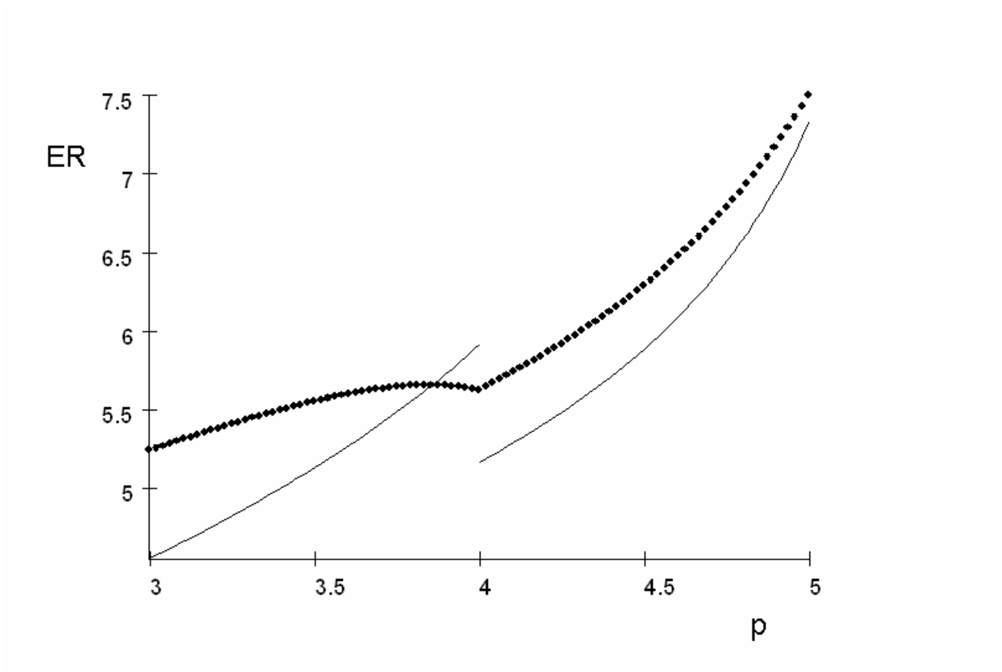

Figure 1: Expected Revenue Functions

The discontinuity of the schedule of $ER_1(p)$ is driven by the following observation. When bidder i drops out at a price between 3 and 4, this indicates $y_i = 1$ (with $w_i = 0$ and $s_i \in [1,2]$). When bidder i drops out at a *higher* price between 4 and 5, this indicates $y_i = 0$ (with $w_i = 1$ and $s_i \in [4,5]$). Thus, the lower dropout can be better news about the valuations (of the object) for the other bidders, so bidding by the remaining two bidders is more aggressive following a dropout between 3 and 4 than following a dropout between 4 and 5. The discontinuity at 4 arises because a dropout at $4 - \varepsilon$ indicates a high y_i and a dropout at $4 + \varepsilon$ indicates a low y_i, while the probability of a second dropout between $4 - \varepsilon$ and $4 + \varepsilon$ is essentially zero for small ε.

In standard English auction models where a dropout is bad news about the values for the other bidders, the bidders could benefit due to the reduced competition. However, for our example, the *seller* could benefit from a dropout, in spite of the reduced competition.

Due to the discontinuity of the schedule of $ER_1(p)$, it is now less surprising to see that the schedule of $ER(p)$ exhibits some non-monotonicity: the expected revenue as a function of the current clock price, conditional on all three bidders remaining in the auction, is decreasing for prices between about 3.85 and 4.

4 Discussion

4.1 Revenue Equivalence

One might consider some other auction formats, such as the first-price sealed-bid and second-price sealed-bid auctions. For example, it can be verified that under a second-price sealed bid auction, the equilibrium bid function is characterized by $B(s) = s + E(Y|S = s) + E(Y|S \leq s)$, which, in our case, is given as follows.[10]

$$B(s) = \begin{cases} s & \text{for} \quad s \in [0,1] \\ 2 + s - \frac{1}{s} & \text{for} \quad s \in [1,2] \\ s + \frac{1}{s-2} & \text{for} \quad s \in [4,5] \\ 2 + s - \frac{2}{s-2} & \text{for} \quad s \in [5,6] \end{cases}$$

Given that reversals occur in the English auction, one might conjecture that the English auction does not generate the same expected revenue as the other auctions do. This is, however, not the case. In all the standard auctions, the symmetric equilibrium is based on the effective type, $s = w + x + y$, and is strictly increasing. As a result, the bidder with the highest summary statistic always wins, which fixes the same allocation rule. Combined with the fact that the bidder with the lowest possible type ($s = 0$) receives the same expected profit, expected payoff to each bidder is the same across all the standard auctions.[11] Payoff equivalence, in turn, implies revenue equivalence, because the expected total surplus is the same across all the standard auctions.

4.2 Aggregation of Regularity Conditions

We now come back to a point mentioned in the introduction, that regularity conditions may not aggregate in auctions with multi-dimensional signals. The particular information structure of our model allows us to aggregate multi-dimensional signals into a single-dimensional scalar index that is sufficient for equilibrium analysis. However, even in this case, the regularity properties of the signals are not inherited by the index. To see this, we can write the value function for bidder 1 as follows.

[10] Again, the equilibrium bid function reflects the "marginal winner" argument. Also note that although neither the monotonicity of $E(Y|S = s)$ nor the monotonicity of $E(Y|S \leq s)$ is guaranteed in our case, the bid function as a whole is strictly increasing in s.

[11] This follows from the envelope theorem, a version of which can be found in Milgrom (2004).

$$U_1(s_1, s_2, s_3) = \begin{cases} s_1 & \text{for} \quad s_2, s_3 \in [0,1] \cup [4,5] \\ s_1 + 1 & \text{for} \quad s_2 \in [0,1] \cup [4,5] \text{ and } s_3 \in [1,2] \cup [5,6] \\ s_1 + 1 & \text{for} \quad s_2 \in [1,2] \cup [5,6] \text{ and } s_3 \in [0,1] \cup [4,5] \\ s_1 + 2 & \text{for} \quad s_2, s_3 \in [1,2] \cup [5,6] \end{cases}$$

Affiliation is trivially satisfied, because the indices are independent. But the value function is not monotonic in s_2 and s_3. For example, we have $U_1(s_1, 1.8, 4.2) = s_1 + 1$, and $U_1(s_1, 4.2, 4.2) = s_1$.

Our analysis in the previous section suggests that the monotonicity of $E(Y|S = s)$ is quite crucial. In fact, the discontinuity of the schedule $ER_1(p)$ at $p = 4$ is exactly because $E(Y|S = s)$ fails to be increasing over the entire range of s.[12] It is intuitive to see that if $E(Y|S = s)$ is increasing in s, then the schedule of $ER_1(p)$, and hence the schedule of $ER(p)$, will also be increasing. In other words, the monotonicity of $E(Y|S = s)$ is one sufficient condition to rule out the reversal phenomenon. Note that reversals cannot arise in Goeree and Offerman's analysis, because their logconcavity condition is sufficient to guarantee the monotonicity of $E(Y|S = s)$. In multi-dimensional applications, one of the dimensions might reflect a binary variable, such as whether a buyer is a local or global player, whether a buyer has a secondary use for the object, and so on. For these natural applications, logconcavity is unlikely to be satisfied.

5 Concluding Remarks

We have constructed an example of an English auction with three dimensional signals, where signals are affiliated and the value function for each bidder, u_i, is nondecreasing in w_i, x_i, and y_j (for $j = 1, 2, 3$). The example is well behaved, because it satisfies all the regularity conditions in Milgrom and Weber (1982). Moreover, we show that in terms of a summary statistic or scalar index, equilibrium exists and exhibits the usual monotonicity. The English auction generates the same expected revenue as in the other standard auctions (revenue equivalence).

Despite all of the regularities mentioned above, we have demonstrated that the expected sale price is not always increasing in the first dropout level, or in the current clock price conditional on no dropouts. In other words, the traditionally perceived "good news" (late dropouts) may become "bad

[12] In particular, we have $E(Y|S = 1.8) = 1$ and $E(Y|S = 4.2) = 0$.

news" (lower revenue) for the seller, and the traditionally perceived "bad news" (early dropouts) may be "good news" (higher revenue) for the seller.

Since our only departure from the traditional auction model is the introduction of the multi-dimensional signals, we thus conclude that stronger regularity conditions, such as the logconcavity condition identified in Goeree and Offerman (2003), are needed with multiple dimensions in order to restore all the usual properties of auction outcomes.

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Appendix

Derivations of the equilibrium strategies specified in Table 1:

If no one drops, the dropping point is computed as follows.

$$B_0(s) = s + 2E(Y|S = s) = \begin{cases} s & \text{for} \quad s \in [0,1] \\ s+2 & \text{for} \quad s \in [1,2] \\ s & \text{for} \quad s \in [4,5] \\ s+2 & \text{for} \quad s \in [5,6] \end{cases}$$

Thus the first-dropout range on equilibrium path is $[0,1], [3,4], [4,5]$, or $[7,8]$.

1. If one drops in $[0,1]$, her underlying common value signal is 0. Hence,

$$B_1(s,[0,1]) = s + E(Y|S = s) = \begin{cases} s & \text{for} \quad s \in [0,1] \\ s+1 & \text{for} \quad s \in [1,2] \\ s & \text{for} \quad s \in [4,5] \\ s+1 & \text{for} \quad s \in [5,6] \end{cases}$$

2. If one drops in $[3,4]$, her underlying common value signal is 1. Hence,

$$B_1(s,[3,4]) = s + E(Y|S = s) + 1 = \begin{cases} s+2 & \text{for} \quad s \in [1,2] \\ s+1 & \text{for} \quad s \in [4,5] \\ s+2 & \text{for} \quad s \in [5,6] \end{cases}$$

3. If one drops in $[4,5]$, her underlying common value signal is 0. Hence,

$$B_1(s,[4,5]) = s + E(Y|S = s) = \begin{cases} s & \text{for} \quad s \in [4,5] \\ s+1 & \text{for} \quad s \in [5,6] \end{cases}$$

4. If one drops in $[7,8]$, her underlying common value signal is 1. Hence,

$$B_1(s,[7,8]) = s + E(Y|S = s) + 1 = \quad s+2 \quad \text{for} \quad s \in [5,6]$$

Given the off-equilibrium beliefs, $B_1(s,[1,3])$ and $B_1(s,[5,7])$ can be analogously computed, which is skipped here.

Proof of Proposition 2:

For ease of notation, we define four types of bidders in terms of the signals w and y they possess:

Type a: $w = 0, y = 0$; Type A: $w = 1, y = 0$

Type b: $w = 0, y = 1$; Type B: $w = 1, y = 1$

We first compute the expected revenue conditional on the first dropouts.

First dropout in $[3, 4)$**:** In this case, the remaining two bidders are either type b, A, or B. We report all the possible cases, the corresponding probabilities, and the expected second dropouts (the expected revenues) in the following table:

Case	Probability	Expected Revenue
bb	$\pi_1 = \pi_b^2$	$ER_1 = \frac{1}{3}(4 + 2p)$
bA	$\pi_2 = 2\pi_b\pi_A$	$ER_2 = \frac{1}{2}(4 + p)$
bB	$\pi_3 = 2\pi_b\pi_B$	$ER_3 = \frac{1}{2}(4 + p)$
AA	$\pi_4 = \pi_A^2$	$ER_4 = 5\frac{1}{3}$
AB	$\pi_5 = 2\pi_A\pi_B$	$ER_5 = 5\frac{1}{2}$
BB	$\pi_6 = \pi_B^2$	$ER_6 = 7\frac{1}{3}$

where $\pi_b = \frac{1}{2}\frac{4-p}{2}$, $\pi_A = \pi_B = \frac{1}{4}$.

Let $\pi_T = \sum_{i=1}^{6} \pi_i$. The expected revenue conditional on the first-dropout price is computed as follows.

$$ER_1(p) = \sum_{i=1}^{6} \frac{\pi_i}{\pi_T} ER_i = \frac{1}{3} \frac{123 - 54(p-3) + 2(p-3)^3}{9 - 6(p-3) + (p-3)^2} \text{ for } p \in [3, 4) \quad (2)$$

First dropout in $(4, 5]$**:** In this case, the remaining bidders are either type A or B. We report all the possible cases, the corresponding probabilities, and the expected second dropouts (the expected revenues) in the following table:

Case	Probability	Expected Revenue
AA	$\pi_1 = \pi_A^2$	$ER_1 = \frac{1}{3}(5 + 2p)$
AB	$\pi_2 = 2\pi_A\pi_B$	$ER_2 = \frac{1}{2}(5 + p)$
BB	$\pi_3 = \pi_B^2$	$ER_3 = 7\frac{1}{3}$

where $\pi_A = \frac{1}{2}\frac{5-p}{2}$, $\pi_B = \frac{1}{4}$.

Let $\pi_T = \sum_{i=1}^{3} \pi_i$. The expected revenue conditional on the first dropout price is computed as follows.

$$ER_1(p) = \sum_{i=1}^{3} \frac{\pi_i}{\pi_T} ER_i = \frac{2}{3} \frac{31 - 24(p-4) + 3(p-4)^2 + (p-4)^3}{4 - 4(p-4) + (p-4)^2} \text{ for } p \in (4,5] \tag{3}$$

Now we compute the expected revenue as a function of the clock prices.

Clock price in $[3,4]$**:** If no one drops until the clock price reaches $p \in [3,4]$, there are 10 possible events, which are reported as follows.

Case	Probability	Expected Revenue
bbb	$\pi_1 = \pi_b^3$	$ER_1 = \frac{1}{2}(4+p)$
bbA	$\pi_2 = 3\pi_b^2 \pi_A$	$ER_2 = \frac{2}{3}(4+2p)$
bbB	$\pi_3 = 3\pi_b^2 \pi_B$	$ER_3 = \frac{2}{3}(4+2p)$
bAA	$\pi_4 = 3\pi_b \pi_A^2$	$ER_4 = 5\frac{1}{3}$
bAB	$\pi_5 = 6\pi_b \pi_A \pi_B$	$ER_5 = 5\frac{1}{2}$
bBB	$\pi_6 = 3\pi_b \pi_B^2$	$ER_6 = 7\frac{1}{3}$
AAA	$\pi_7 = \pi_A^3$	$ER_7 = 4\frac{1}{2}$
AAB	$\pi_8 = 3\pi_A^2 \pi_B$	$ER_8 = 4\frac{2}{3}$
ABB	$\pi_9 = 3\pi_A \pi_B^2$	$ER_9 = 6\frac{1}{3}$
BBB	$\pi_{10} = \pi_B^3$	$ER_{10} = 7\frac{1}{2}$

where $\pi_b = \frac{1}{2}\frac{4-p}{2}, \pi_A = \frac{1}{4}, \pi_B = \frac{1}{4}$.

Let $\pi_T = \sum_{i=1}^{10} \pi_i$. The expected revenue conditional on clock price is computed as follows.

$$ER(p) = \sum_{i=1}^{10} \frac{\pi_i}{\pi_T} ER_i = \frac{1}{2} \frac{-1021 + 246p - 54(p-3)^2 + (p-3)^4}{-108 + 27p - 9(p-3)^2 + (p-3)^3} \text{ for } p \in [3,4] \tag{4}$$

Clock price in $[4,5]$**:** If no one drops until the clock price reaches $p \in [4,5]$, there are 4 possible events, which is reported as follows.

Case	Probability	Expected Revenue
AAA	$\pi_1 = \pi_A^3$	$ER_1 = \frac{1}{2}(5+p)$
AAB	$\pi_2 = 3\pi_A^2 \pi_B$	$ER_2 = \frac{2}{3}(5+2p)$
ABB	$\pi_3 = 3\pi_A \pi_B^2$	$ER_3 = 6\frac{1}{3}$
BBB	$\pi_4 = \pi_B^3$	$ER_4 = 7\frac{1}{2}$

where $\pi_A = \frac{1}{2}\frac{5-p}{2}$, $\pi_B = \frac{1}{4}$.

Let $\pi_T = \sum_{i=1}^{4} \pi_i$. The expected revenue conditional on clock price is computed as follows.

$$ER(p) = \sum_{i=1}^{4} \frac{\pi_i}{\pi_T} ER_i = \frac{1}{2} \frac{-562 + 118p - 48(p-4)^2 + 4(p-4)^3 + (p-4)^4}{-56 + 12p - 6(p-4)^2 + (p-4)^3} \text{ for } p \in [4, 5] \qquad (5)$$

(2)-(5) give rise to the the schedules of $ER_1(p)$ and $ER(p)$ plotted in Figure 1. Note that $ER_1(p)$ has a jump at $p = 4$, and $ER(p)$ is decreasing over the interval $[3.85, 4]$.